Beating Classical and Quantum Limits in Optics

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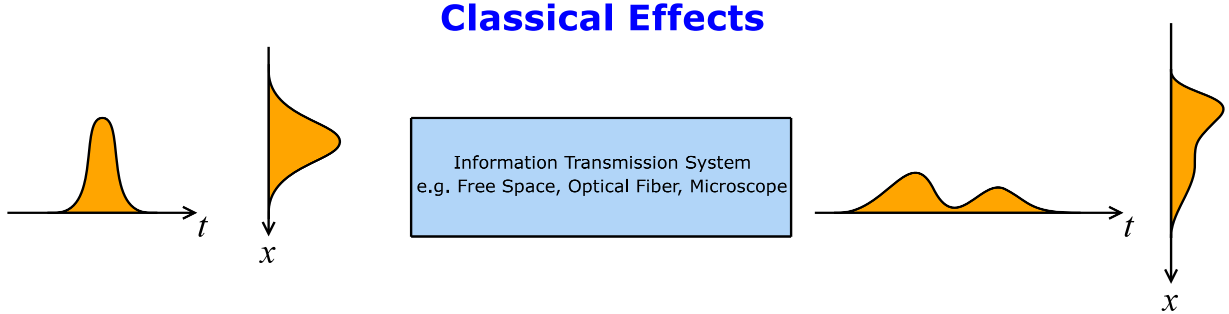
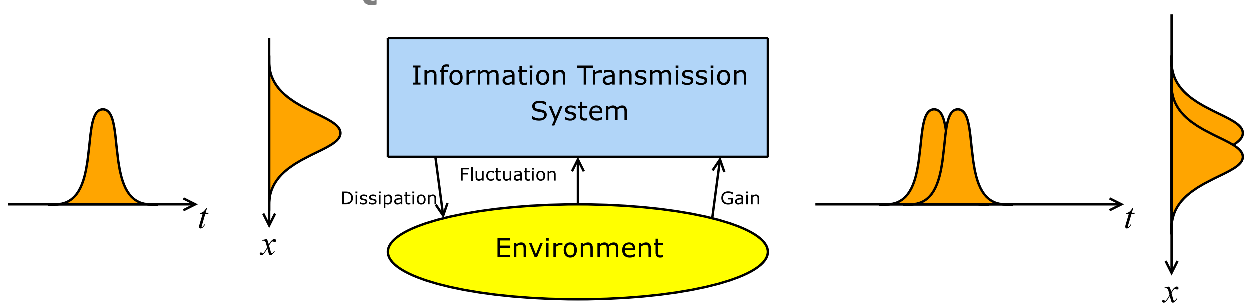
Classical and Quantum Distortions

Classical Effects



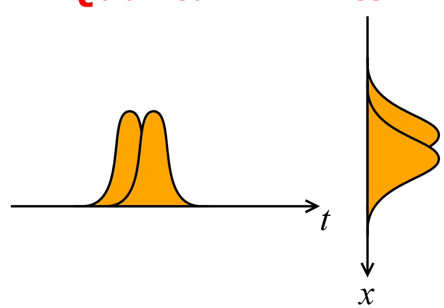
Dispersion, Diffraction, Rayleigh Criterion, Loss, Nonlinearity, ...

Quantum Decoherence



Langevin Noise, Amplified Spontaneous Emission, Gordon-Haus Timing Jitter, ...

Quantum Limits

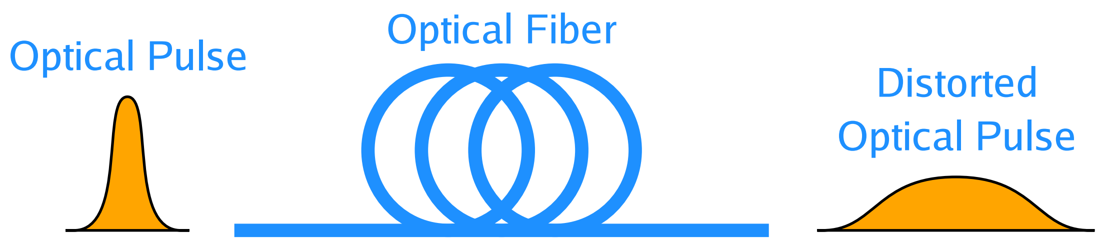


Standard Quantum Limits, Heisenberg Uncertainty Principle

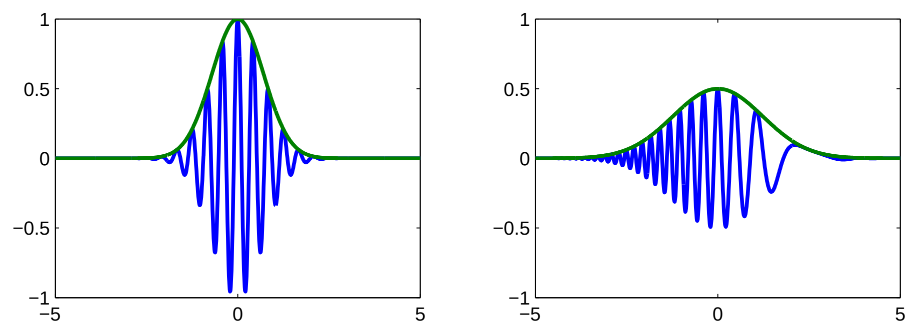
Outline

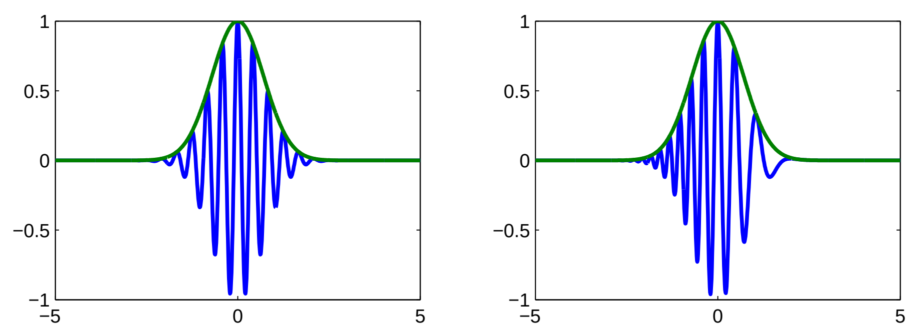
- Dispersion and nonlinearity compensation by Spectral Phase Conjugation
- Beating temporal quantum limits by Quantum Soliton Control
- Beating spatial quantum limits by Self-Focusing
- Nonlinear Optics and Fluid Dynamics
- Beating resolution limits by Dielectric Slabs

Ultrashort Pulse Propagation Effects



Group-Velocity Dispersion

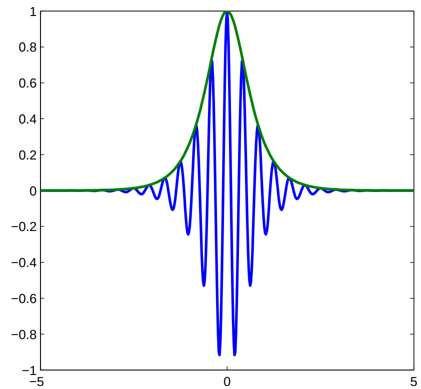


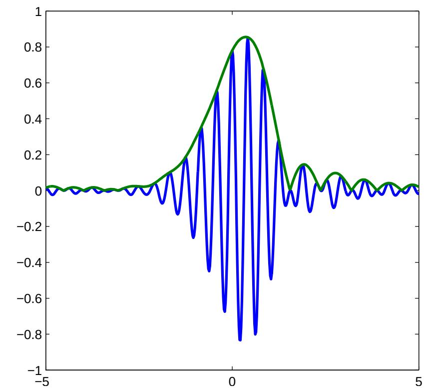


Kerr Nonlinearity ($\Delta n(t) \sim I(t)$)

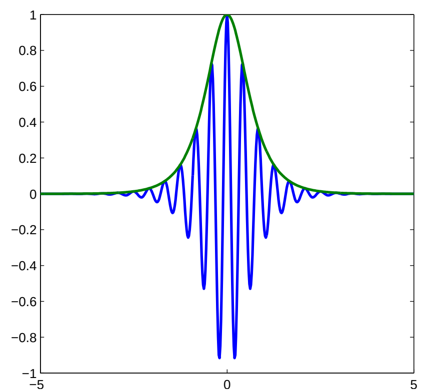
High-Order Effects

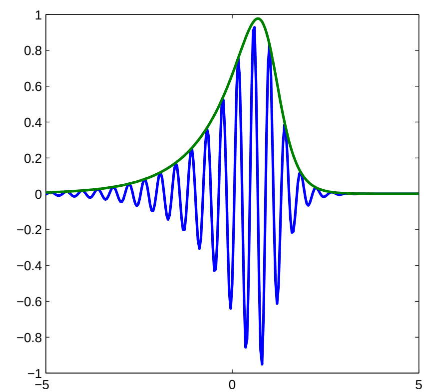
Third-Order Dispersion





Self-Steepening





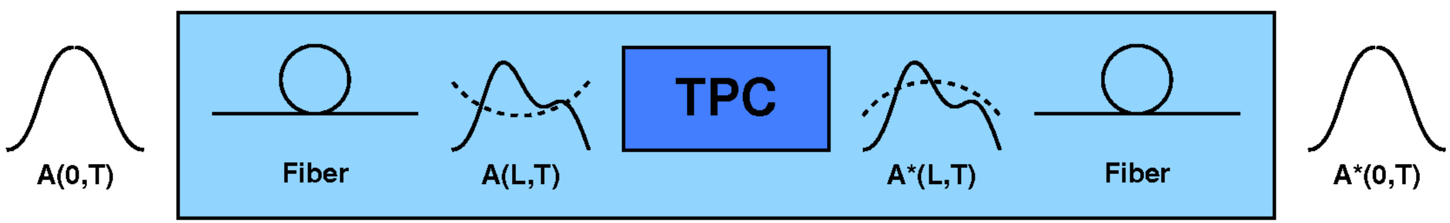
Stimulated Raman Scattering, ...

Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, 2001)

Ultrashort Pulse Propagation

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A - \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3} + i\gamma\left[|A|^2 A + \frac{i}{\omega_0}\frac{\partial}{\partial T}(|A|^2 A) - T_R A\frac{\partial |A|^2}{\partial T}\right] \quad (1)$$

- Perform $a(\omega) \rightarrow a^*(2\omega_0 - \omega)$, or equivalently $A(T) \rightarrow A^*(T)$, to compensate for GVD and Kerr effect and if loss, higher-order dispersion, and self-steepening can be neglected.
- Temporal Phase Conjugation (TPC)



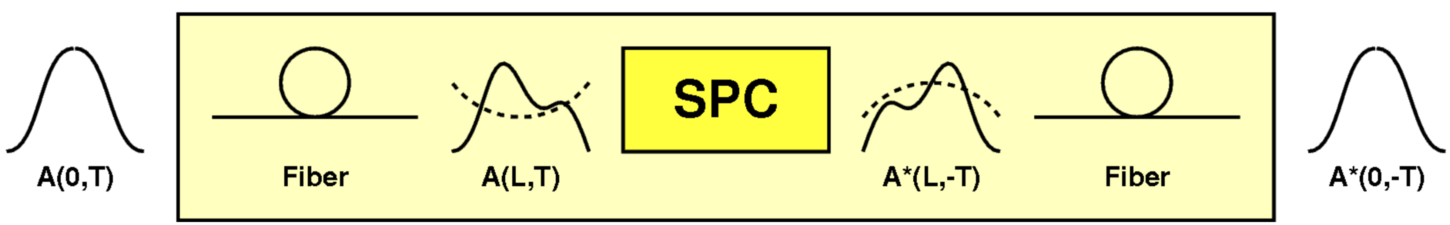
Yariv, Fekete, and Pepper, Optics Letters, **4**, 52 (1979),

Fisher, Suydam, and Yevick, Optics Letters, **8**, 611 (1983).

Spectral Phase Conjugation

- $$\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A - \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + \sum_{n=3}^{\infty}\frac{i\beta_n}{n!}\left(i\frac{\partial}{\partial T}\right)^n A$$
$$+ i\gamma\left[|A|^2 A + \frac{i}{\omega_0}\frac{\partial}{\partial T}(|A|^2 A) - T_R A\frac{\partial |A|^2}{\partial T}\right] \quad (2)$$

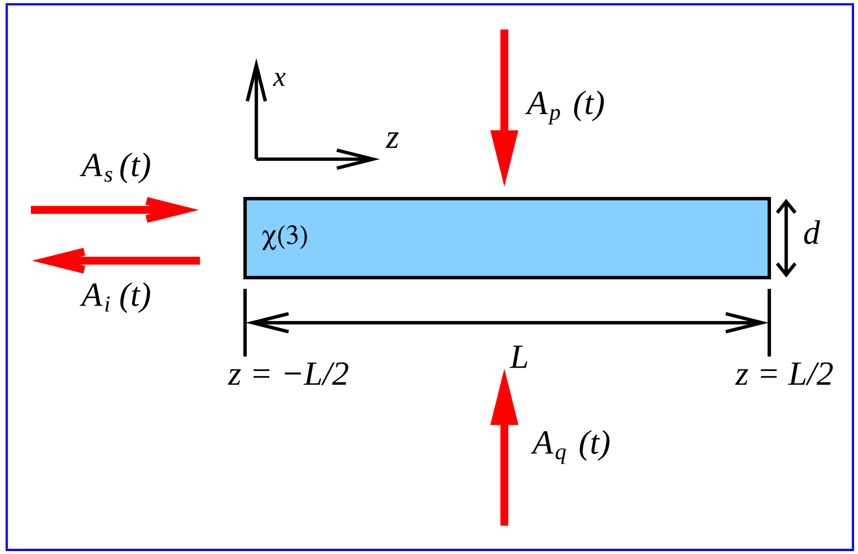
- Perform $a(\omega) \to a^*(\omega)$, or equivalently $A(T) \to A^*(T_0 - T)$, to compensate for dispersion of all orders, Kerr effect, and self-steepening if loss and stimulated Raman scattering can be neglected.
- Spectral Phase Conjugation (SPC)



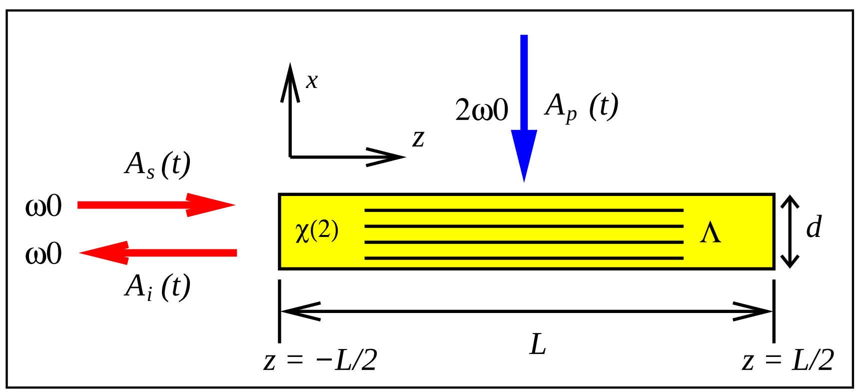
Tsang and Psaltis, Optics Letters **28**, 1558 (2003)

Methods of Performing SPC

- Tsang and Psaltis, Optics Express, **12**, 2207 (2004)



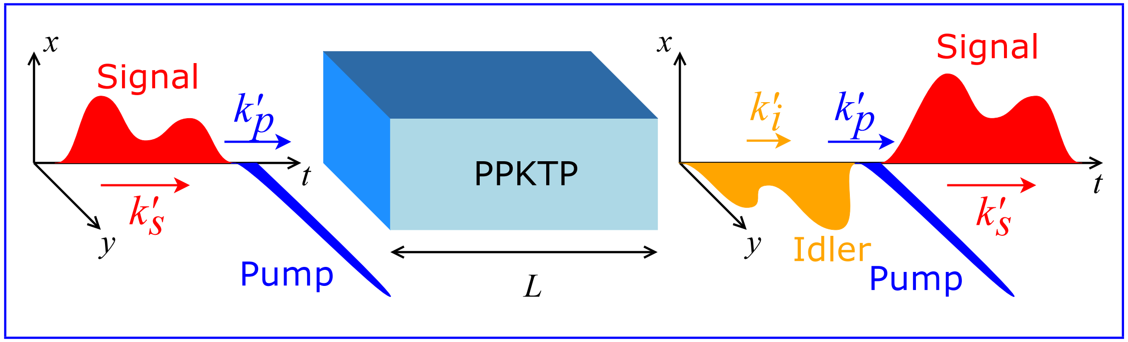
- Tsang and Psaltis, Optics Communications, **242**, 659 (2004)



- Marom et al., Optics Letters, **25**, 132 (2000)

SPC via Extended Phase Matching

- Tsang, JOSA B **23**, 861 (2006)



- Extended Phase Matching: Quasi-Phase Matching + Group Velocity Matching
- Quasi-Phase Matching satisfied by periodic poling of nonlinear crystals
- Group Velocity Matching satisfied by material dispersion, such as KTP at 1584 nm.

Coupled-Mode Equations

- z derivatives can be neglected if the pump pulse is short enough:

$$\frac{\partial A_s}{\partial z} + k_s' \frac{\partial A_s}{\partial t} = j\chi A_p(t - k_p' z) A_i^* \quad (3)$$

$$\frac{\partial A_i^*}{\partial z} + k_i' \frac{\partial A_i^*}{\partial t} = -j\chi A_p(t - k_p' z) A_s \quad (4)$$

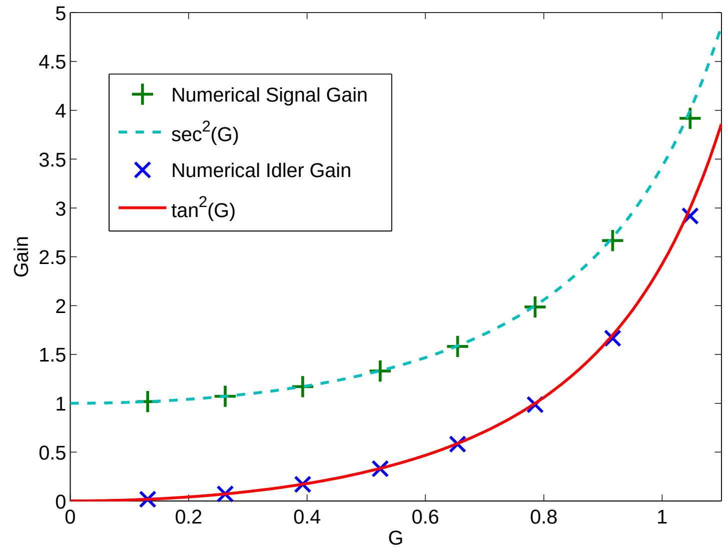
- Approximate solutions:

$$A_s(L,t) = A_s(0,t)\sec(G) + jA_i^*(0,-t)\tan(G) \quad (5)$$

$$A_i(L,t) = A_i(0,t)\sec(G) + jA_s^*(0,-t)\tan(G) \quad (6)$$

$$G = \frac{\chi}{\gamma}\int A_p(\tau)d\tau = \Big(\frac{1}{1 - k_p'/k_s'}\Big)\chi\int A_p(\tau)v\,d\tau \quad (7)$$

Mirrorless Parametric Oscillation

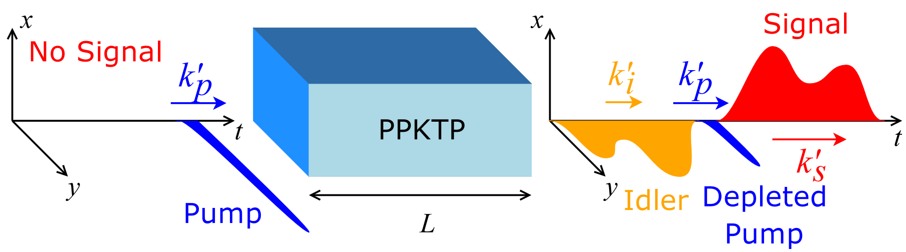


$$A_s(L,t) = A_s(0,t)\sec(G) + jA_i^*(0,-t)\tan(G) \quad (8)$$

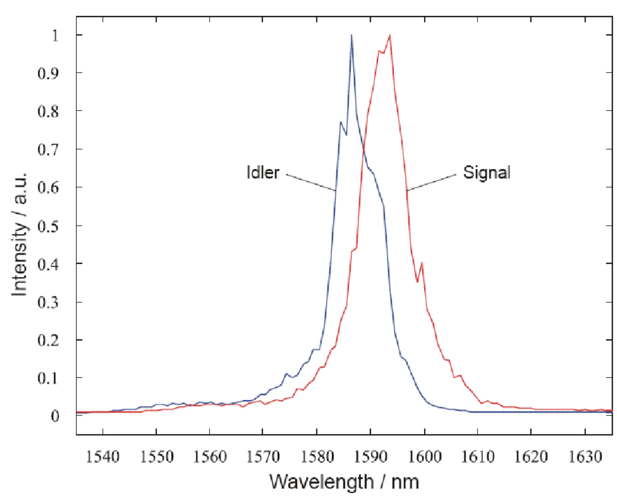
$$A_i(L,t) = A_i(0,t)\sec(G) + jA_s^*(0,-t)\tan(G) \quad (9)$$

- What happens when $G = \pi/2$, and $\sec(G), \tan(G) = \infty$?
- z derivatives can no longer be neglected, gain increases exponentially with respect to z.
- Analogous to mirrorless optical parametric oscillation

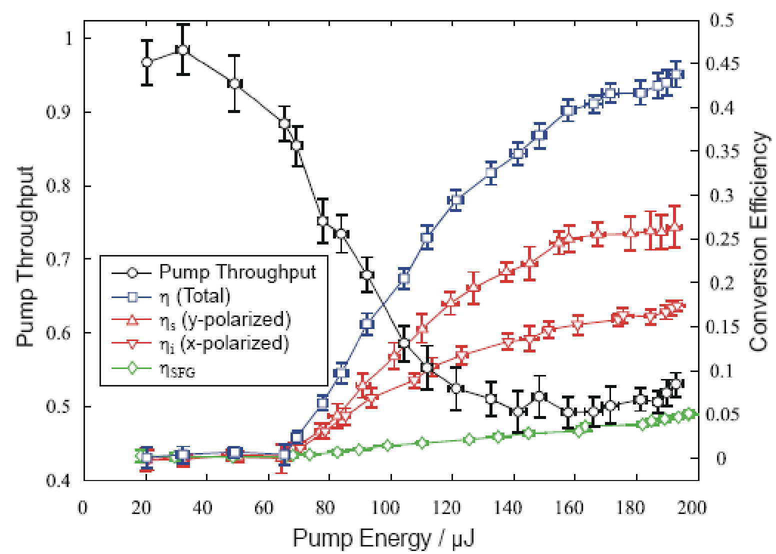
Experimental Demonstration of Mirrorless OPO



- 3cm periodically-poled KTP crystal from Raicol Crystals, dispersed femtosecond pump pulse at 792nm
- 43% down conversion efficiency, 140 dB equivalent gain

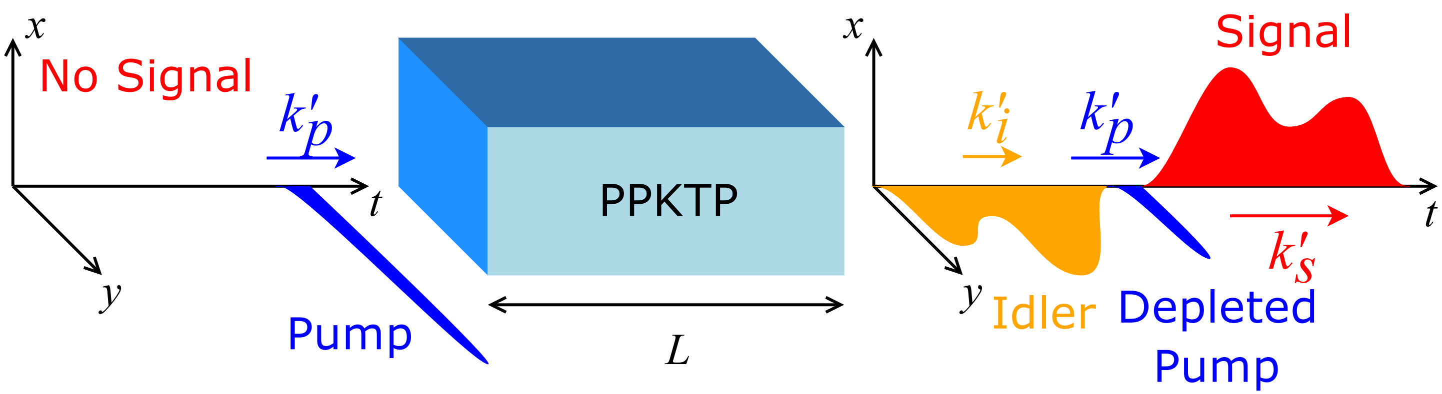


(a)



Pu, Wu, Tsang, and Psaltis, under preparation

Spontaneous Parametric Down Conversion



- Classical theory predicts zero output for zero input:

$$\frac{\partial A_s}{\partial z} + k_s' \frac{\partial A_s}{\partial t} = j\chi A_p(t - k_p' z) A_i^* \tag{10}$$

$$\frac{\partial A_i^*}{\partial z} + k_i' \frac{\partial A_i^*}{\partial t} = -j\chi A_p(t - k_p' z) A_s \tag{11}$$

- Quantum theory predicts entangled photon pair generation even for zero input.

Giovannetti *et al.* (MIT), Physical Review Letters,**88**, 183602 (2002),

Kuzucu *et al.* (MIT), Physical Review Letters, **94**, 083601 (2005)

Comparison with Kuzucu et al.'s experiment

- Number of generated photon pairs per pump pulse is given by $\tan^2(G)$.
  [Tsang, JOSAB **23**, 861 (2006)]
- Using their parameters, $\lambda_0 = 1584$ nm, $\chi^{(2)} = 7.3$ pm/V, $n_0 = 2$, $\gamma = 1.5 \times 10^{-10}$ s/m, $T_p = 100$ fs, average pump power $= 350$ mW, diameter $= 200$ μm, and pump repetition rate $f_r = 80$ MHz, the spontaneously generated photon pairs per second is theoretically given by

$$G = 0.2, \tag{12}$$

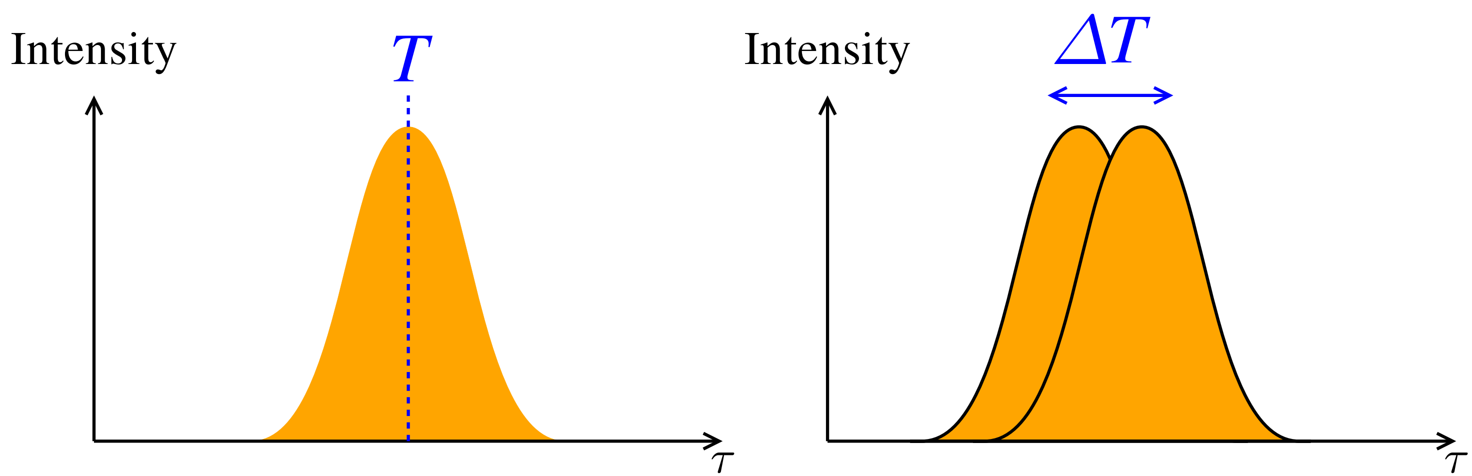
$$f_r \tan^2(G) = 3.6 \times 10^6\text{/s}. \tag{13}$$

Kuzucu *et al.* (MIT), Physical Review Letters, **94**, 083601 (2005):

coupling efficiency into the PM fiber. From the detection efficiencies and our measurement duty cycle we estimate a single spatial fiber-optic mode pair production rate of $\sim 4 \times 10^6$/s at 350 mW of pump power.

What's Special about These Photon Pairs?

- The entangled photons are frequency correlated and time anti-correlated.
- One-way autocompensating cryptography [Walton et al., PRA **67**, 062309 (2003)], Quantum enhancement of timing accuracy [Giovannetti et al., Nature **412**, 417 (2001)].



$$T = \frac{\int_{-\infty}^{\infty} t|A(t)|^2 dt}{\int_{-\infty}^{\infty} |A(t)|^2 dt} \qquad (14)$$

- Time-anti-correlated photons can achieve a lower uncertainty in T for the same bandwidth.
- Analogous to how mutual funds work: selecting negatively-correlated stocks reduces risk.

Multiphoton Enhancement

Giovannetti, Lloyd, and Maccone, Nature **412**, 417 (2001)

- N independent photons:

$$\Delta T \geq \frac{1}{2\sqrt{N}\Delta\omega} \quad \text{(Standard Quantum Limit)} \tag{15}$$

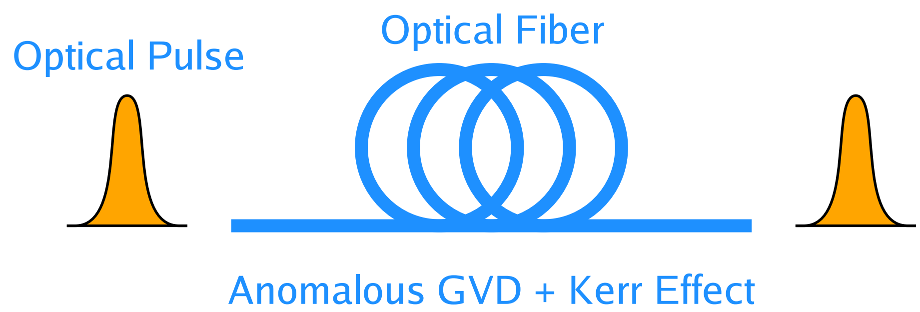
- e.g.: $W = 100$ ps, $N = 10^{10}$, $\Delta T = 1$ fs
- N negatively-time-correlated photons:

$$\Delta T \geq \frac{1}{2N\Delta\omega} \quad \text{(Ultimate Quantum Limit)} \tag{16}$$

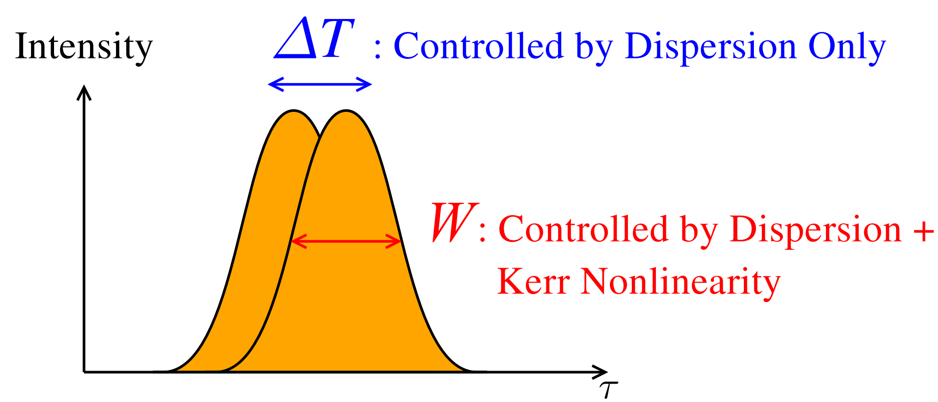
- $N = 2$ is quite useless compared to $N \gg 1$
- How to create multiphoton time anti-correlation with $N \gg 1$?

Quantum Theory of Optical Fiber Soliton

- Classical theory: soliton is a stable solitary wave due to balance between anomalous GVD and Kerr effect



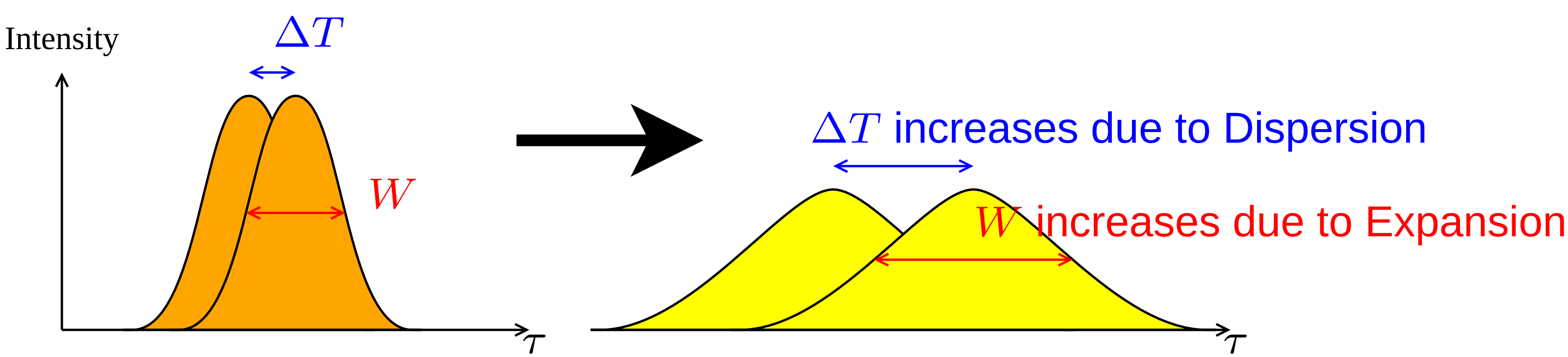
- Quantum theory: Stable pulse shape and bandwidth due to balance between GVD and Kerr effect, but the average position of the pulse is affected by dispersion only.

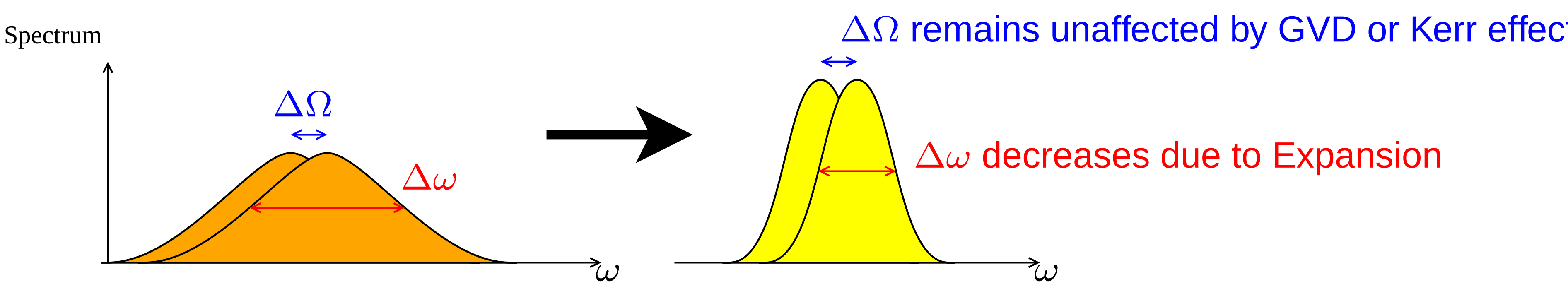


Tsang, Physical Review Letters **97**, 023902 (2006)

Adiabatic Soliton Expansion

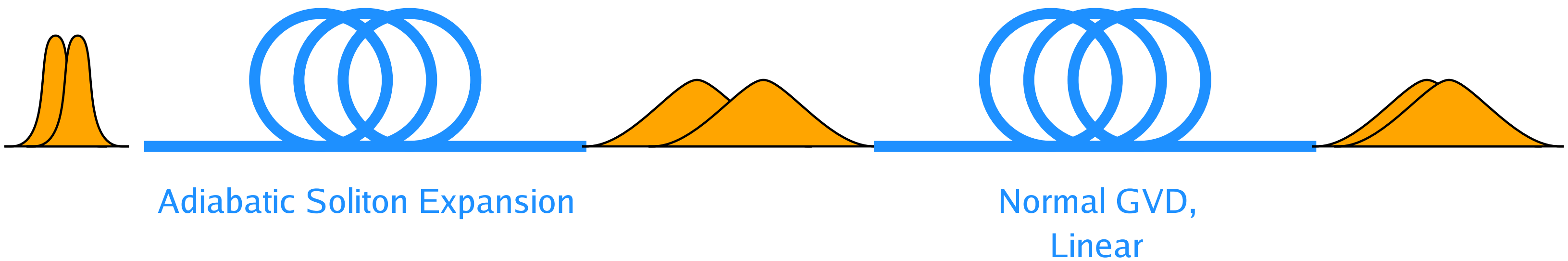
- 1. Adiabatically reduce the Kerr nonlinearity or increase the group-velocity dispersion along the fiber

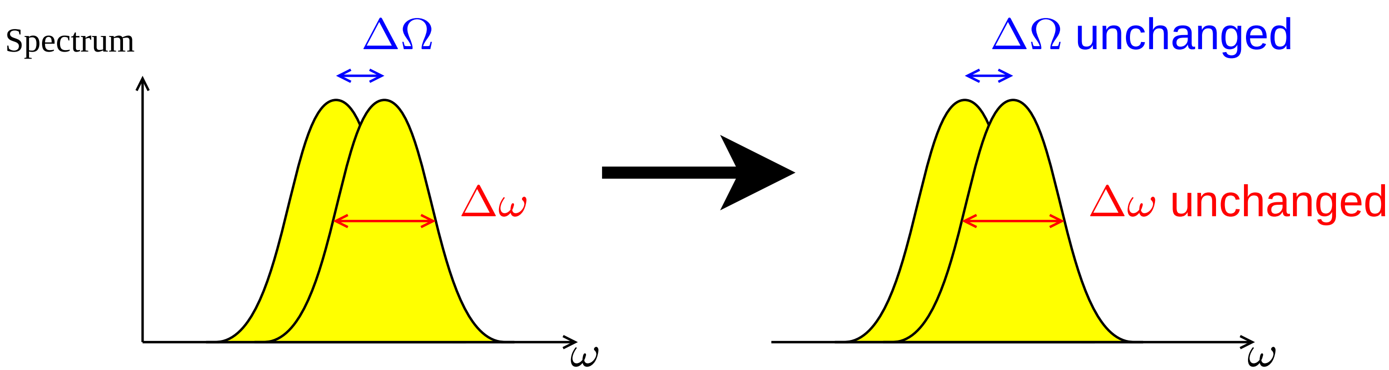




Quantum Dispersion Compensation

- 2. Compensate for dispersion of T in a second fiber with $b'L' = -bL$

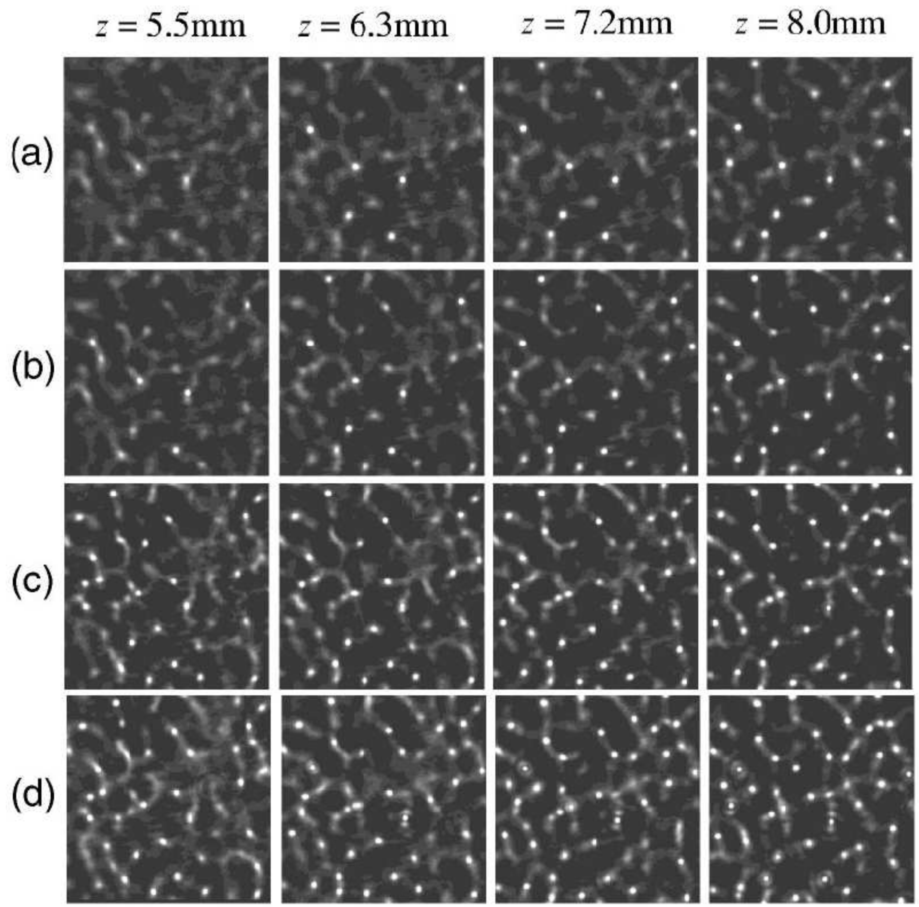




- ΔT is the same as the input, but $\Delta\omega$ is reduced, so $\Delta T < 1/(2\sqrt{N}\Delta\omega)$.
- Subfemtosecond timing jitter detection can be performed by cross-correlation measurements via sum-frequency generation or balanced homodyne measurements with a reference local oscillator pulse.

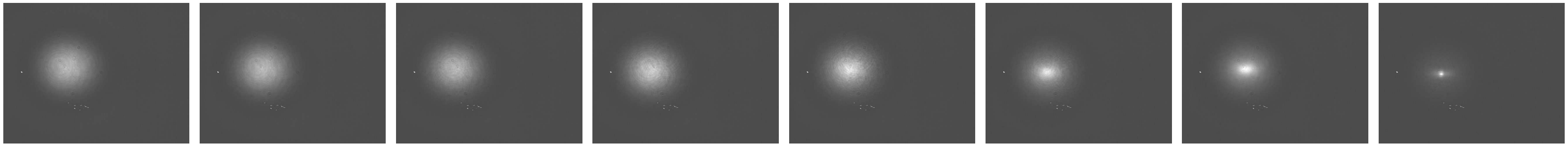
2D Self-Focusing Collapse

Balance between diffraction and Kerr effect is unstable.

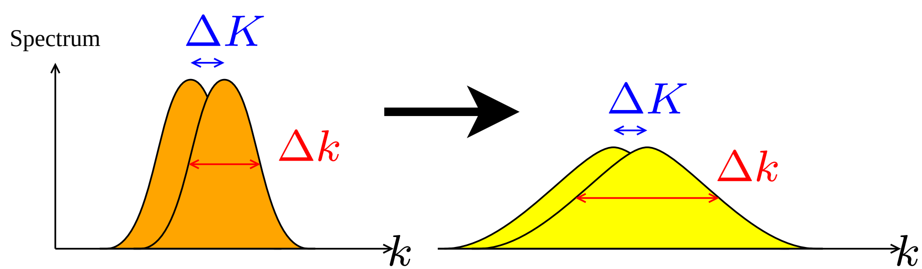


Centurion, Pu, Tsang, and Psaltis, Physical Review A **71**, 063811 (2005)

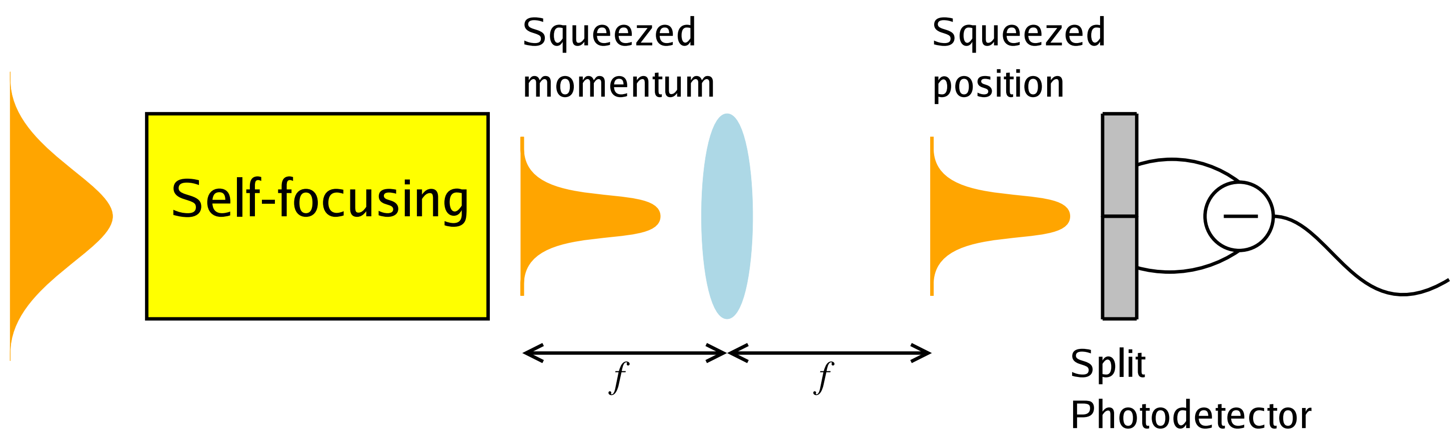
Spatial Quantum Enhancement by Self-Focusing



- increasing pulse energy $\rightarrow$

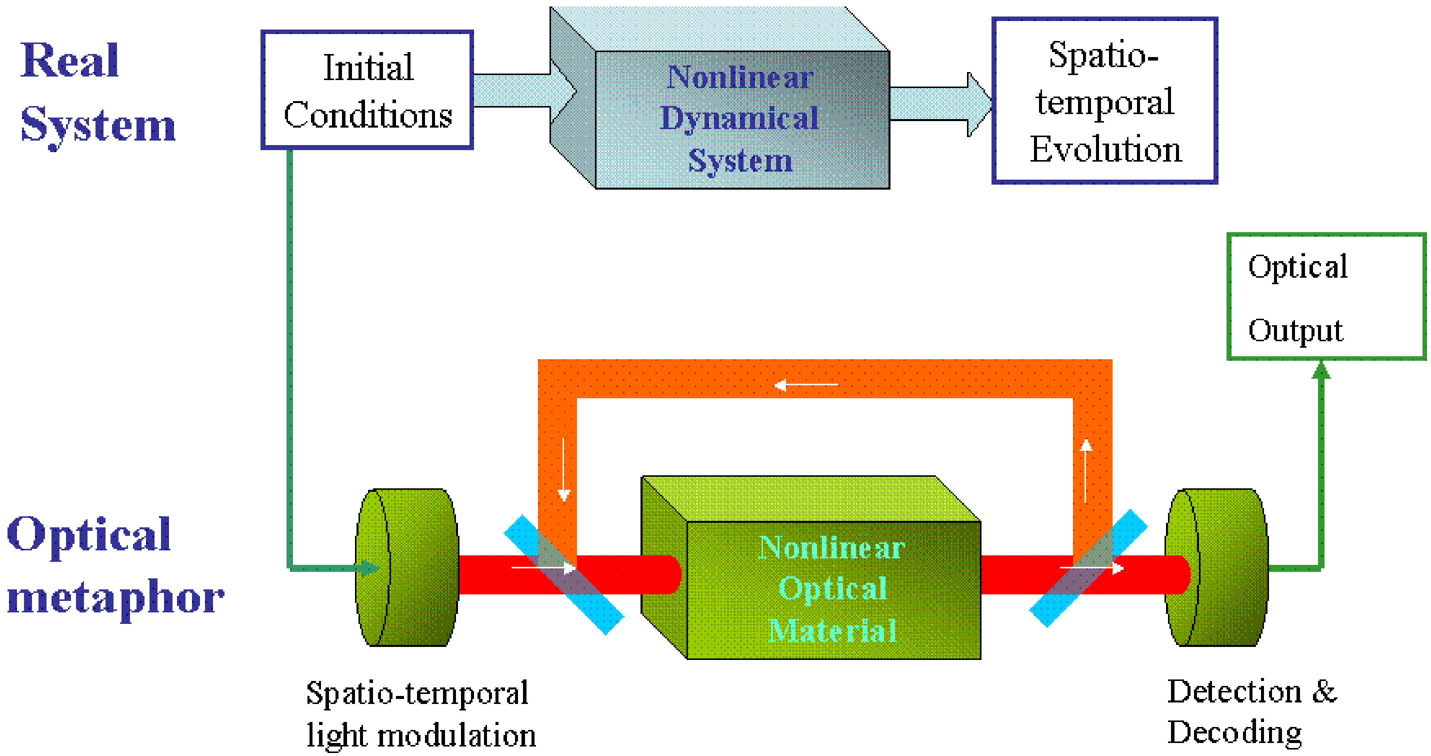


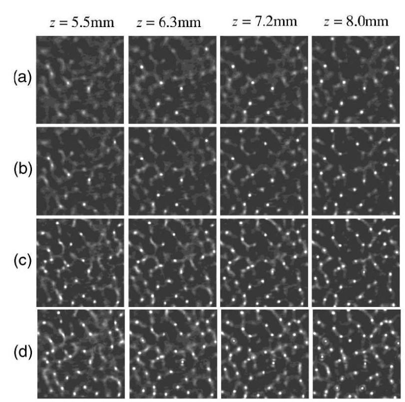
- Use a Fourier-transform lens to transform to real space



- e.g. $W = 3$ mm, $N = 10^{10}$, $\Delta X = 30$ nm

Demetri’s Grand Vision of Optical Computing





Nonlinear Optics and Fluid Dynamics

- (3+1)D Nonlinear Schrödinger equation:

$$\frac{\partial A}{\partial z} = \frac{i}{2n_0 k_0}\nabla_\perp^2 A - \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + ik_0 n_2 |A|^2 A \qquad (17)$$

- If $\beta_2 < 0$ and $n_2 < 0$ (self defocusing), and we make the "Madelung transformation" $\rho = |A|^2$ and $\mathbf{v} = \nabla Arg(A)$, We can obtain equations that resemble hydrodynamic equations:

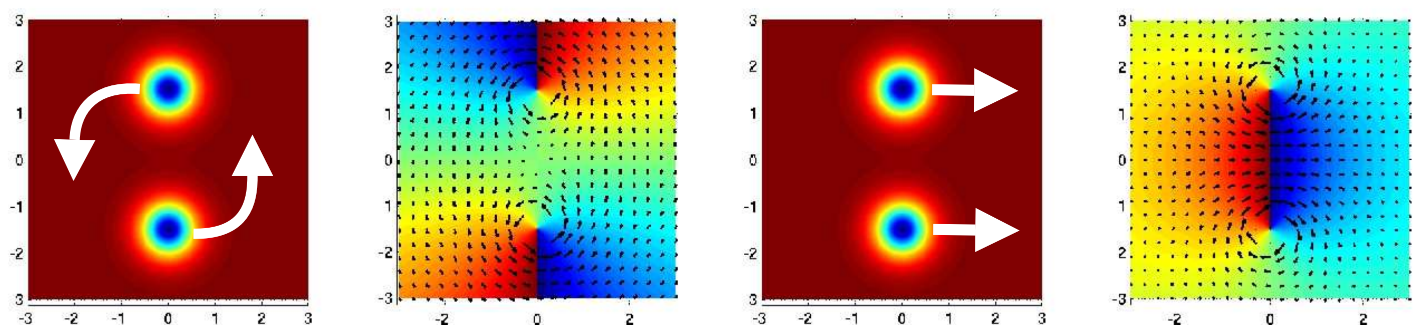
$$k_0 \frac{\partial \rho}{\partial z} + \nabla \cdot (\rho \mathbf{v}) = 0, \qquad (18)$$

$$k_0 \frac{\partial \rho \mathbf{v}}{\partial z} + \nabla \cdot (\rho \mathbf{v}\mathbf{v}) = -\nabla P + \nabla \cdot \mathbf{T}^q, \qquad (19)$$

- $P \propto I^2$ is the pressure, n_2 needs to be negative for the pressure to have the correct sign
- $\mathbf{T}^q \propto \partial_i \sqrt{\rho}\partial_j\sqrt{\rho} - \sqrt{\rho}\partial_i\partial_j\sqrt{\rho}$ is the so-called quantum pressure that is not present in ordinary fluid dynamics equations, but can be neglected if the nonlinearity is high enough

Vorticity and Optical Vortex Solitons

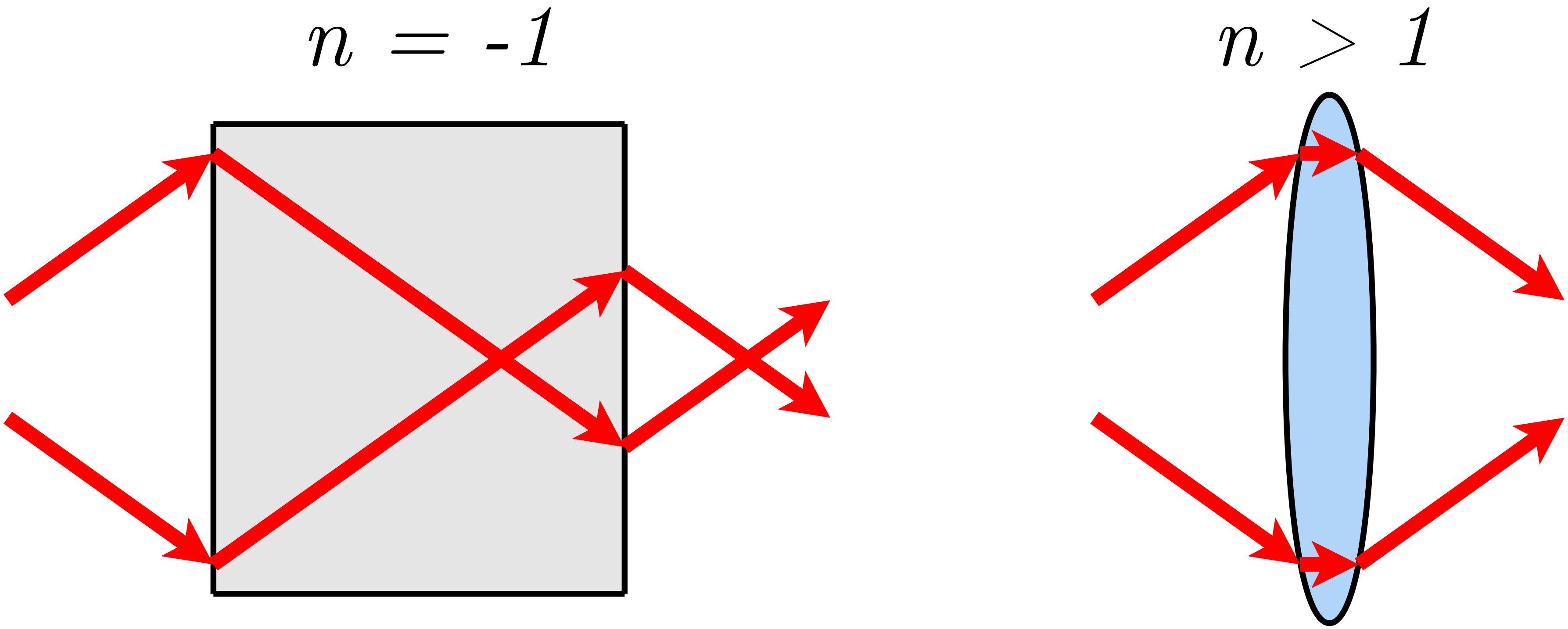
- Most interesting fluid dynamics depends on vorticity, or $\omega = \nabla \times \mathbf{v}$.
- But for the Madelung transformation, $\mathbf{v} = \nabla Arg(A)$ and $\nabla \times \mathbf{v} = 0$
- fluid vorticity can be represented by optical vortex solitons, where $\rho = 0$ and the Madelung transformation does not apply



- A large number of optical vortex solitons can approximate continuous vorticity and therefore inviscid fluid dynamics
- Still need optical analogues of viscosity (quantum-noise-induced random walk of vortex solitons?) and no-slip boundary conditions for the correspondence to be complete.

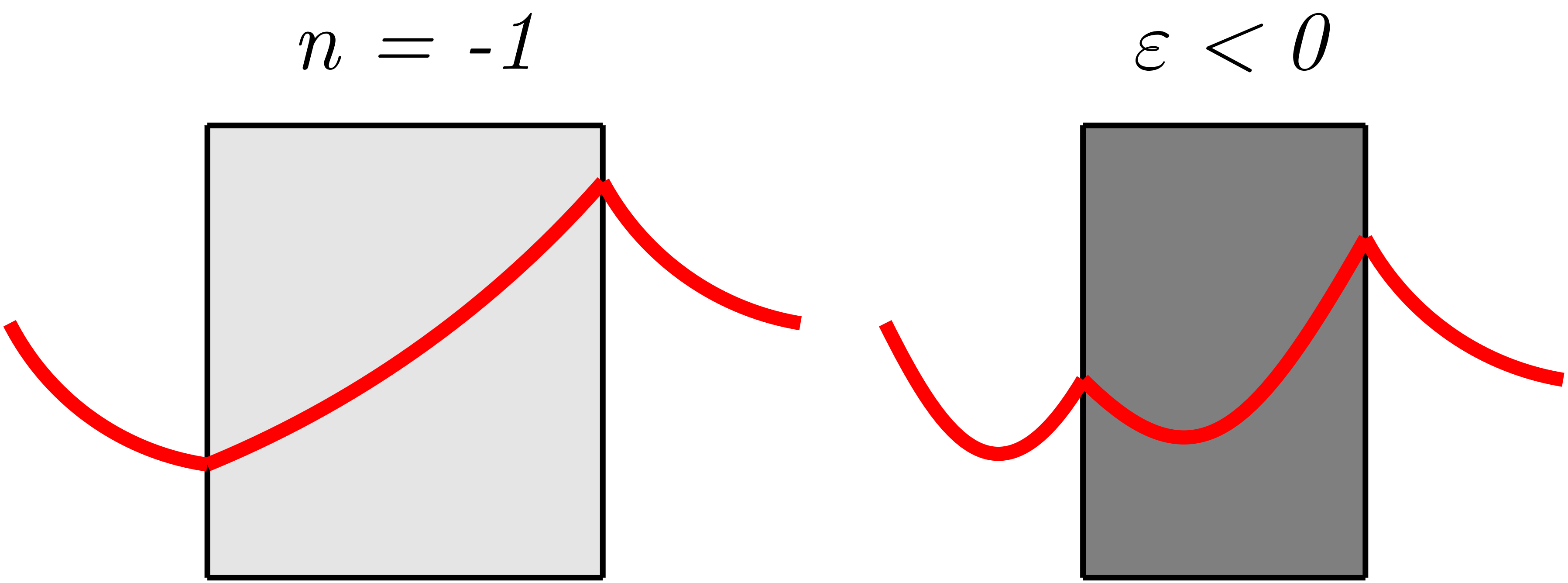
Tsang and Psaltis, e-print physics/0604149.

Veselago: Negative Refraction



Veselago, Sov. Phys. Usp. **10**, 509 (1968)

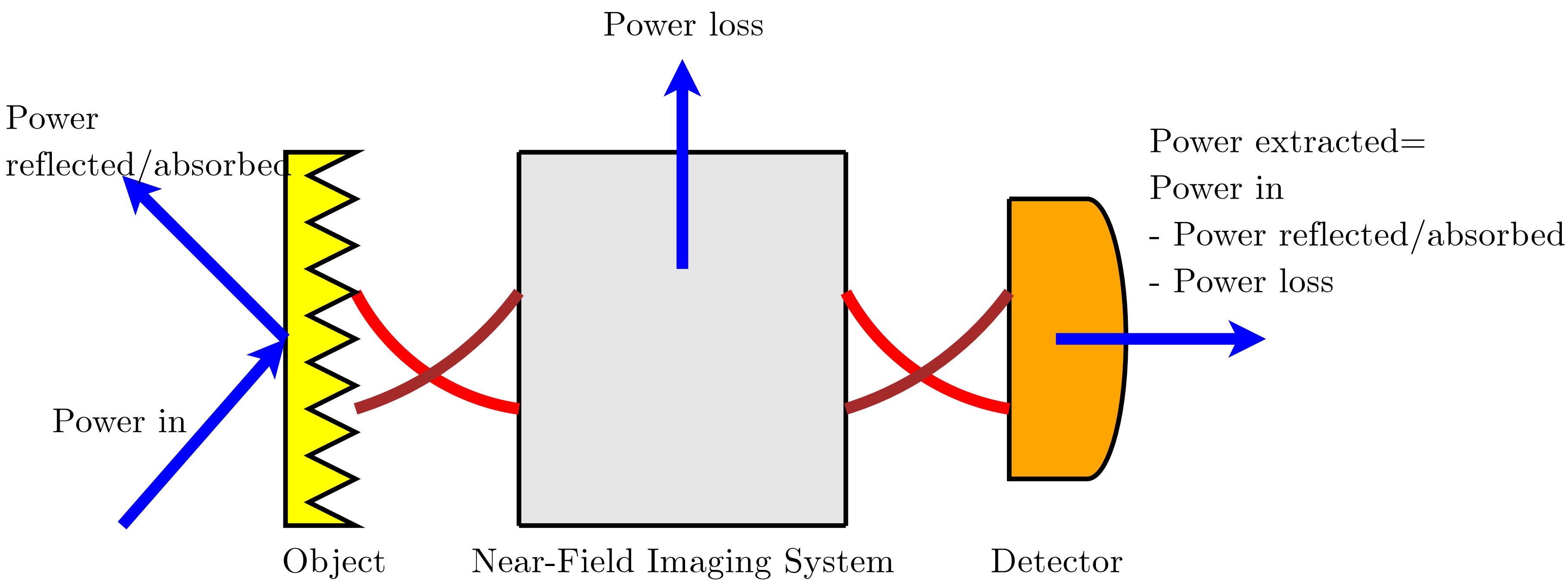
Pendry: Evanescent Wave Amplification



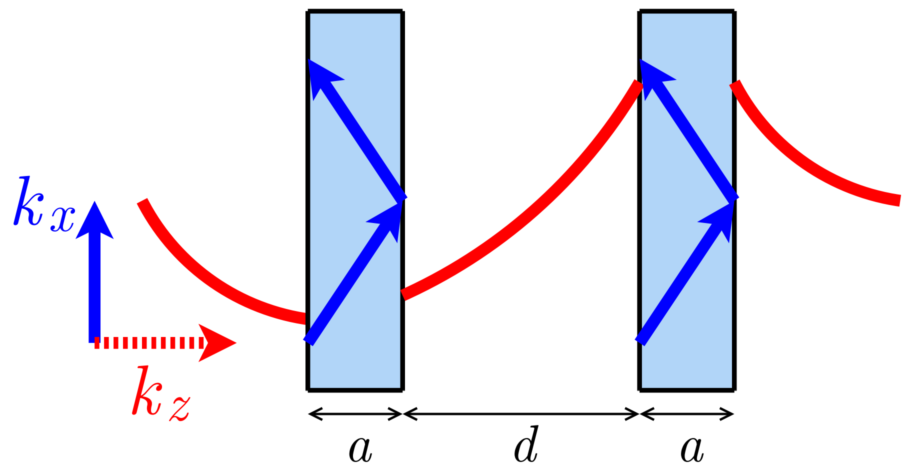
$$T \to \exp(-ik_z d), \quad R \to 0 \tag{20}$$

Pendry, Phys. Rev. Lett. **85**, 3966 (2000)

Importance of Low Loss



Two Dielectric Slabs



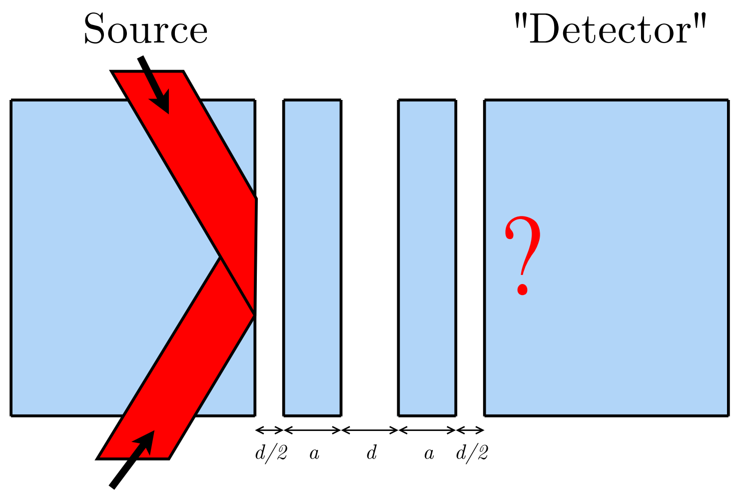
$$R = \Gamma + \frac{\tau^2\Gamma\exp(2ik_zd)}{1-\Gamma^2\exp(2ik_zd)} = 0, \quad T = \frac{\tau^2\exp(ik_zd)}{1-\Gamma^2\exp(2ik_zd)} = -\exp(-ik_zd) \tag{21}$$

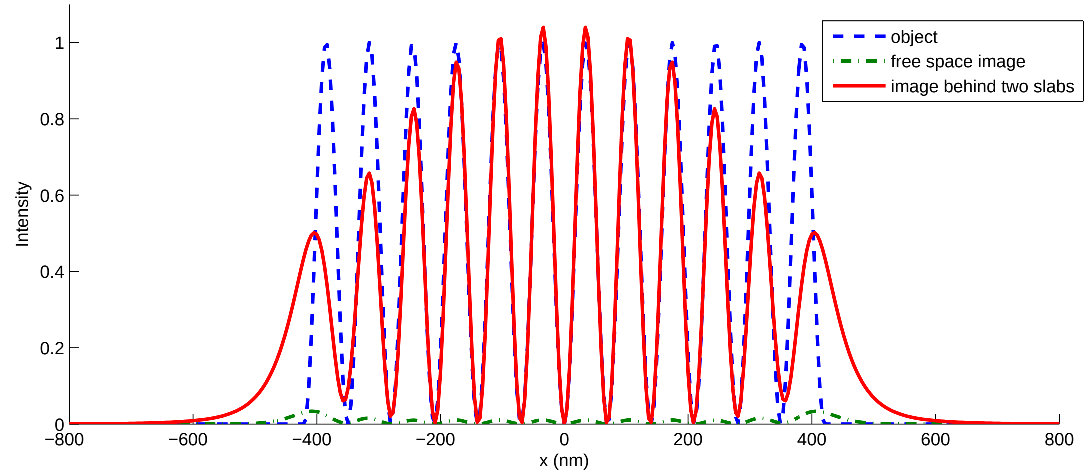
for some k_x.

Tsang and Psaltis, Optics Letters, **31**, 2741 (2006), Erratum: **32**, 86 (2006).

Numerical Example

- $\lambda = 230$ nm, $n = 2.7$, $a = 20$ nm, $d = 20$ nm, TE polarization,





- Low loss
- Many spatial modes
- High refractive index material available (transparent down to $\lambda = 230$ nm, $n = 2.7$ for diamond)
- non-contact imaging, suitable for lithography and bio-imaging

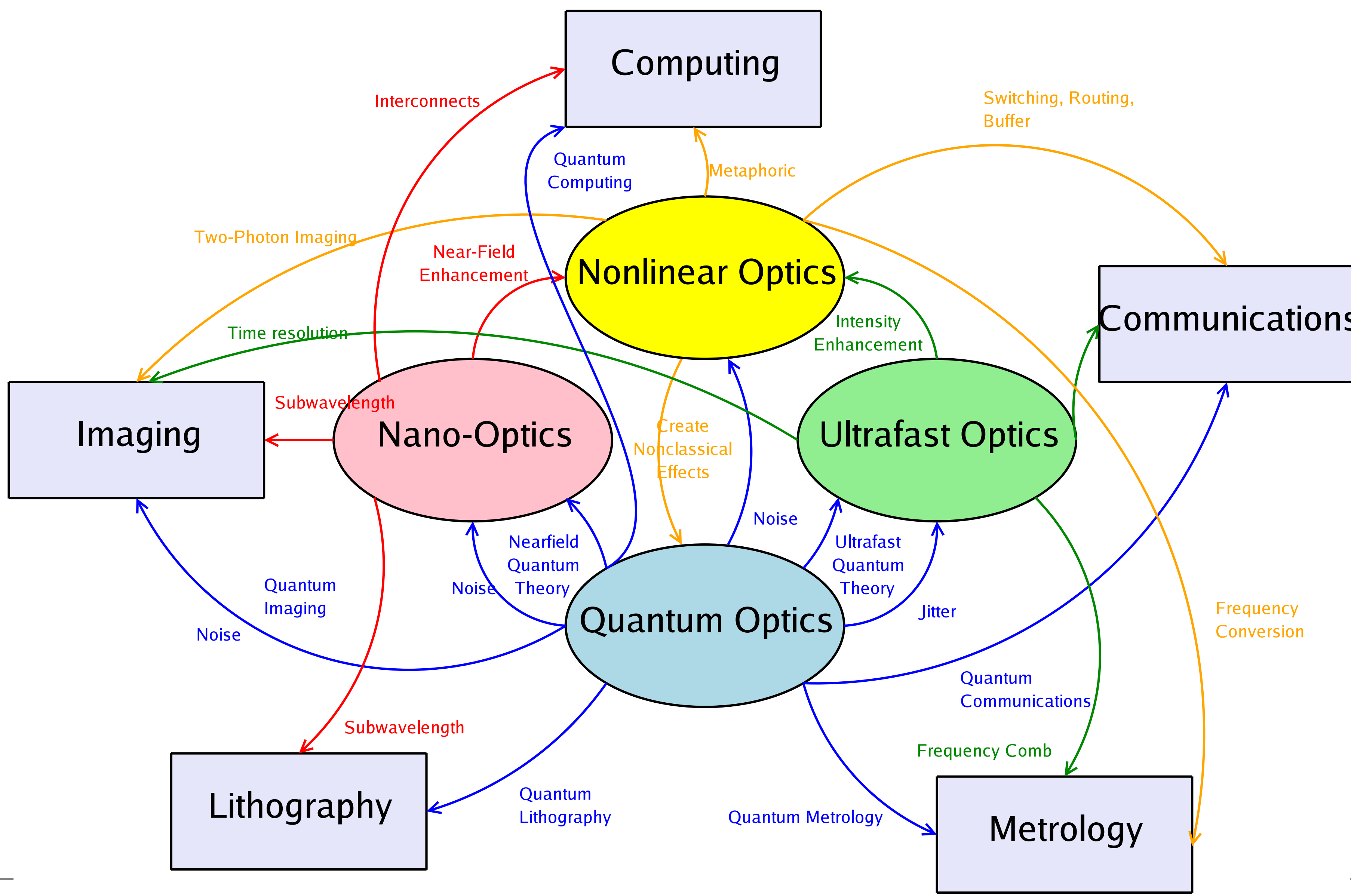
Miscellaneous

- Reverse propagation of femtosecond pulses in optical fiber (collaboration with Fiorenzo Omenetto at Tufts)
  Tsang, Psaltis, and Omenetto, Optics Letters **28**, 1873 (2003)
- Spontaneous spectral phase conjugation for coincident frequency entanglement
  Tsang and Psaltis, Physical Review A **71**, 043806 (2005)
- Quantum temporal imaging
  Tsang and Psaltis, Physical Review A **73**, 013822 (2006)
- Trade-off between resolution enhancement and multiphoton absorption rate in quantum lithography
  Tsang, e-print quant-ph/0607114 [accepted by Physical Review A].
- Electro-optical solitons
  collaboration with Prof. Hajimiri's Electronics group at Caltech
- Compensation of random scattering by phase conjugation
  collaboration with Prof. Yang's Biophotonics group at Caltech

Future Work

- Quantum theory of mirrorless optical parametric oscillators
- Quantum information processing via scalar and vector solitons
- Spatial quantum information processing via spatial solitons
- Effect of loss and decoherence
- Quantum limits on spatial, temporal, and spectral information capacity of optical fields
- Beating the resolution limit of λ/n by the use of dielectrics, e.g. photonic crystals, coupled resonators
- Quantum near-field optics
- Correspondence between nonlinear optics and viscous fluid dynamics
- Application to Bose-Einstein condensates and superfluids
- Experiments

Quantum Optical Engineering



Publications

1. M. Tsang and D. Psaltis, “Reflectionless evanescent-wave amplification by two dielectric planar waveguides,” Optics Letters **31**, 2741 (2006) [Erratum: **32**, 86 (2006)].
2. M. Tsang, “Quantum temporal correlations and entanglement via adiabatic control of vector solitons,” Physical Review Letters **97**, 023902 (2006).
3. M. Tsang, “Spectral phase conjugation via extended phase matching,” Journal of the Optical Society of America B **23**, 861 (2006).
4. M. Tsang and D. Psaltis, “Propagation of temporal entanglement,” Physical Review A **73**, 013822 (2006).
5. M. Centurion, Y. Pu, M. Tsang, and D. Psaltis, “Dynamics of filament formation in a Kerr medium,” Physical Review A **71**, 063811 (2005).
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7. M. Tsang and D. Psaltis, “Spectral phase conjugation by quasi-phase-matched three-wave mixing,” Optics Communications **242**, 659 (2004).
8. M. Tsang and D. Psaltis, “Spectral phase conjugation with cross-phase modulation compensation,” Optics Express **12**, 2207 (2004).
9. M. Tsang, D. Psaltis, and F. G. Omenetto, “Reverse propagation of femtosecond pulses in optical fibers,” Optics Letters **28**, 1873 (2003).
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Conferences:

1. M. Tsang and D. Psaltis, "Reflectionless evanescent wave amplification by two dielectric slabs," Oral Presentation, OSA Annual Meeting, Oct 2006, paper FMB4.
2. M. Tsang and D. Psaltis, “Quantum lithography has a reduced multiphoton absorption rate,” Oral Presentation, OSA Annual Meeting, Oct 2006, paper LWH4.
3. M. Tsang and D. Psaltis, “Quantum temporal imaging,” Oral Presentation, CLEO/QELS, May 2006, paper QWB5.
4. M. Tsang and D. Psaltis, “Metaphoric optical computing of fluid dynamics,” Oral Presentation, CLEO/QELS, May 2005, paper QML6.
5. M. Tsang and D. Psaltis, “Metaphoric optical computing for fluid dynamics,” Invited Paper, Proceedings of SPIE, 5735, 1 (Apr 2005).
6. M. Tsang and D. Psaltis, “Spectral phase conjugation with cross-phase modulation compensation,” Poster, OSA Annual Meeting, Oct 2004, paper FWH44.

Preprints:

1. M. Tsang, “On the relationship between resolution enhancement and multiphoton absorption rate in quantum lithography,” e-print quant-ph/0607114 [accepted by Physical Review A].
2. M. Tsang, “Metaphoric optical computing of fluid dynamics,” e-print physics/0604149.
3. Y. Pu, J. Wu, M. Tsang, and D. Psaltis, “Optical parametric generation in periodically poled $KTiOPO_4$ via extended phase matching,” submitted to CLEO/QELS, under preparation.

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