

## Resonantly Enhanced Microwave Photonics \*

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$$i\hbar\psi = H\psi$$

# Quantum Measurement Theory

- Two foundations of quantum mechanics:  
**Schrödinger's equation**



(<https://www.flickr.com/photos/sababa-dan/396970817/>)

## Born's rule

### I.2 ON THE QUANTUM MECHANICS OF COLLISIONS

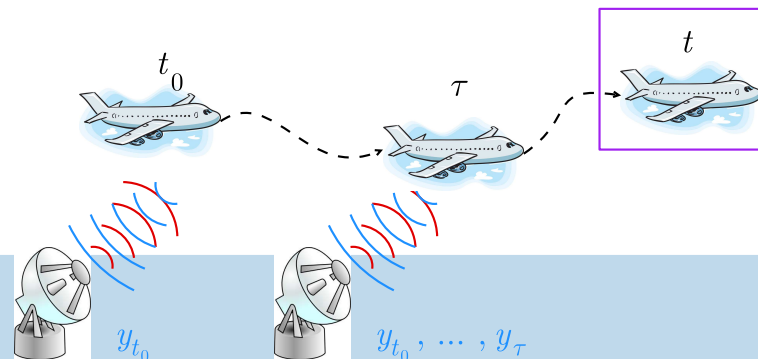
[Preliminary communication]†

MAX BORN

\* Addition in proof: More careful consideration shows that the probability is proportional to the square of the quantity  $\Phi_{n,m}$ .

$$\Pr(x) = |\psi(x)|^2$$

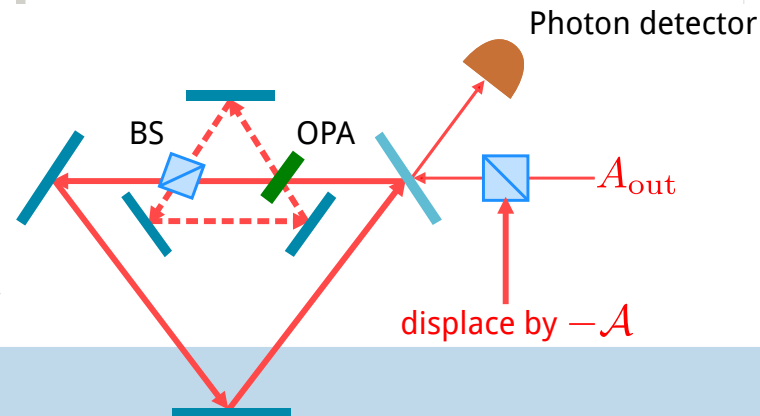
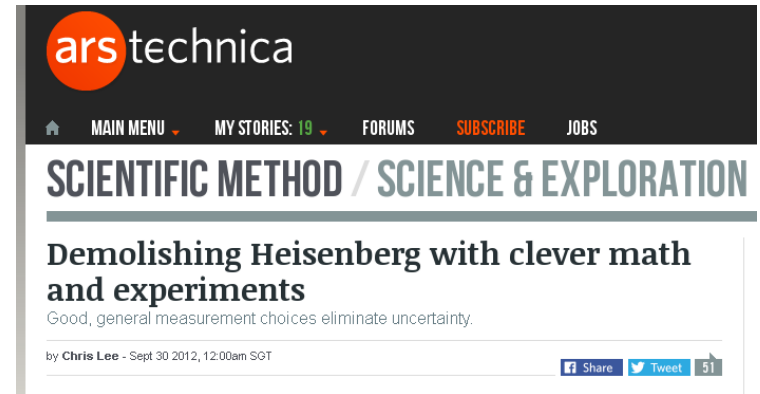
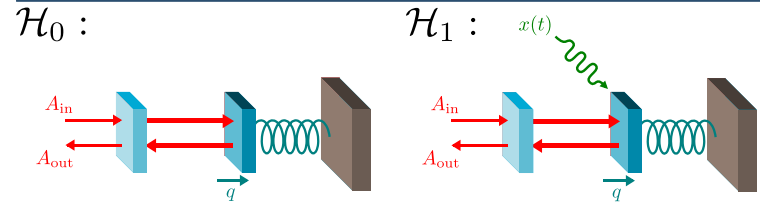
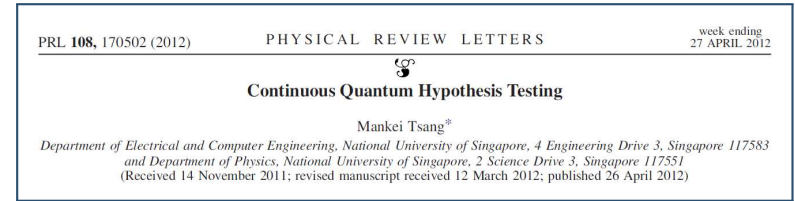
- Impose **fundamental uncertainties** to information processing, communication, computing, and sensing.
- Experimental technology** is catching up (see 2012 Nobel Prize), especially in **quantum optics**, **optomechanics**.
- Use **engineering techniques** (Bayesian inference, information theory, control theory, etc.) to address the challenges.



$$i\hbar\psi = H\psi$$

# Quantum Sensing Estimation, Control, and Fundamental Limits

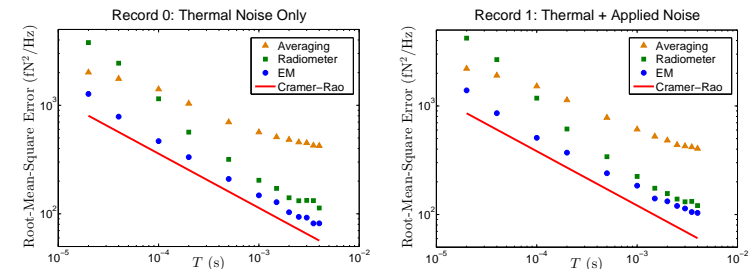
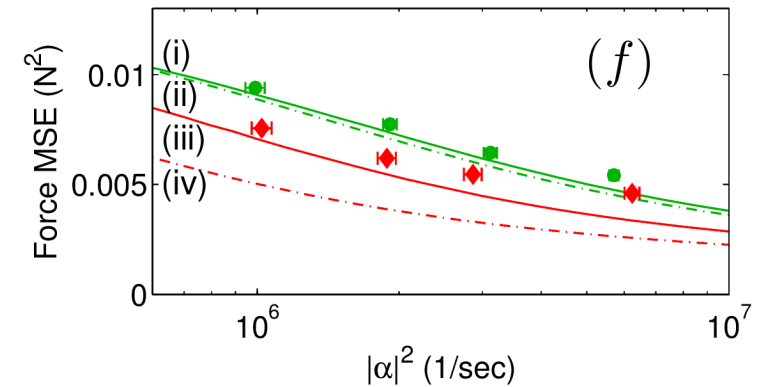
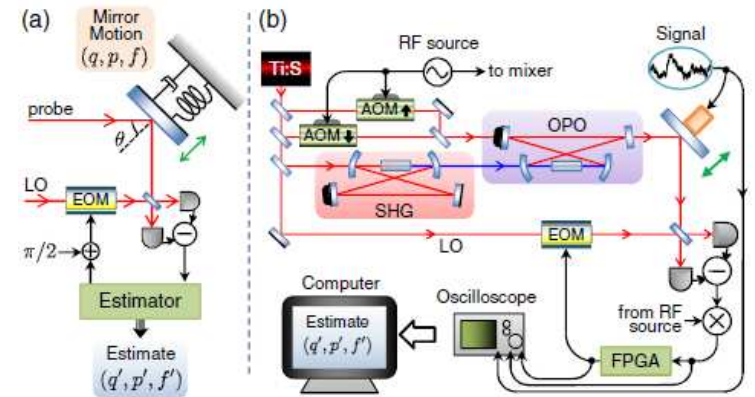
- Optimal hypothesis testing for quantum dynamical systems [M. Tsang, Phys. Rev. Lett. **108**, 170502 (2012)]
  - ◆ Detection of mirror motion, force in cavity optomechanics
  - ◆ Detection of magnetic fields with atomic spins, diamond-defect spins
  - ◆ Qubit readout for quantum computing [S. Ng and M. Tsang, Phys. Rev. A **90**, 022325 (2014)].
  
- Evading quantum backaction noise [M. Tsang, C. M. Caves, Phys. Rev. X **2**, 031016 (2012)]
  - ◆ Ongoing experimental collaboration with Michele Heurs, Max Planck Institute for Gravitational Physics, Germany
  
- Fundamental quantum limits to sensing accuracy
  - ◆ Parameter Estimation: M. Tsang, Phys. Rev. Lett. **107**, 270402 (2012); M. Tsang, Phys. Rev. Lett. **108**, 230401 (2012), etc.
  - ◆ Detection: M. Tsang and R. Nair, Phys. Rev. A **86**, 042115 (2012); M. Tsang, Phys. Rev. A (Rapid Commun.) **88**, 021801(R) (2013); M. Tsang, New J. Phys. **15**, 103028 (2013), etc.



$$i\hbar\dot{\psi} = H\psi$$

## Experimental Demonstrations

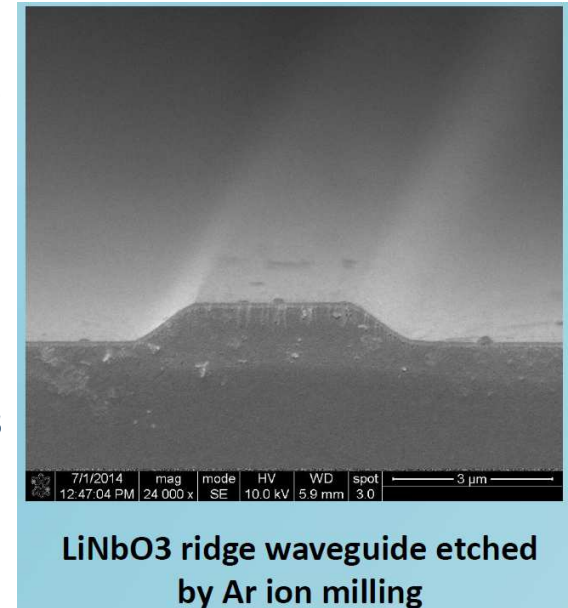
- Optomechanical force estimation near fundamental quantum limit [K. Iwasawa *et al.*, Phys. Rev. Lett. **111**, 163602 (2013)]
  - ◆ Collaboration with U. Tokyo, Japan and UNSW, Australia
  - ◆ Quantum-limited mirror motion measurement with laser beam/squeezed light.
  - ◆ Based on my earlier theoretical work [M. Tsang, Phys. Rev. Lett. **102**, 250403 (2009); M. Tsang, H. M. Wiseman, C. M. Caves, Phys. Rev. Lett. **106**, 090401 (2011)]
  
- Optomechanical parameter estimation [S. Z. Ang *et al.*, New J. Phys. **15**, 103028 (2013)]
  - ◆ Collaboration with U. Queensland, Australia
  - ◆ Efficient expectation-maximization algorithm



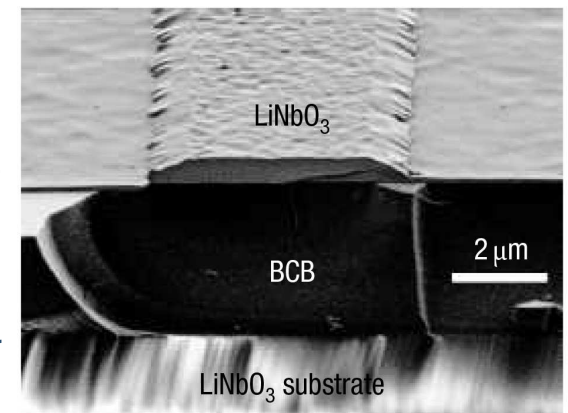
$$i\hbar\psi = H\psi$$

## Outlook

- Ongoing theoretical work on optical interferometry, optomechanical sensors, magnetometry, etc.
- Ongoing experimental collaborations on optomechanics with Bowen (U. Queensland), Huntington (UNSW), Heurs (MPI), etc.
- **Microwave Photonics** (with Aaron Danner's group at NUS)
  - ◆ Quantum optical measurement of microwave signals [M. Tsang, Phys. Rev. A **84**, 043845 (2011)]
  - ◆ Apply atomic/mechanical physics to microwave photonics
  - ◆ Ideal platform for quantum measurement demonstrations
  - ◆ Applications in communications, optical interconnects, clock synchronization, etc.
- **Quantum Imaging**
  - ◆ Early work: M. Tsang, Phys. Rev. Lett. **102**, 253601 (2009).
  - ◆ Application of quantum multi-parameter estimation and multi-hypothesis testing.
  - ◆ Fundamental quantum limits beyond Rayleigh-Abbe.
- Support by the Singapore National Research Foundation under NRF Award No. NRF-NRFF2011-07 is gratefully acknowledged.



Unpublished work by Saha, Yohanes, Deng

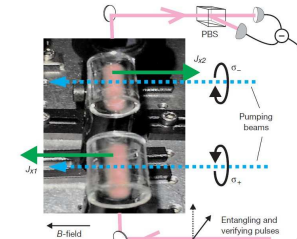
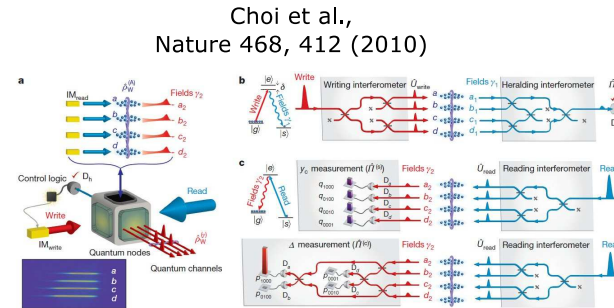
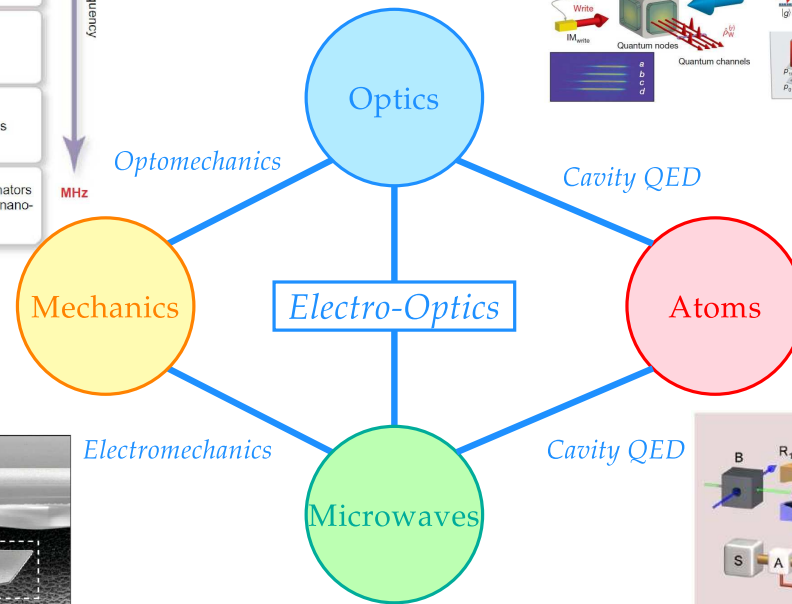
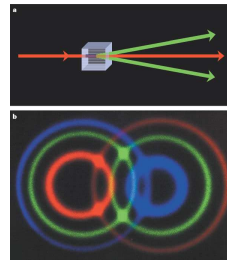
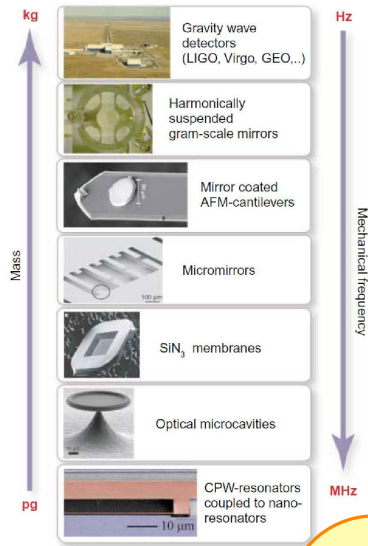


Guarino *et al.*, Nature Photonics **1**, 407 (2007).

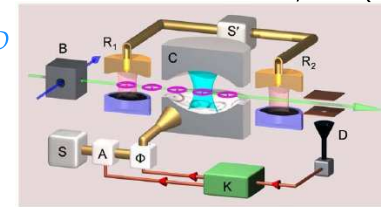
$$i\hbar\dot{\psi} = H\psi$$

# Hybrid Quantum Systems

Kippenberg and Vahala, Science 321, 1172 (2008)

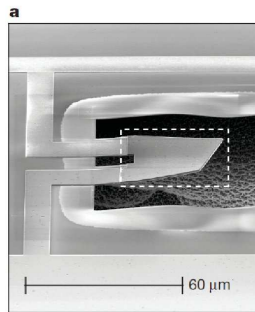


Julsgaard, Kozhekin, Polzik, Nature 413, 400 (2001)

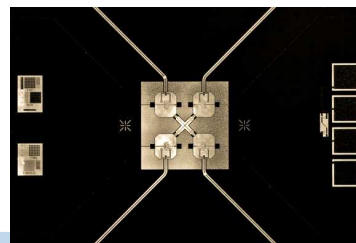


Sayrin et al., Nature 477, 73 (2011)

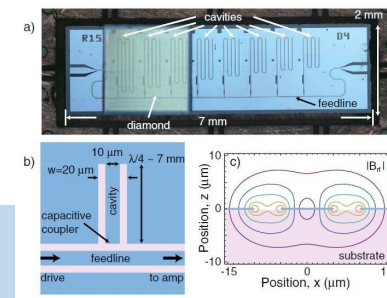
Haroche and Raimond, Exploring the Quantum



O'Connell et al., Nature 464, 697 (2010)

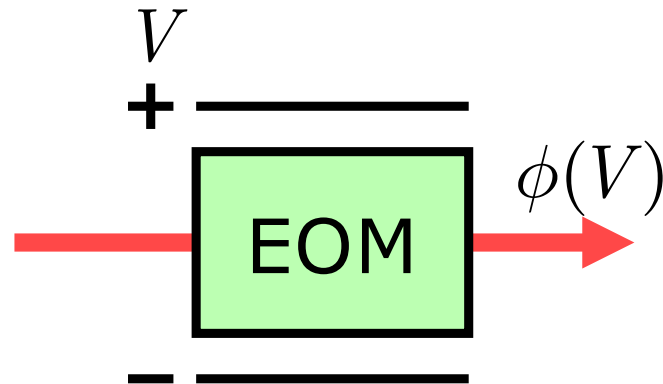


Neeley et al., Nature 467, 570 (2010)



Schuster et al., PRL 105, 140501 (2010)

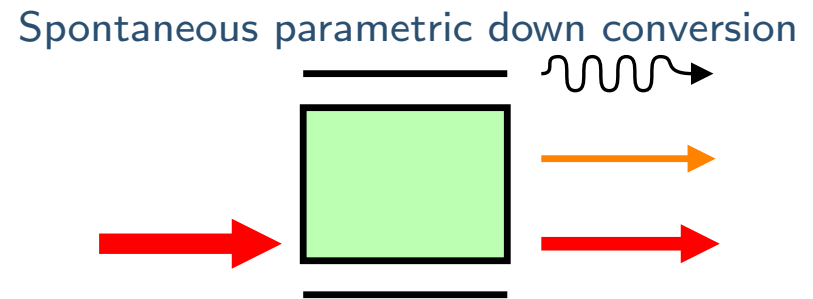
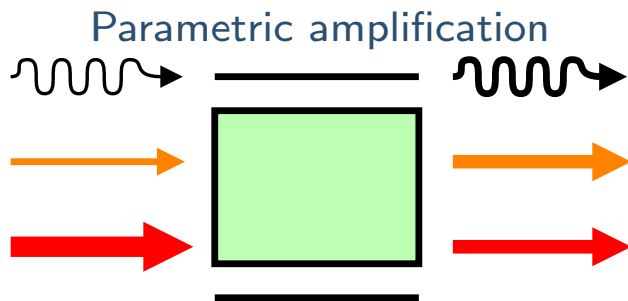
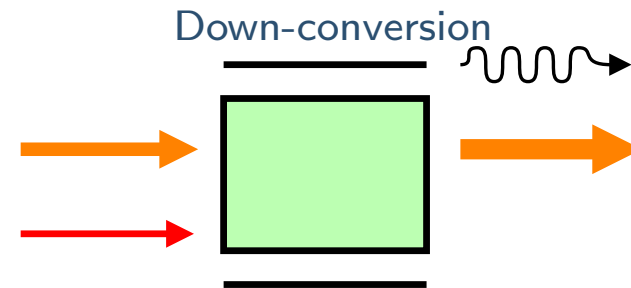
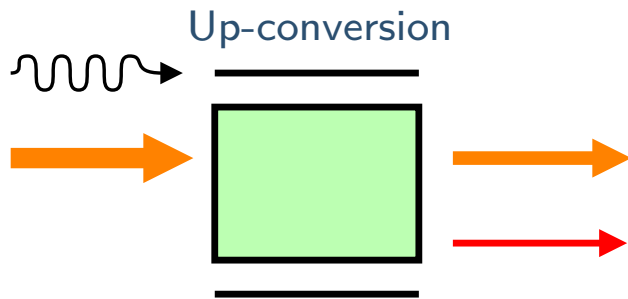
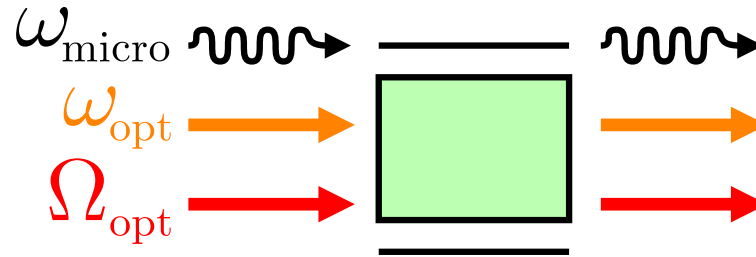
Kubo et al., PRL 105, 140502 (2010)



- $\epsilon = \epsilon_0 (1 + \chi^{(1)} + \chi^{(2)} \mathbf{E} + \chi^{(3)} : \mathbf{E}\mathbf{E} + \dots)$
- $\chi^{(2)}$  (Pockels):  $\Delta\phi(V) \propto V$ : e.g., lithium niobate ( $\text{LiNbO}_3$ )
- Optical:
  - ◆ transparent between 350nm-5 $\mu\text{m}$
  - ◆ intrinsic  $Q \sim 10^6$  resonator at 1.55 $\mu\text{m}$  [Ilchenko *et al.*, JOSAB 20, 333 (2003)]
  - ◆ 10dB squeezing [Vahlbruch *et al.* PRL 100, 033602 (2008)]
- Microwave:
  - ◆ intrinsic  $\epsilon_l \approx 28$ ,  $\epsilon_t \approx 45$ ,  $Q \approx 2.3 \times 10^3$  at 9GHz, 300K [Bourreau *et al.*, EL 22, 399 (1986)], loss should decrease with temp.
  - ◆ Cu half-wave resonator:  $Q \approx 100$  at 9GHz, 300K [Ilchenko *et al.*]
  - ◆ 26.5GHz EOM with Nb electrode on  $\text{LiNbO}_3$  at 4.2K [Yoshida *et al.*, IEEE TMTT 47, 1201 (1999)]

# Three-Wave Mixing

$$P^{(NL)} \propto \chi^{(2)} : EE, \quad \omega_{\text{micro}} + \omega_{\text{opt}} = \Omega_{\text{opt}}$$



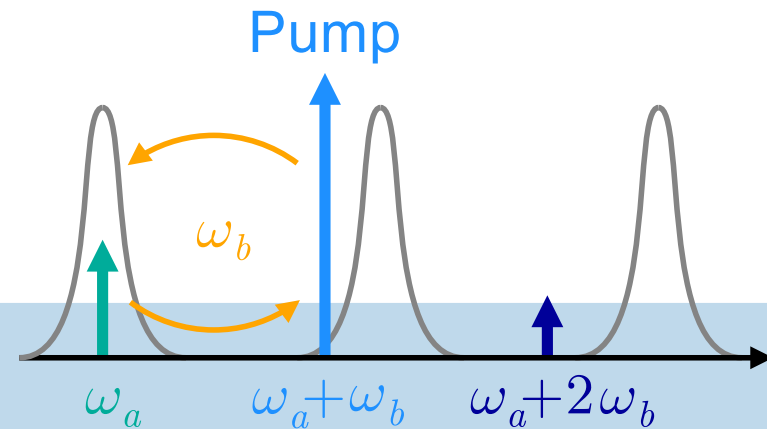
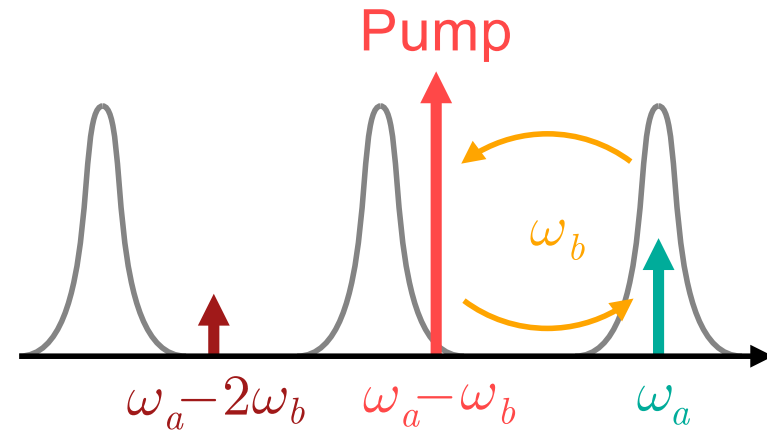
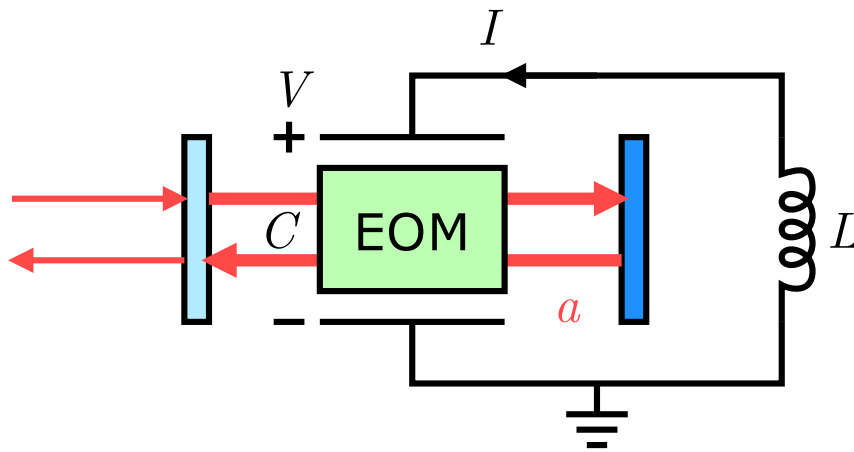
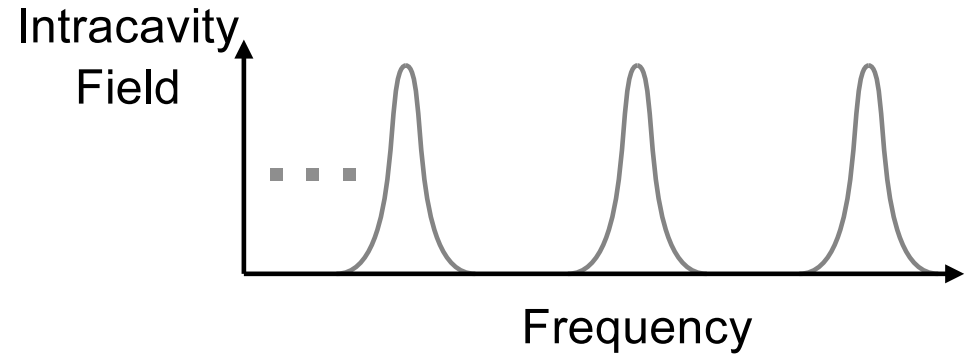
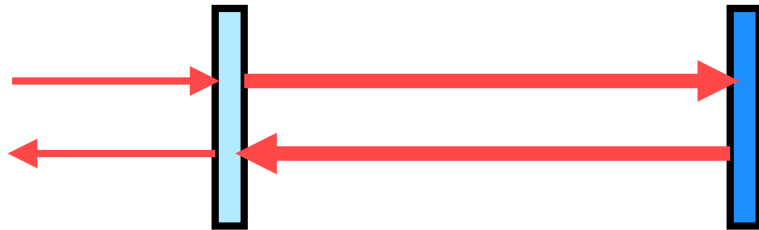
- Spatial mode matching



$$i\hbar\psi = H\psi$$

# Resonant Enhancement

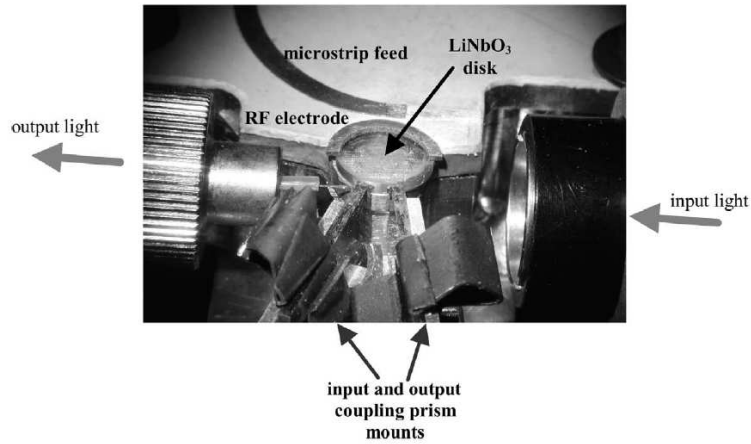
- How to enhance desired processes and suppress parasitic ones?



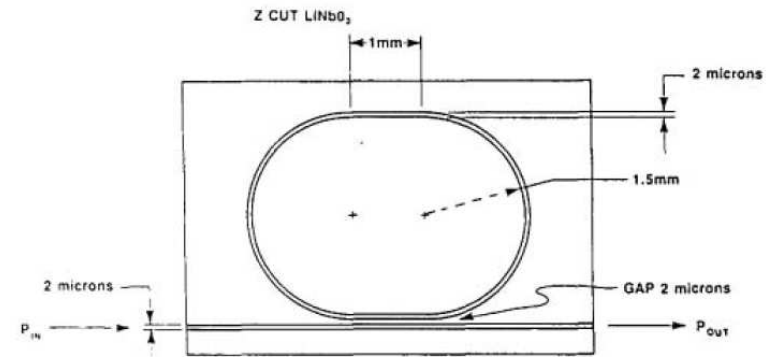
- Similar with microwave resonator

$$i\hbar\psi = H\psi$$

# Device Geometry



Cohen *et al.* (USC), "High- $Q$  microphotonic electro-optic modulator," *Solid-State Electronics* **45**, 1557 (2001)



Mahapatra and Robinson, "Integrated-optic ring resonators made by proton exchange in lithium niobate," *Appl. Opt.* **24**, 2285 (1985).

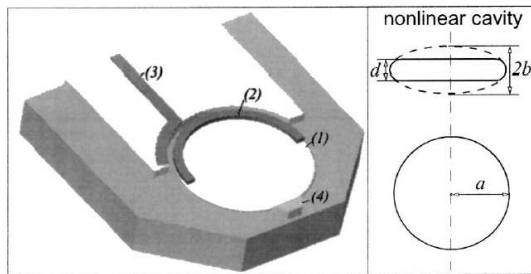
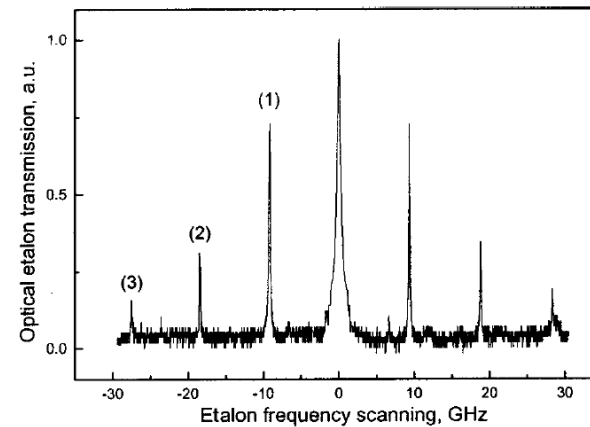


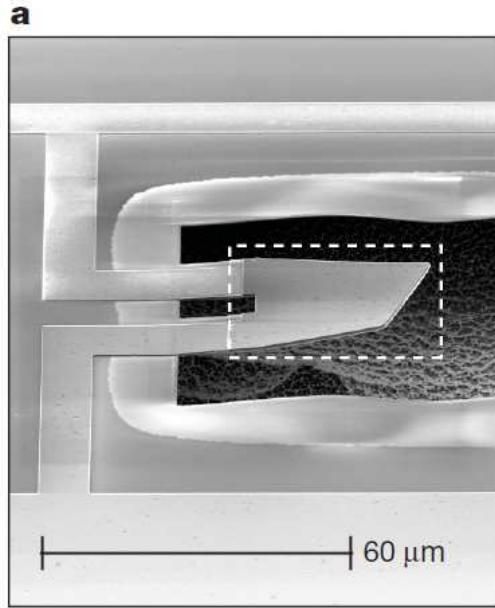
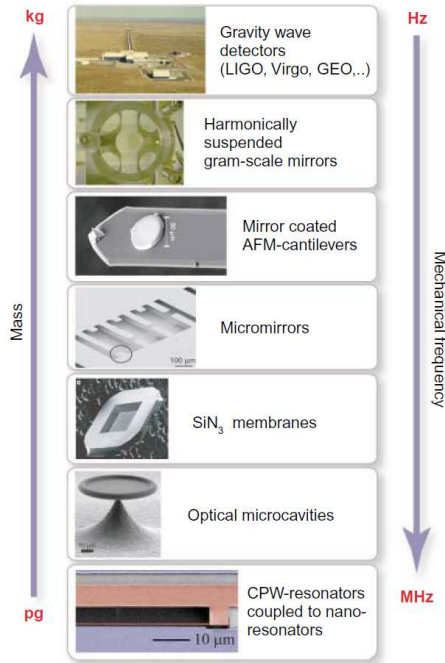
Fig. 1. Experimental setup: (1) LiNbO<sub>3</sub> optical cavity, (2) microwave resonator, (3) microwave feeding strip line, and (4) diamond coupling prism. Inset: geometric characteristics of the nonlinear optical cavity.

Ilichenko *et al.* (JPL), "Whispering-gallery-mode electro-optic modulator and photonic microwave receiver," *J. Opt. Soc. Am. B* **20**, 333 (2003),  $r = 2.4$  mm,  $d = 150$   $\mu$ m, half-wave 9 GHz resonator



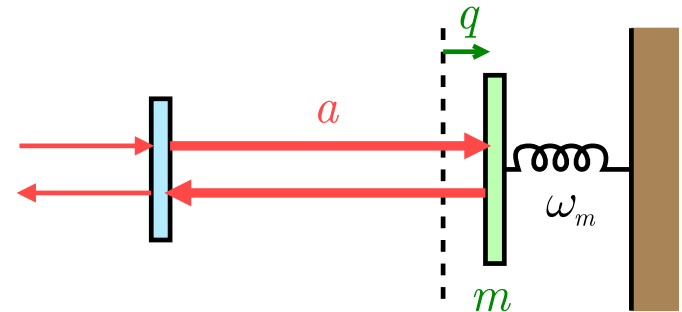
$$i\hbar\psi = H\psi$$

# Analogy with Cavity Optomechanics

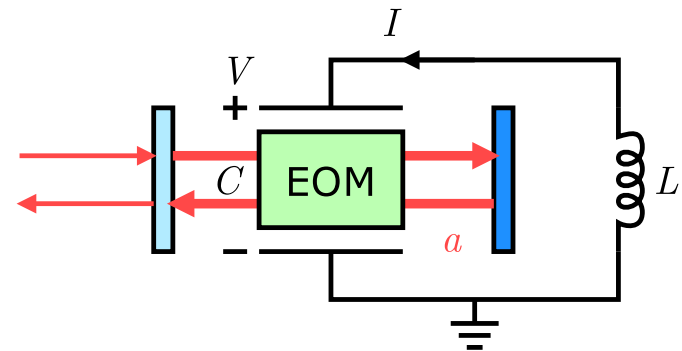


O'Connell *et al.*, Nature **464**, 697 (2010)

Optical/microwave photons  $\leftrightarrow$  microwave/RF phonons:

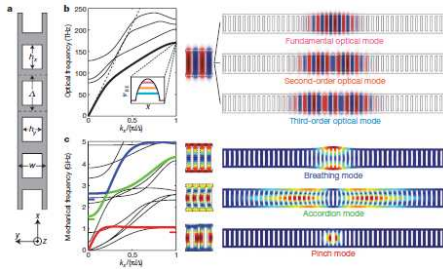


optical photons  $\leftrightarrow$  microwave/RF photons:

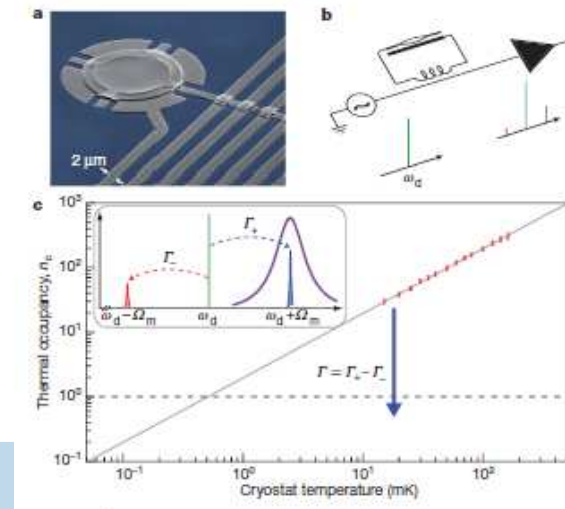


Tsang, Phys. Rev. A **81**, 063837 (2010); e-print arXiv:1108.1829 (2011)

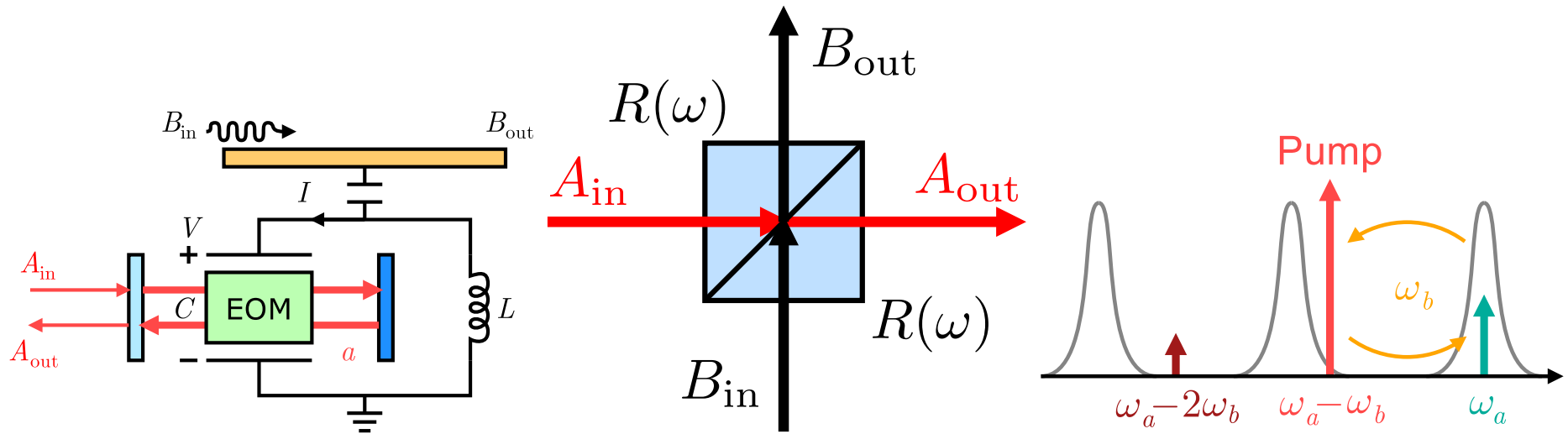
Kippenberg and Vahala, Science **321**, 1172 (2008)



Eichenfield *et al.*, Nature **462**, 78 (2009)



Teufel *et al.*, Nature **464**, 697 (2010)



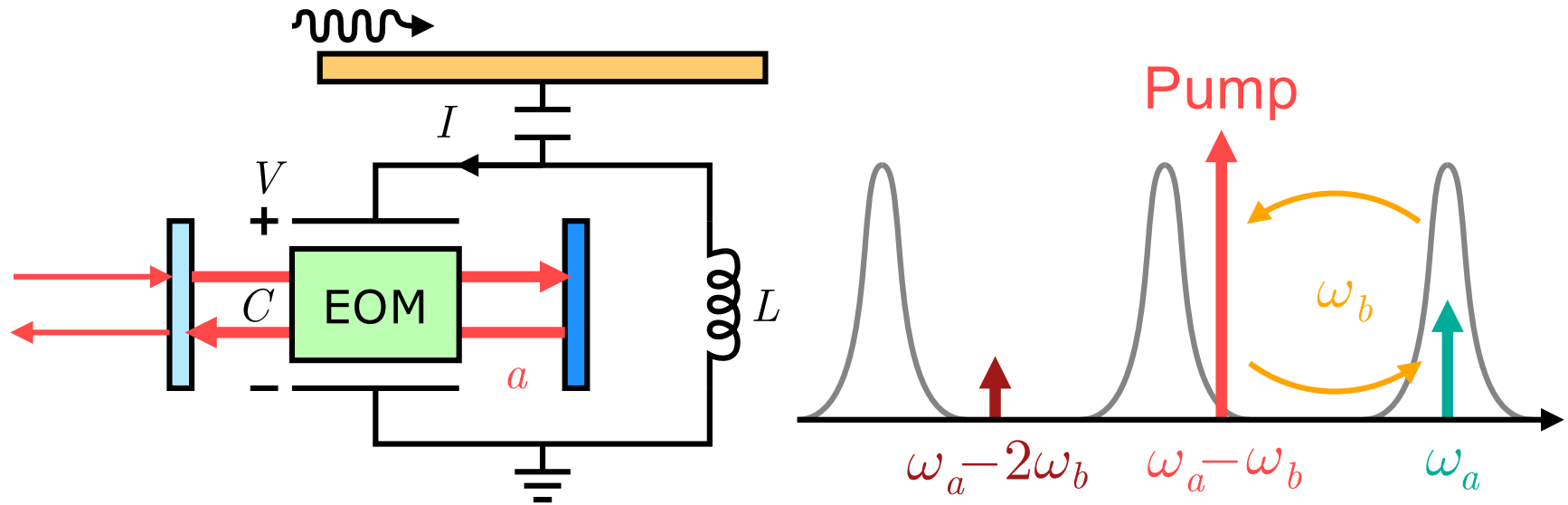
$$\frac{da}{dt} = ig\alpha b - \frac{\Gamma_a}{2}a + \sqrt{\gamma_a}A_{in} + \sqrt{\gamma'_a}A', \quad (1)$$

$$\frac{db}{dt} = ig\alpha^*a - \frac{\Gamma_b}{2}b + \sqrt{\gamma_b}B_{in} + \sqrt{\gamma'_b}B', \quad (2)$$

$$A_{out} = \sqrt{\gamma_a}a - A_{in}, \quad (3)$$

$$B_{out} = \sqrt{\gamma_b}b - B_{in}. \quad (4)$$

- Effective microwave resonator temperature  $\propto \langle b^\dagger b \rangle$



$$H_I \approx g\sqrt{N_{\text{pump}}} (a^\dagger b + ab^\dagger) \quad (5)$$

$$g = \nu \frac{\omega_a n^3 r}{2d} \sqrt{\frac{\hbar\omega_b}{2C}}, \quad (6)$$

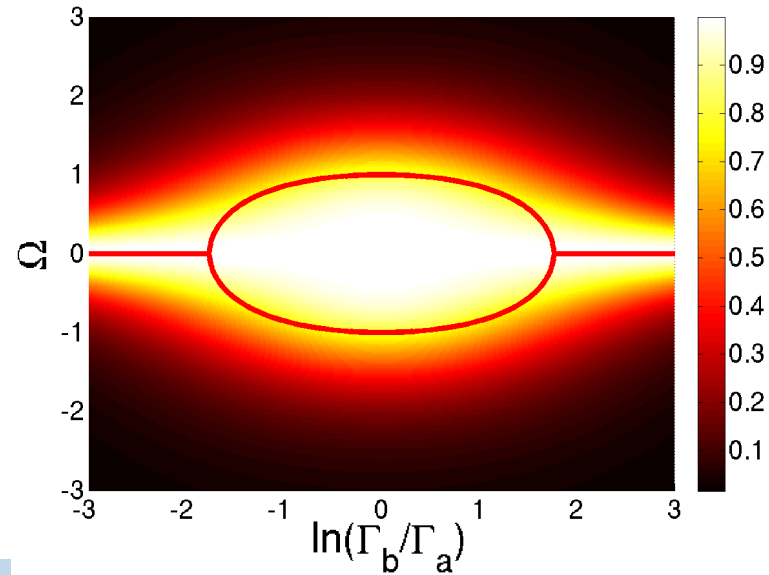
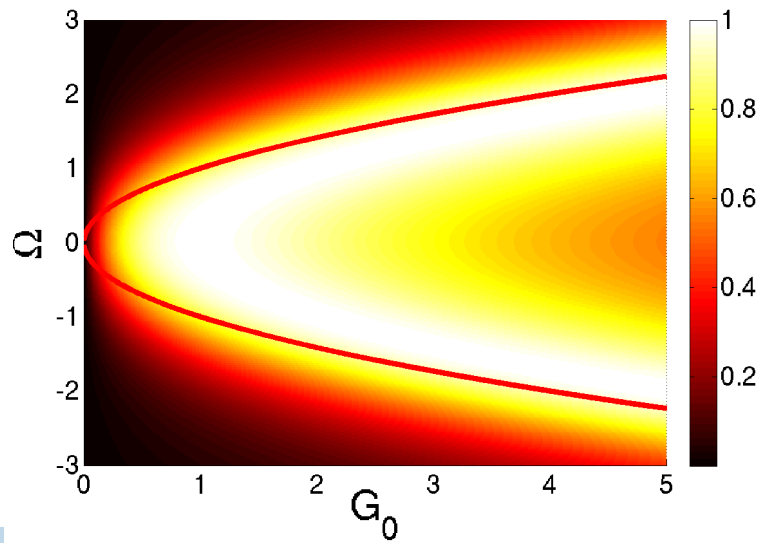
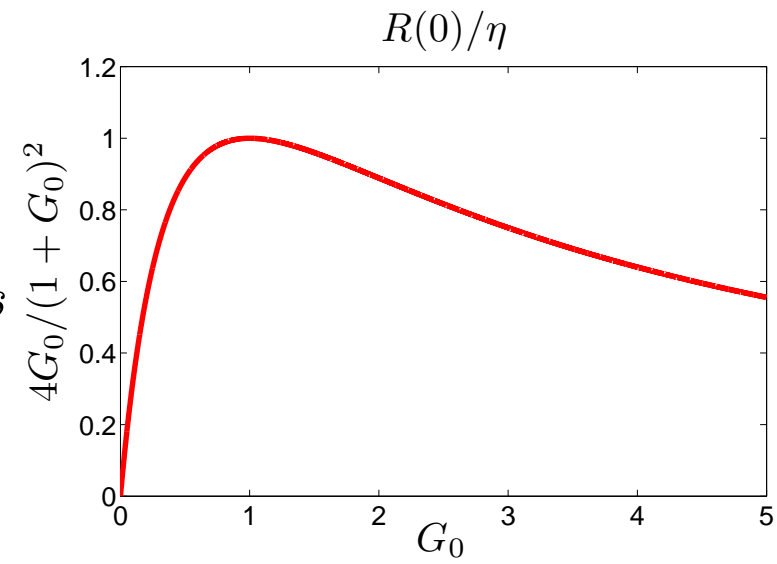
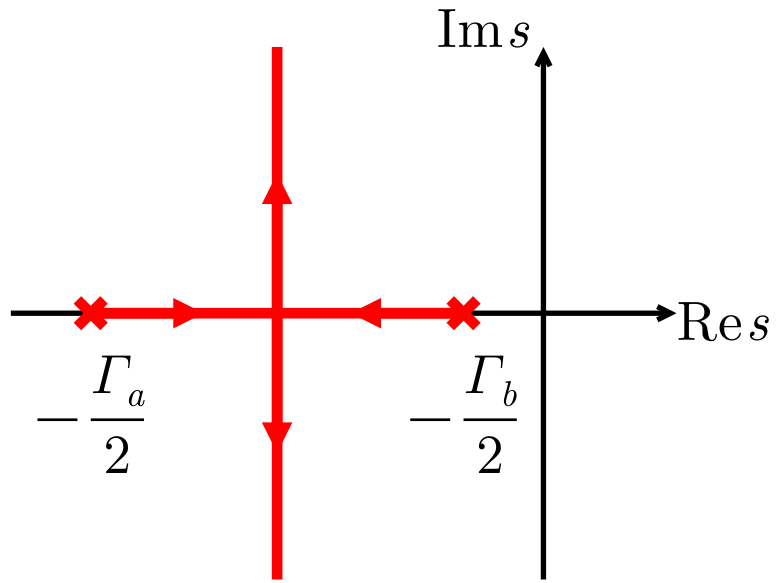
$$G \equiv \frac{g^2 N_{\text{pump}}}{\Gamma_a \Gamma_b} \quad (7)$$

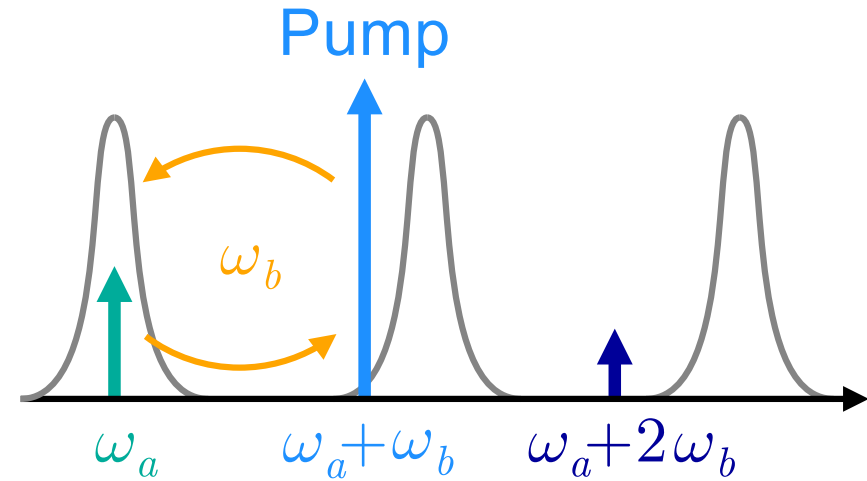
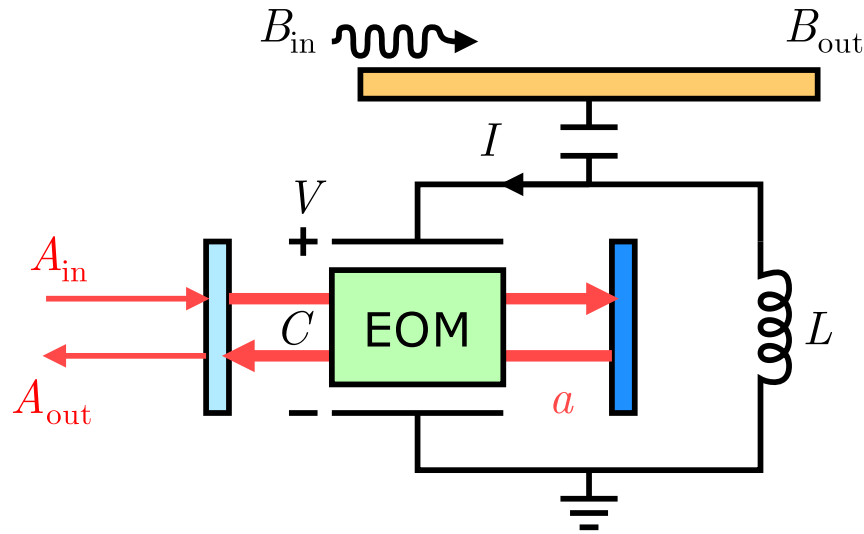
$$\text{Cooling : } G \gg 1 \quad (8)$$

$$\text{Conversion : } G = 1 \quad (9)$$

$$i\hbar\psi = H\psi$$

## Plots



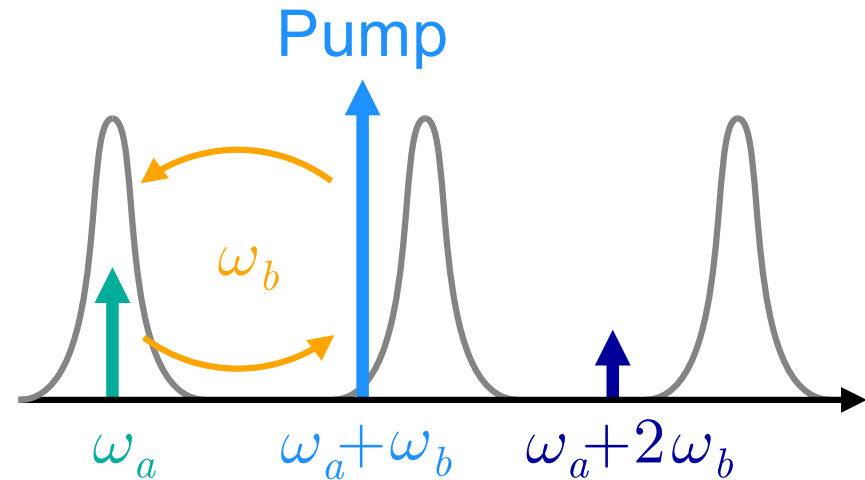
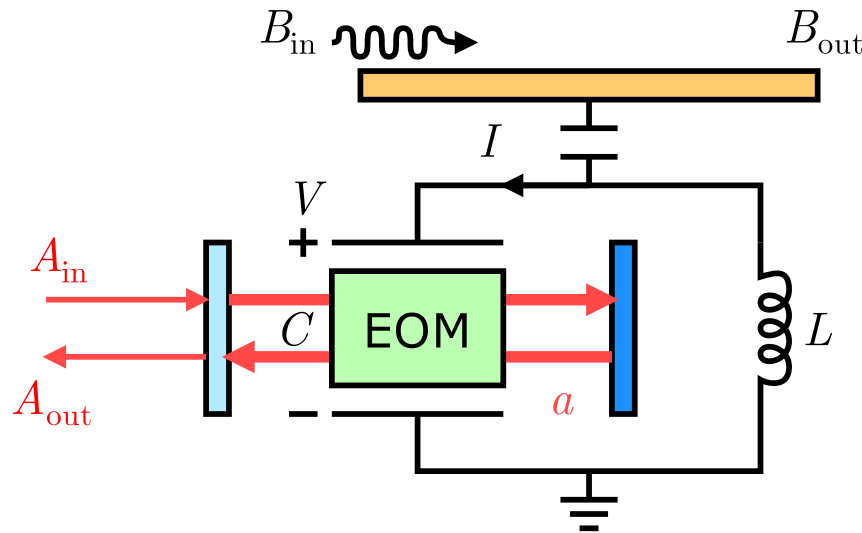


$$\frac{da}{dt} = ig\alpha b^* - \frac{\Gamma_a}{2}a + \sqrt{\gamma_a}A_{in} + \sqrt{\gamma'_a}A', \quad (10)$$

$$\frac{db}{dt} = ig\alpha a^* - \frac{\Gamma_b}{2}b + \sqrt{\gamma_b}B_{in} + \sqrt{\gamma'_b}B', \quad (11)$$

$$A_{out} = \sqrt{\gamma_a}a - A_{in}, \quad (12)$$

$$B_{out} = \sqrt{\gamma_b}b - B_{in}. \quad (13)$$



$$H_I \approx g\sqrt{N_{\text{pump}}} (a^\dagger b^\dagger + ab),$$

$$G \equiv \frac{g^2 N_{\text{pump}}}{\Gamma_a \Gamma_b}$$

(14)

Oscillation :  $G \geq 1,$

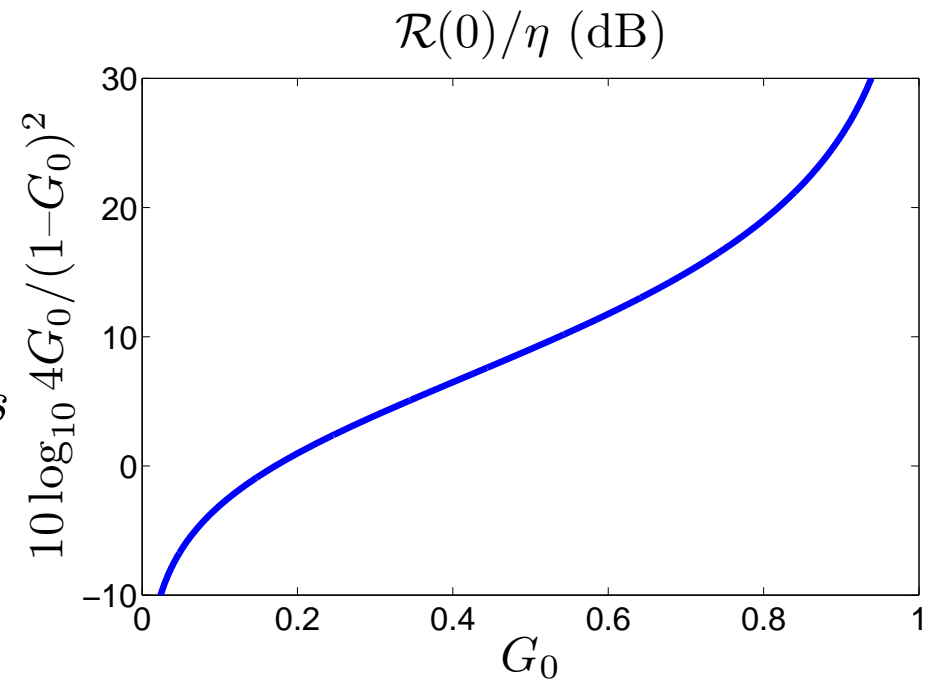
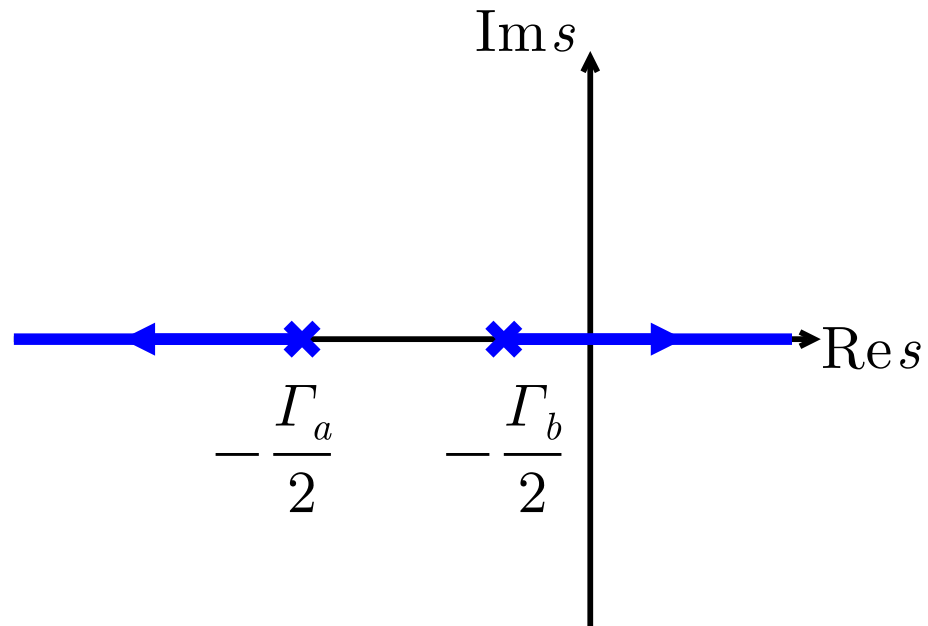
(15)

Spontaneous Down Conversion/Entangled Photons :  $G \ll 1$

(16)

- Double-sideband pumping: backaction-evading **microwave quadrature** measurement
- $\chi^{(3)}$  (Kerr):  $\phi(V) \propto V^2$ , backaction-evading **microwave energy** measurement





$$G = \frac{g^2 N_{\text{pump}}}{\gamma_a \gamma_b}, \quad g = \nu \frac{\omega_a n^3 r}{2d} \sqrt{\frac{\hbar \omega_b}{2C}}. \quad (17)$$

- Ilchenko *et al.*, JOSAB 20, 333 (2003) ( $\gamma_a \approx 2\pi \times 90$  MHz,  $\gamma_b \approx 2\pi \times 50$  MHz,  $d \approx 150\mu\text{m}$ ):

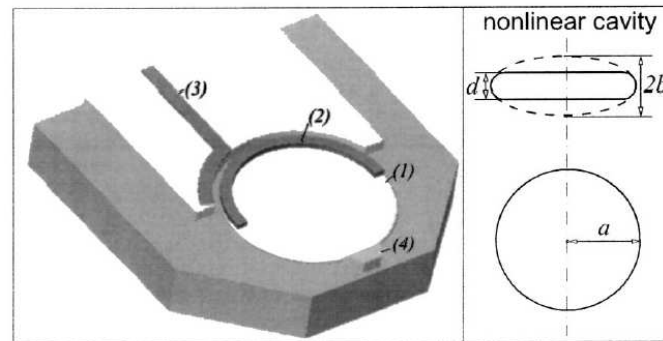


Fig. 1. Experimental setup: (1) LiNbO<sub>3</sub> optical cavity, (2) microwave resonator, (3) microwave feeding strip line, and (4) diamond coupling prism. Inset: geometric characteristics of the nonlinear optical cavity.

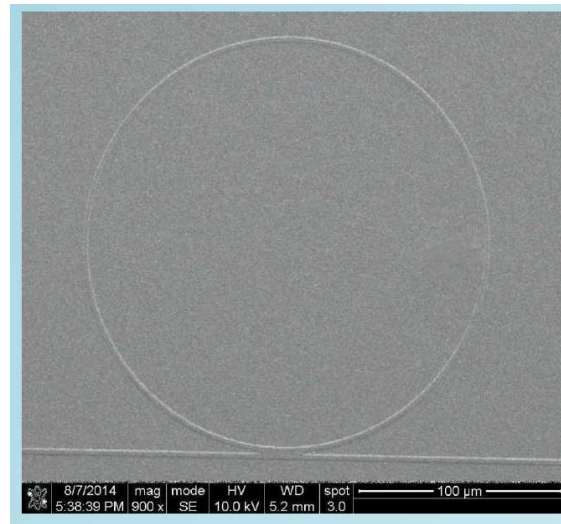
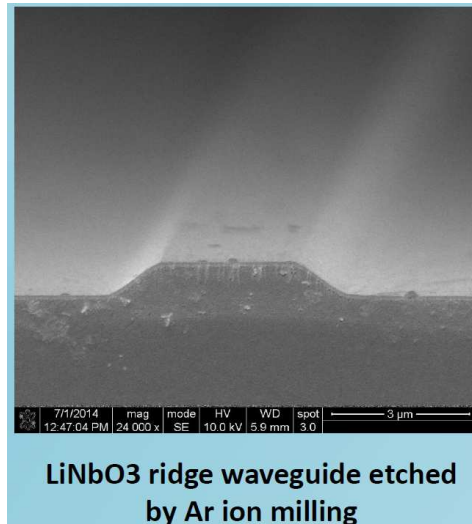
$$g \approx 20 \text{ Hz}, \quad G \approx 2 \times 10^{-5} \text{ at } 2 \text{ mW pump} \quad (18)$$

- $g$  can be improved by  $\sim 10^1 - 10^2$ ,  $\gamma_b$  reduced by  $\sim 10^3$  using superconducting microwave resonator
- $r$  in BaTiO<sub>3</sub> and KTN is higher than LiNbO<sub>3</sub> by  $10^1 - 10^2$

$$i\hbar\psi = H\psi$$

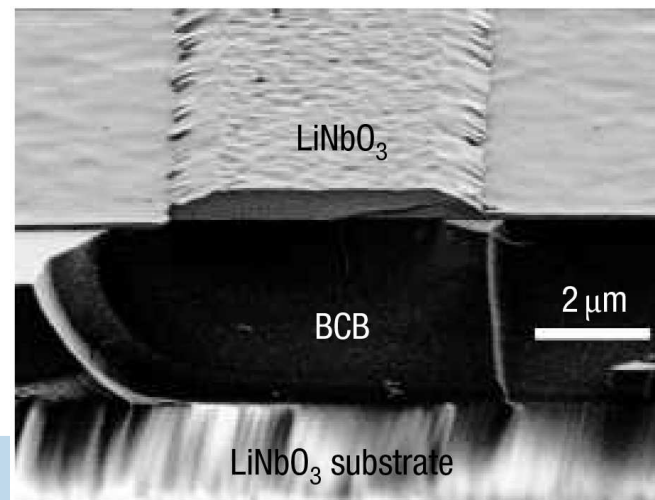
## Experiment

- Collaboration with Aaron Danner's group at NUS
- Ridge waveguide/resonator in  $\text{LiNbO}_3$



Unpublished work by Saha, Yohanes, Deng

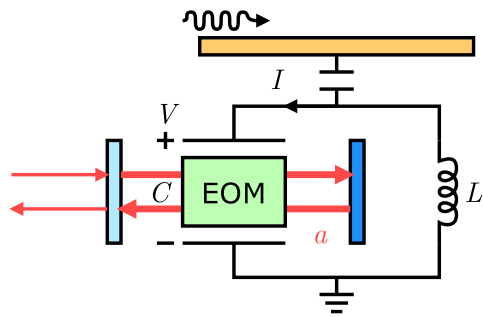
- State-of-the-art:



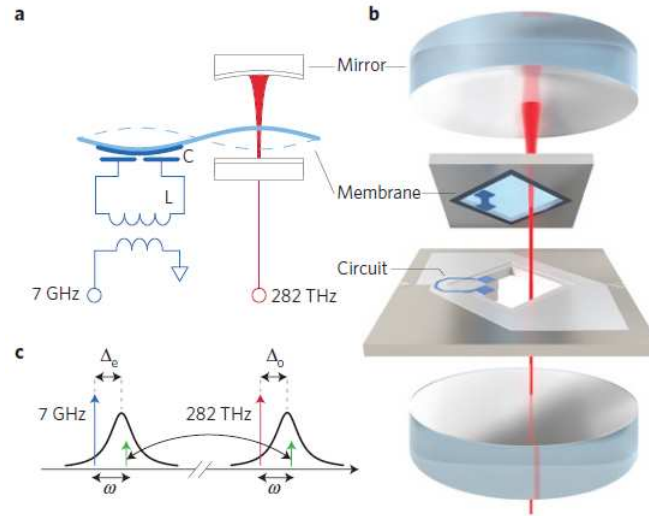
Guarino *et al.*, Nature Photonics 1, 407 (2007).

$$i\hbar\dot{\psi} = H\psi$$

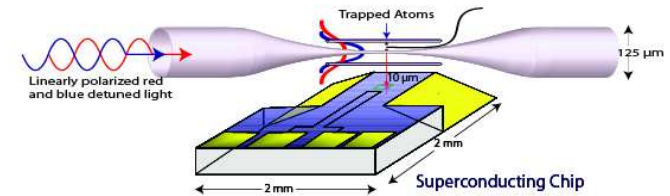
# Competition: electro-optomechanics, atoms



M. Tsang, "Cavity quantum electro-optics," *Phys. Rev. A* **81**, 063837 (2010); **84**, 043845 (2011).



Andrews *et al.*, *Nature Physics* **10**, 321 (2014); News & Views article: Tsang, *Nature Phys.* **10**, 245 (2014)



Hoffman *et al.*, arXiv:1108.4153 (2011); Hafezi *et al.*, *PRA* **85**, 020302(R) (2012)

	Electro-optics	Mechanics	Atoms
Effect	$\chi^{(2)}$	$\chi^{(3)}$	$\chi^{(3)}$
Pumps	optical	optical + microw.	optical + microw.
Resonators	microw. + optical	microw. + optical + mech.	microw. + optical + atoms

$$i\hbar\psi = H\psi$$

## Summary

- Cooling, frequency conversion, parametric amplification/oscillation, entangled photons, BAE quadrature/energy measurements, ...
- Quantum apps require  $G \sim 1$ , technology not there yet but not impossible
- Classical apps: microwave photonics, sensing, metrology, etc.
- M. Tsang, "Cavity quantum electro-optics," *Phys. Rev. A* **81**, 063837 (2010); **84**, 043845 (2011).
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