Supplementary Material for Quantum Transition-Edge Detectors

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Consider a degenerate optical parametric oscillator (OPO) [1]. The equation of motion for the optical-mode analytic signal is

$$\frac{da(t)}{dt} = -\frac{\gamma}{2}a(t) - i\omega_m a(t) + 2\lambda a^*(t) + \sqrt{\gamma}A(t), \tag{1}$$

where γ is the coupling rate, ω_m is the resonance frequency, λ is the pump coefficient, and $A(t)$ is the input field. The output field is given by

$$A_{\text{out}}(t) = \sqrt{\gamma}a(t) - A(t) + A'(t), \tag{2}$$

where A' is an excess noise. Suppose that A_{out} is measured by continuous heterodyne detection, and A and A' are white phase-insensitive noises with noise powers S_{in} and S'. After some lengthy but standard calculations, the output power spectral density is given by

$$S(\omega) = [1 + 2V(\omega)]S_{\text{in}} + S', \tag{3}$$

where $V(\omega)$ is the idler gain. In terms of normalized frequency and parameters,

$$V(\Omega) = \frac{\Gamma^2}{[\Omega^2 - (g^{-2} - 1 - \Gamma^2/4)]^2 + (g^{-2} - 1)\Gamma^2}, \tag{4}$$

$$\Omega \equiv \frac{\omega}{2|\lambda|}, \quad g \equiv \frac{2|\lambda|}{\omega_m}, \quad \Gamma \equiv \frac{\gamma}{2|\lambda|}. \tag{5}$$

To compute the Fisher information for estimating ω_m from A_{out}, we start with the Bhattacharyya distance [2]:

$$B(g, g') = 2|\lambda|t \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \ln \frac{S(\Omega|g) + S(\Omega|g')}{2\sqrt{S(\Omega|g)S(\Omega|g')}}, \tag{6}$$

and find the Fisher information through the identity [2]:

$$\mathcal{G}(\omega_m) = 4\left(\frac{\partial g}{\partial \omega_m}\right)^2 \frac{\partial^2}{\partial g^2} B(g, g')\Big|_{g'=g}. \tag{7}$$

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If the noise powers are quantum-limited,

$$S_{\rm in} = S' = 0.5, \tag{8}$$

$$S(\Omega) = V(\Omega) + 1. \tag{9}$$

After more algebra, we get

$$\mathcal{G}(\omega_m) = \frac{8|\lambda|^3 t}{\omega_m^4}\int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}\left(\frac{\partial V}{\partial g}\right)^2\frac{1}{(V+1)^2}. \tag{10}$$

Focusing on the OPO threshold, which occurs at

$$g = \left(1-\Gamma^2/4\right)^{-1/2}, \tag{11}$$

we obtain

$$\mathcal{G} = \frac{16\omega_m^2 t}{\gamma^3}\int_{-\infty}^{\infty}\frac{dx}{2\pi}\frac{1}{(x^2+1)^2}\frac{1}{[1+\Gamma^2x^2(x^2+1)]^2}. \tag{12}$$

To obtain an analytic result, suppose $\Gamma < 2$, such that we can lower-bound $\mathcal{G}$:

$$\mathcal{G} > \frac{16\omega_m^2 t}{\gamma^3}\int_{-\infty}^{\infty}\frac{dx}{2\pi}\frac{1}{(x^2+1)^2}\frac{1}{[1+4x^2(x^2+1)]^2} = \frac{1.532\omega_m^2 t}{\gamma^3}. \tag{13}$$

In the limit of $\Gamma \to 0$, on the other hand, $\mathcal{G} \to 4\omega_m^2 t/\gamma^3$, so

$$\frac{4\omega_m^2 t}{\gamma^3} > \mathcal{G} > \frac{1.532\omega_m^2 t}{\gamma^3}, \quad 0 < \Gamma < 2, \tag{14}$$

which is the result quoted in the main text. This result suggests that the parameter estimation accuracy can be improved significantly if γ is reduced.

Below threshold, the Fisher information can be investigated by numerical integration using this formula:

$$\mathcal{G} = \frac{16\omega_m^2 t}{\gamma^3\Gamma}\int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}\left[\Omega^2-\left(g^{-2}-1+\Gamma^2/4\right)\right]^2\frac{V^4}{(V+1)^2}. \tag{15}$$

For example, Fig. 1 plots the normalized $\mathcal{G}$ versus g on logarithmic scale for $\Gamma = 0.01$. The plot demonstrates significant enhancement near $g = 1$.

So far all the results are derived for below-threshold operations. If the perturbation causes the threshold to be exceeded, the system becomes unstable, and we can no longer rely on the frequency-domain analysis. The analysis in the main text hints that instability should improve the sensitivity even further, however.

[1] C. W. Gardiner and P. Zoller, *Quantum Noise* (Springer-Verlag, Berlin, 2004).

[2] H. L. Van Trees, *Detection, Estimation, and Modulation Theory, Part III: Radar-Sonar Signal Processing and Gaussian Signals in Noise* (John Wiley & Sons, New York, 2001).

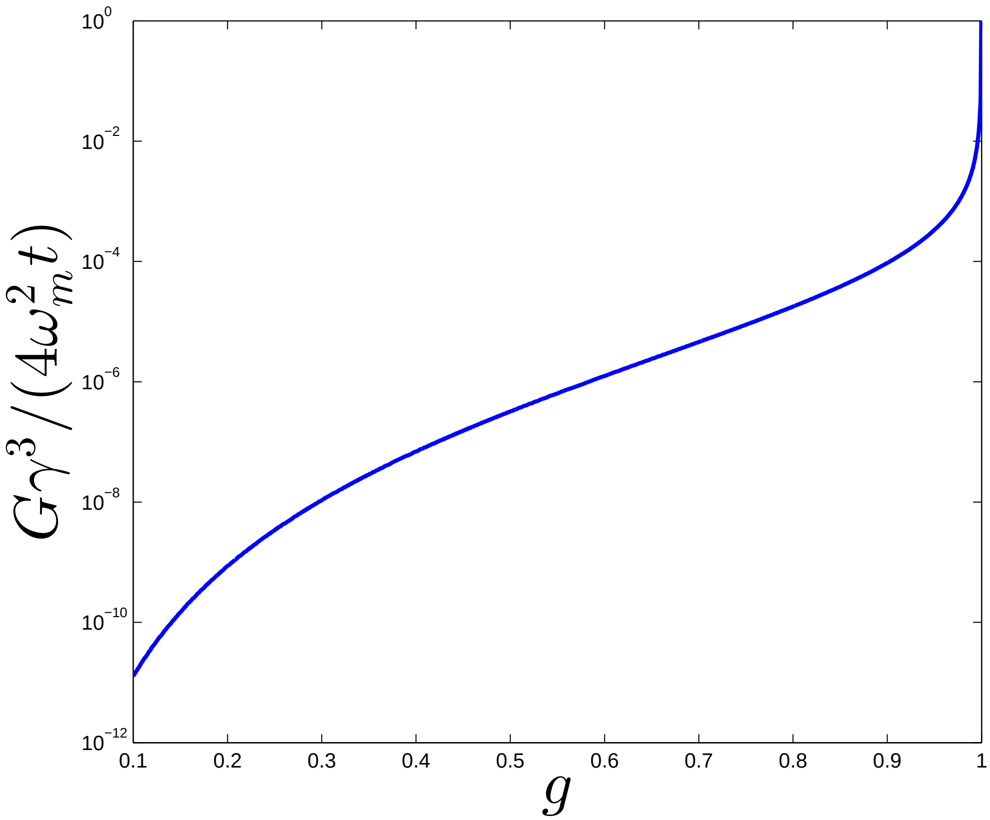

FIG. 1. Normalized Fisher information versus g on logarithmic scale for $\Gamma = 0.01$.