Quantum Nonlocality in Weak-Thermal-Light Interferometry: Supplementary Material

Mankei Tsang*

Department of Electrical and Computer Engineering,

National University of Singapore, 4 Engineering Drive 3, Singapore 117583

Department of Physics, National University of Singapore,

2 Science Drive 3, Singapore 117551 and

Center for Quantum Information and Control,

University of New Mexico, MSC07–4220,

Albuquerque, New Mexico 87131-0001, USA

This Supplementary Material contains supportive calculations and discussions that complement the main text. Section I derives the positive operator-valued measure (POVM) for direct detection, Sec. II derives the POVM for shared-entanglement interferometry and the resulting Fisher information, Sec. III calculates a bound on the total Fisher information for multiple adaptive measurements, Secs. IV and V calculate the Fisher information for heterodyne and homodyne detection, Sec. VI investigates the performances of direct and heterodyne detection for strong thermal light, and Sec. VII discusses the origin of the quantum nonlocality in terms of the semiclassical photodetection picture. The reference list at the end is identical to the one in the main text for easier cross-referencing.

I. POVM FOR DIRECT DETECTION

In direct detection, the optical modes a and b are combined by a beam splitter and photon-counting is performed at the two output ports. Let U be the unitary operator that corresponds to the operation of the beam splitter on the bipartite quantum state ρ. The observation probability distribution is

$$P(n,m|g) = \langle n,m|U\rho U^\dagger|n,m\rangle, \tag{S1}$$

*Electronic address: `eletmk@nus.edu.sg`

where $|n,m\rangle$ is a Fock state. We can then write the POVM as

$$E(n,m) = U^\dagger |n,m\rangle\langle n,m| U, \tag{S2}$$

which propagates the Fock-state projection back to the time when the state of light is ρ.

With at most one photon in the quantum state, we are interested in $(n,m) = (0,0),(1,0),(0,1)$ only. Applying the unitary to the Fock states,

$$\begin{aligned}
U^\dagger|0,0\rangle &= |0,0\rangle, &\text{(S3)}\\
U^\dagger|1,0\rangle &= U^\dagger a^\dagger U U^\dagger |0,0\rangle = U^\dagger a^\dagger U|0,0\rangle &\text{(S4)}\\
&= \frac{1}{\sqrt{2}}\left(a^\dagger + e^{-i\delta} b^\dagger\right)|0,0\rangle &\text{(S5)}\\
&= \frac{1}{\sqrt{2}}\left(|1,0\rangle + e^{-i\delta}|0,1\rangle\right), &\text{(S6)}\\
U^\dagger|0,1\rangle &= \frac{1}{\sqrt{2}}\left(|1,0\rangle - e^{-i\delta}|0,1\rangle\right). &\text{(S7)}
\end{aligned}$$

The POVM is hence

$$\begin{aligned}
E(0,0) &= |0,0\rangle\langle 0,0|, &\text{(S8)}\\
E(1,0) &= \frac{1}{2}\left(|1,0\rangle + e^{-i\delta}|0,1\rangle\right)\left(\langle 1,0| + e^{i\delta}\langle 0,1|\right), &\text{(S9)}\\
E(0,1) &= \frac{1}{2}\left(|1,0\rangle - e^{-i\delta}|0,1\rangle\right)\left(\langle 1,0| - e^{i\delta}\langle 0,1|\right). &\text{(S10)}
\end{aligned}$$

II. SHARED-ENTANGLEMENT INTERFEROMETRY

Assuming an entangled ancilla in two modes c and d given by

$$|\delta\rangle \equiv \frac{1}{\sqrt{2}}\left(|0,1\rangle_{c,d} + e^{i\delta}|1,0\rangle_{c,d}\right), \tag{S11}$$

with each mode sent to the sites of a and b modes for separate interference measurements [9], the POVM for photon counts (n,m,n',m') is

$$E(n,m,n',m') = \langle\delta|U_{ac}^\dagger \otimes U_{bd}^\dagger|n,m,n',m'\rangle\langle n,m,n',m'|U_{ac}\otimes U_{bd}|\delta\rangle, \tag{S12}$$

where U_{ac} denotes the beam-splitting unitary on modes a and c and U_{bd} denotes the same unitary on modes b and d. The calculation is more involved but similar to the one for direct detection. The final result is

$$E(y_0) = |0,0\rangle\langle 0,0|, \tag{S13}$$

$$E(y_1) = \frac{1}{2}|0,1\rangle\langle 0,1|, \tag{S14}$$

$$E(y_2) = \frac{1}{2}|1,0\rangle\langle 1,0|, \tag{S15}$$

$$E(y_3) = \frac{1}{4}\left(|1,0\rangle + e^{-i\delta}|0,1\rangle\right)\left(\langle 1,0| + e^{i\delta}\langle 0,1|\right), \tag{S16}$$

$$E(y_4) = \frac{1}{4}\left(|1,0\rangle - e^{-i\delta}|0,1\rangle\right)\left(\langle 1,0| - e^{i\delta}\langle 0,1|\right), \tag{S17}$$

where each y_j corresponds to a set of (n, m, n', m') that produce the same POVM. When applied to the quantum state ρ, only observations y_3 and y_4 contribute to the Fisher information about g. Since $E(y_3) = E(1,0)/2$ and $E(y_4) = E(0,1)/2$, the Fisher information for shared-entanglement interferometry is simply that for direct detection reduced by a factor of 2:

$$F = \frac{\epsilon}{2[1 - \mathrm{Re}(ge^{-i\delta})^2]}\begin{pmatrix} \cos^2\delta & \sin\delta\cos\delta \\ \sin\delta\cos\delta & \sin^2\delta \end{pmatrix}. \tag{S18}$$

III. ADAPTIVE MEASUREMENTS

For M measurements, the joint observation probability distribution can be written as

$$P(y_M, \ldots, y_1|g) = P(y_M|g, y_{M-1}, \ldots, y_1)P(y_{M-1}, \ldots, y_1|g). \tag{S19}$$

Each element of the total Fisher information matrix for M measurements, using an alternate form of the Fisher matrix [15], becomes

$$F_{jk}^{(M)} \equiv -\sum_{y_1,\ldots,y_M} P(y_M, \ldots, y_1|g)\frac{\partial^2}{\partial g_j \partial g_k}\ln P(y_M, \ldots, y_1|g) \tag{S20}$$

$$= -\sum_{y_1,\ldots,y_M} P(y_M|g, y_{M-1}, \ldots, y_1)P(y_{M-1}, \ldots, y_1|g) \times \left[\frac{\partial^2}{\partial g_j \partial g_k}\ln P(y_M|g, y_{M-1}, \ldots, y_1) + \frac{\partial^2}{\partial g_j \partial g_k}\ln P(y_{M-1}, \ldots, y_1|g)\right] \tag{S21}$$

$$= \sum_{y_1,\ldots,y_{M-1}} P(y_{M-1}, \ldots, y_1|g)F_{Mjk}(y_{M-1}, \ldots, y_1) + F_{jk}^{(M-1)}, \tag{S22}$$

where F_M denotes the conditional Fisher information of the Mth measurement:

$$F_{Mjk}(y_{M-1}, \ldots, y_1) \equiv -\sum_{y_M} P(y_M|g, y_{M-1}, \ldots, y_1)\frac{\partial^2}{\partial g_j \partial g_k}\ln P(y_M|g, y_{M-1}, \ldots, y_1). \tag{S23}$$

Applying the subadditivity property of matrix norms,

$$||F^{(M)}|| \leq \sum_{y_1,\ldots,y_{M-1}} P(y_{M-1},\ldots,y_1|g)||F_M|| + ||F^{(M-1)}|| \tag{S24}$$

$$\leq \max_{y_1,\ldots,y_{M-1}} ||F_M|| + ||F^{(M-1)}||, \tag{S25}$$

and by induction,

$$||F^{(M)}|| \leq \sum_{m=1}^{M} \max_{y_1,\ldots,y_{m-1}} ||F_m||. \tag{S26}$$

This proves that the norm of the total Fisher information cannot exceed the sum of the maximized single-measurement values.

For the mth quantum measurement with outcome y_m conditioned upon previous observations, we can write

$$P(y_m|g, y_{m-1},\ldots,y_1) = \mathrm{tr}\left[E(y_m|y_{m-1},\ldots,y_1)\rho\right]. \tag{S27}$$

This means that the bound given by Eq. (23) in the main text for spatial-LOCC measurements in the case of $\epsilon \ll 1$ is also applicable to $||F_m||$:

$$||F_m|| \leq \epsilon^2 + O(\epsilon^3). \tag{S28}$$

The total Fisher information is hence bounded by

$$||F^{(M)}|| \leq M\left[\epsilon^2 + O(\epsilon^3)\right], \tag{S29}$$

which generalizes the bound to the case of multiple spatiotemporal-LOCC measurements and proves that no adaptive strategy can improve the scaling $||F^{(M)}|| \sim M\epsilon^2$.

IV. HETERODYNE DETECTION

The POVM for heterodyne detection is [14]

$$E(\mu,\nu) = \frac{1}{\pi^2}|\mu,\nu\rangle\langle\mu,\nu|, \tag{S30}$$

where $|\mu,\nu\rangle$ is a coherent state and the normalization is $\int d^2\mu d^2\nu E(\mu,\nu) = I$, the identity operator. The relevant POVM matrix elements are

$$E_{00,00}(\mu,\nu) \equiv \frac{1}{\pi^2}\left|\langle 0,0|\mu,\nu\rangle\right|^2 \tag{S31}$$
$$= \frac{1}{\pi^2}\exp\left(-|\mu|^2-|\nu|^2\right), \tag{S32}$$
$$E_{01,01}(\mu,\nu) \equiv \frac{1}{\pi^2}\left|\langle 0,1|\mu,\nu\rangle\right|^2 \tag{S33}$$
$$= \frac{1}{\pi^2}\exp\left(-|\mu|^2-|\nu|^2\right)|\nu|^2, \tag{S34}$$
$$E_{10,10}(\mu,\nu) \equiv \frac{1}{\pi^2}\left|\langle 1,0|\mu,\nu\rangle\right|^2 \tag{S35}$$
$$= \frac{1}{\pi^2}\exp\left(-|\mu|^2-|\nu|^2\right)|\mu|^2, \tag{S36}$$
$$E_{01,10}(\mu,\nu) \equiv \frac{1}{\pi^2}\langle 0,1|\mu,\nu\rangle\langle\mu,\nu|1,0\rangle \tag{S37}$$
$$= \frac{1}{\pi^2}\exp\left(-|\mu|^2-|\nu|^2\right)\mu^*\nu. \tag{S38}$$

The Fisher information is hence

$$F = \frac{\epsilon^2}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + O(\epsilon^3), \tag{S39}$$
$$||F|| = \epsilon^2 + O(\epsilon^3). \tag{S40}$$

This shows that the performance of heterodyne detection is already the optimum allowed by quantum mechanics for any local measurement according to the bound given by Eq. (23) in the main text.

V. HOMODYNE DETECTION

For homodyne detection,

$$E(x,y) = |x,y\rangle\langle x,y|, \tag{S41}$$

where $|x,y\rangle$ is a quadrature eigenstate:

$$\frac{1}{\sqrt{2}}\left(ae^{-i\delta_a}+a^\dagger e^{i\delta_a}\right)|x,y\rangle = x|x,y\rangle, \tag{S42}$$
$$\frac{1}{\sqrt{2}}\left(be^{-i\delta_b}+b^\dagger e^{i\delta_b}\right)|x,y\rangle = y|x,y\rangle, \tag{S43}$$

δ_a and δ_b are local-oscillator phases, and the normalization is $\int dxdyE(x,y) = I$. The relevant POVM elements are

$$E_{00,00}(x,y) = \frac{1}{\pi}\exp\left(-x^2 - y^2\right), \tag{S44}$$

$$E_{01,01}(x,y) = \frac{2}{\pi}\exp\left(-x^2 - y^2\right)y^2, \tag{S45}$$

$$E_{10,10}(x,y) = \frac{2}{\pi}\exp\left(-x^2 - y^2\right)x^2, \tag{S46}$$

$$E_{10,01}(x,y) = \frac{2}{\pi}e^{i\delta}\exp\left(-x^2 - y^2\right)xy, \tag{S47}$$

where $\delta \equiv \delta_a - \delta_b$. The Fisher information becomes

$$F = \epsilon^2 \begin{pmatrix} \cos^2\delta & \sin\delta\cos\delta \\ \sin\delta\cos\delta & \sin^2\delta \end{pmatrix} + O(\epsilon^3), \tag{S48}$$

$$||F|| = \epsilon^2 + O(\epsilon^3). \tag{S49}$$

Homodyne detection is also able to saturate the bound given by Eq. (23) in the main text, but each measurement gives information about only one quadrature of g and δ should be varied over measurements to estimate both quadratures.

Qualitatively, the inferior Fisher information for heterodyne and homodyne detection can be attributed to the non-zero vacuum fluctuations even when no photon is coming in to provide information about the unknown parameters. Nonlocal measurements are able to perfectly discriminate against this case and discard the useless observations, but heterodyne or homodyne detection is unable to do so and forced to include vacuum fluctuations as potentially useful observations, resulting in a substantially worse estimation accuracy in the long run.

VI. THERMAL LIGHT WITH ARBITRARY ϵ

For $\epsilon \gtrsim 1$, it is necessary to use the full thermal state given by Eq. (1) in the main text. First consider the observation probability density of heterodyne detection:

$$P(\mu,\nu|g) = \int d^2\alpha d^2\beta\,\Pi(\mu,\nu|\alpha,\beta)\Phi(\alpha,\beta|g), \tag{S50}$$

$$\Pi(\mu,\nu|\alpha,\beta) \equiv \langle\alpha,\beta|E(\mu,\nu)|\alpha,\beta\rangle \tag{S51}$$

$$= \frac{1}{\pi^2}\exp\left(-|\mu-\alpha|^2 - |\nu-\beta|^2\right). \tag{S52}$$

We can interpret these expressions using a semiclassical photodetection picture [1]: The heterodyne detection statistics obey $\Pi(\mu,\nu|\alpha,\beta)$ for given classical fields (α,β), but the fields from the source also have a statistical distribution given by $\Phi(\alpha,\beta)$, so the marginal observation density is taken to be Π averaged over Φ. The resulting convolution of the two Gaussians can be calculated analytically and given by

$$P(\mu,\nu|g) = \frac{1}{\pi^2 \det \Gamma'} \exp\left[-\begin{pmatrix} \mu^* & \nu^* \end{pmatrix} \Gamma'^{-1} \begin{pmatrix} \mu \\ \nu \end{pmatrix}\right], \tag{S53}$$

where the new covariance matrix Γ' is

$$\Gamma' = \Gamma + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \epsilon/2+1 & \epsilon g/2 \\ \epsilon g^*/2 & \epsilon/2+1 \end{pmatrix}. \tag{S54}$$

The factors of 1 come from Π and represent detection noise. When $\epsilon \gg 1$, Π is much sharper than Φ, so the convolution essentially reproduces Φ as the marginal observation density and $P(\mu,\nu|g) \approx \Phi(\mu,\nu|g)$. In other words, the inherent thermal noise from the source overwhelms the heterodyne detection noise in the $\epsilon \gg 1$ regime.

To estimate the performance of heterodyne detection, we can consider the signal-to-noise ratio (SNR) [3, 6]. If we take $\mu\nu^*$ as the output signal, $\langle \mu\nu^* \rangle = \epsilon g/2$ directly gives g on average, and the signal energy is

$$S \equiv |\langle \mu\nu^* \rangle|^2 = \frac{\epsilon^2 |g|^2}{4}. \tag{S55}$$

The noise energy is

$$N \equiv \left\langle |\mu\nu^*|^2 \right\rangle - S. \tag{S56}$$

The fourth-order field statistics can be computed with the help of the matrix $G \equiv \Gamma'^{-1}$:

$$\det G = G_{aa}G_{bb} - G_{ab}G_{ba}, \tag{S57}$$

$$\left\langle |\mu\nu^*|^2 \right\rangle = \det G \frac{\partial^2}{\partial G_{aa}\partial G_{bb}} \frac{1}{\det G} \tag{S58}$$

$$= \frac{2G_{aa}G_{bb}}{\det G^2} - \frac{1}{\det G} \tag{S59}$$

$$= \left(1+\frac{\epsilon}{2}\right)^2 + \frac{\epsilon^2 |g|^2}{4}, \tag{S60}$$

$$N = \left(1+\frac{\epsilon}{2}\right)^2. \tag{S61}$$

The SNR is hence

$$\frac{S}{N} = \frac{\epsilon^2 |g|^2}{(2+\epsilon)^2}. \tag{S62}$$

For $\epsilon \ll 1$, $S/N \approx \epsilon^2 |g|^2/4$, but for $\epsilon \gg 1$, the SNR saturates to $S/N \to |g|^2$ and becomes independent of ϵ.

For direct detection,

$$P(n,m|g) = \int d^2\alpha d^2\beta \Pi(n,m|\alpha,\beta)\Phi(\alpha,\beta|g), \tag{S63}$$

$$\Pi(n,m|\alpha,\beta) \equiv \langle \alpha,\beta|E(n,m)|\alpha,\beta\rangle = |\langle n,m|u,v\rangle|^2, \tag{S64}$$

$$\begin{pmatrix} u \\ v \end{pmatrix} \equiv V \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad V \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\delta} \\ 1 & -e^{i\delta} \end{pmatrix}. \tag{S65}$$

Changing the integration variables to (u,v),

$$P(n,m|g) = \int d^2u d^2v \Pi'(n,m|u,v)\Phi'(u,v|g), \tag{S66}$$

$$\Pi'(n,m|u,v) \equiv \exp(-|u|^2)\frac{|u|^{2n}}{n!}\exp(-|v|^2)\frac{|v|^{2m}}{m!}, \tag{S67}$$

$$\Phi'(u,v|g) \equiv \frac{\det K}{\pi^2}\exp\left[-\begin{pmatrix} u^* & v^* \end{pmatrix} K \begin{pmatrix} u \\ v \end{pmatrix}\right], \tag{S68}$$

$$K \equiv V\Gamma^{-1}V^\dagger = \frac{2}{\epsilon(1-|g|^2)}\begin{pmatrix} 1-\mathrm{Re}(ge^{-i\delta}) & i\,\mathrm{Im}(ge^{-i\delta}) \\ -i\,\mathrm{Im}(ge^{-i\delta}) & 1+\mathrm{Re}(ge^{-i\delta}) \end{pmatrix}, \tag{S69}$$

$$\det K = K_{aa}K_{bb} - K_{ab}K_{ba} = \frac{4}{\epsilon^2(1-|g|^2)}. \tag{S70}$$

The averaging of a Poissonian with a Gaussian is difficult to calculate exactly. For $\epsilon \gg 1$, however, the photon counts (n,m) should be large most of the time, and $P(n,m|g)$ may be approximated by a Gaussian. Let us therefore focus on the first and second moments of (n,m) for $P(n,m|g)$. The first moment of n is

$$\langle n\rangle = \sum_{n,m} nP(n,m|g) \tag{S71}$$

$$= \int d^2u d^2v |u|^2 \Phi'(u,v|g) \tag{S72}$$

$$= -\det K \frac{\partial}{\partial K_{aa}}\frac{1}{\det K}. \tag{S73}$$

Similarly,

$$\langle m\rangle = -\det K \frac{\partial}{\partial K_{bb}} \frac{1}{\det K}, \tag{S74}$$

$$\left\langle n^2\right\rangle = \langle n\rangle + \det K \frac{\partial^2}{\partial K_{aa}^2} \frac{1}{\det K}, \tag{S75}$$

$$\left\langle m^2\right\rangle = \langle m\rangle + \det K \frac{\partial^2}{\partial K_{bb}^2} \frac{1}{\det K}, \tag{S76}$$

$$\langle nm\rangle = \det K \frac{\partial^2}{\partial K_{aa}\partial K_{bb}} \frac{1}{\det K}. \tag{S77}$$

This gives

$$\langle n\rangle = \frac{\epsilon}{2}\left[1 + \mathrm{Re}(ge^{-i\delta})\right], \tag{S78}$$

$$\langle m\rangle = \frac{\epsilon}{2}\left[1 - \mathrm{Re}(ge^{-i\delta})\right], \tag{S79}$$

$$\left\langle \Delta n^2\right\rangle = \langle n\rangle + \langle n\rangle^2, \tag{S80}$$

$$\left\langle \Delta m^2\right\rangle = \langle m\rangle + \langle m\rangle^2, \tag{S81}$$

$$\langle \Delta n \Delta m\rangle = \frac{\epsilon^2}{4}\mathrm{Im}(ge^{-i\delta})^2. \tag{S82}$$

A behavior similar to the case of heterodyne detection can be seen here. For $\langle n\rangle, \langle m\rangle \sim \epsilon \gg 1$, the noise covariances scale as ϵ^2 rather than ϵ, indicating that the source thermal noise also overwhelms the Poissonian detection noise.

Since the observation statistics are expected to be approximately Gaussian for $\epsilon \gg 1$, we can similarly consider the SNR as a performance metric. Taking the output signal as $n-m$, the average of which gives $\langle n-m\rangle = \epsilon\,\mathrm{Re}(ge^{-i\delta})$, a quadrature of g, the signal energy is

$$S = \langle n-m\rangle^2 = \epsilon^2\,\mathrm{Re}(ge^{-i\delta})^2, \tag{S83}$$

and the noise energy is

$$N = \left\langle (n-m)^2\right\rangle - S \tag{S84}$$

$$= \left\langle \Delta n^2\right\rangle + \left\langle \Delta m^2\right\rangle - 2\left\langle \Delta n\Delta m\right\rangle \tag{S85}$$

$$= \epsilon + \frac{\epsilon^2}{2}\left[1 + \mathrm{Re}(ge^{-i\delta})^2 - \mathrm{Im}(ge^{-i\delta})^2\right]. \tag{S86}$$

If we perform two measurements, one with $\delta = \delta_1$ and one with $\delta = \delta_1 + \pi/2$ to measure the other quadrature of g, the average signal and noise energies per measurement becomes

$$\bar{S} = \frac{\epsilon^2|g|^2}{2}, \tag{S87}$$

$$\bar{N} = \epsilon + \frac{\epsilon^2}{2}, \tag{S88}$$

and the average SNR is

$$\frac{\bar{S}}{\bar{N}} = \frac{\epsilon|g|^2}{2+\epsilon}. \tag{S89}$$

For $\epsilon \gg 1$, the SNR saturates to $|g|^2$, just like the SNR of heterodyne detection, suggesting that the SNR is dominated by source thermal noise regardless of the detection method and nonlocal measurements do not have an advantage when $\epsilon \gg 1$.

For $\epsilon \ll 1$, the SNR is still a valid performance metric for a large number of measurements, in which case the statistics become approximately Gaussian by the central limit theorem and averaging M observations improves the final SNR by a factor of M. The direct-detection SNR is $\approx M\epsilon|g|^2/2$ and significantly better than the heterodyne SNR $\approx M\epsilon^2|g|^2/4$, a fact well known in astronomy [3, 6] and rigorously generalized in this paper.

VII. QUANTUM ORIGIN OF MEASUREMENT NONLOCALITY IN THE SEMICLASSICAL PHOTODETECTION PICTURE

One may well wonder where quantum mechanics comes in, if both Φ and Π are nonnegative and the whole problem obeys classical statistics in the semiclassical photodetection picture. The answer lies in the fact that the likelihood function

$$\Pi(y|\alpha,\beta) \equiv \langle\alpha,\beta|E(y)|\alpha,\beta\rangle \tag{S90}$$

cannot be an arbitrarily sharp probability distribution in quantum mechanics. It is the Husimi representation [1], more commonly applied to a quantum state but here to a POVM.

If we regard $\Pi(y|\alpha,\beta)$ as a likelihood function of (α,β) for a given observation y, the sharpness of $\Pi(y|\alpha,\beta)$ with respect to (α,β) in phase space characterizes the amount of information about (α,β) contained in the observation y. The Husimi representation has a maximum magnitude and a finite variance for each quadrature, which means that there is a limited amount of information about the fields that an observation can provide.

The information of mutual coherence lies only in the nonlocal second-order field correlation $\alpha\beta^*$ for thermal light, the first-order mean fields of which are zero. If $E(y)$ corresponds to a local measurement and is separable into $E_a(y) \otimes E_b(y)$, the sharpness of $\Pi(y|\alpha,\beta) = \Pi_a(y|\alpha)\Pi_b(y|\beta)$ with respect to $\alpha\beta^*$ would be more limited than that allowed by nonlocal measurements, meaning that local measurements extract less information about the coherence than nonlocal measurements. In this sense, the measurement nonlocality can be regarded as a dual of Einstein-Podolsky-Rosen entanglement [4, 10]; the former is a property of bipartite measurements that can extract more information from certain separable states and the latter a property of bipartite states that can produce higher correlations in certain separable measurements.

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