Quantum Temporal Imaging

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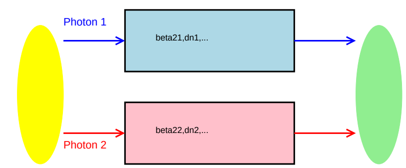
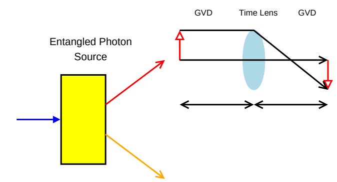
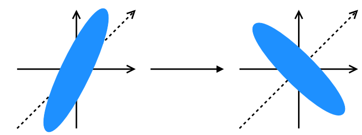
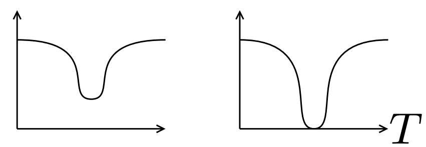
Outline

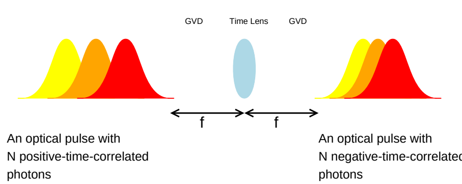








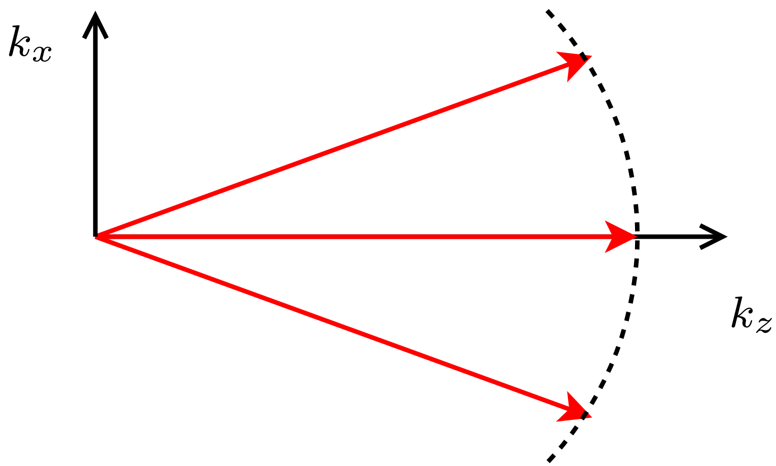
- Basic Principles
- Converting 2 positive-time-correlated photons to 2 negative-time-correlated photons
- Enhancing the temporal indistinguishability of two photons
- Converting N positive-time-correlated photons to N negative-time-correlated photons



Spatial Imaging

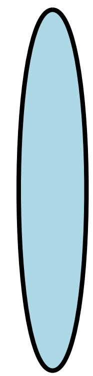
Fresnel Diffraction

$$k_z = \sqrt{k_0^2 - k_x^2} \approx k_0 - \frac{ik_x^2}{2k_0}$$



$$\boxed{\frac{\partial A}{\partial z} = \frac{i}{2k_0}\frac{\partial^2 A}{\partial x^2}}$$

Lens



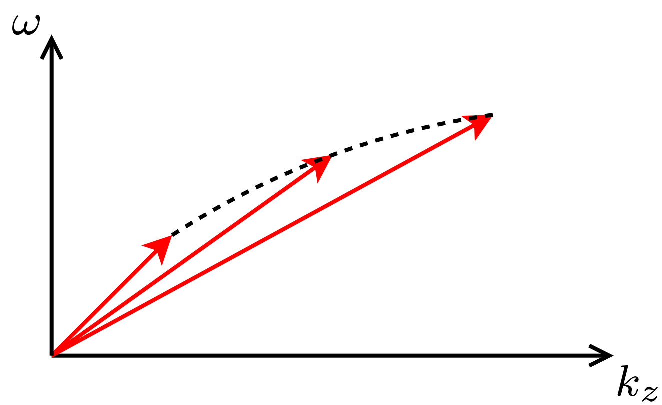
$$\boxed{A(z) = \exp\left(-j\,\frac{k_0}{2f}x^2\right)A(0)}$$

Temporal Imaging

Kolner and Nazarathy, Opt. Lett. **14**, 630 (1989).

Group-Velocity Dispersion

$$k = \beta_0 + \beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2$$

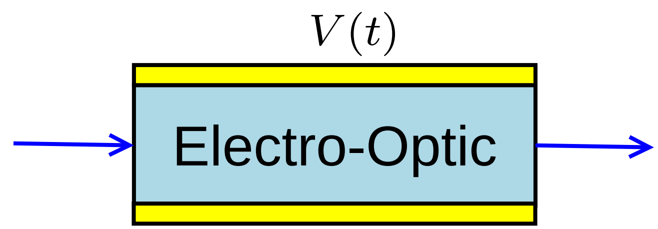


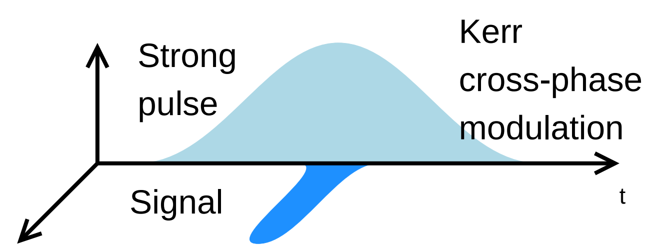
$$\boxed{\frac{\partial A}{\partial z} = -\frac{i\beta_2}{2}\frac{\partial^2 A}{\partial \tau^2}}$$

$$\tau = t - \beta_1 z$$

Time Lens

$$\Delta n(\tau) \sim \tau^2$$





$$\boxed{\frac{\partial A}{\partial z} = ik_0\Delta n(\tau)A}$$

Quantum Temporal Information Processing

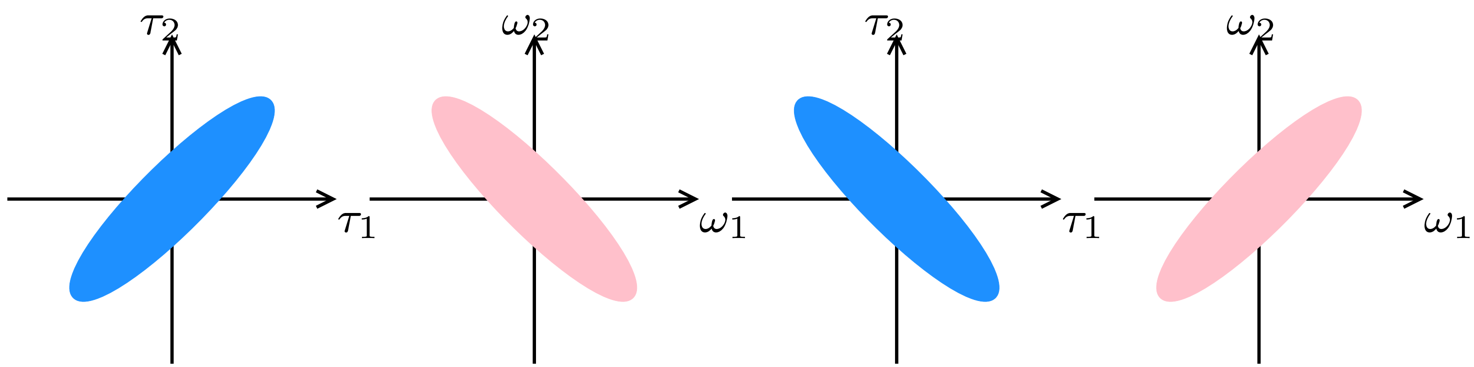
- Instead of manipulating the optical envelope $A(z,\tau)$ classically, we'd like to manipulate the biphoton probability amplitude $\psi(\tau_1,\tau_2)$ of two entangled photons:

$$|\Psi\rangle = \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2\, \psi(\tau_1,\tau_2)\hat{A}_1^\dagger(\tau_1)\hat{A}_2^\dagger(\tau_2)|0,0\rangle \tag{1}$$

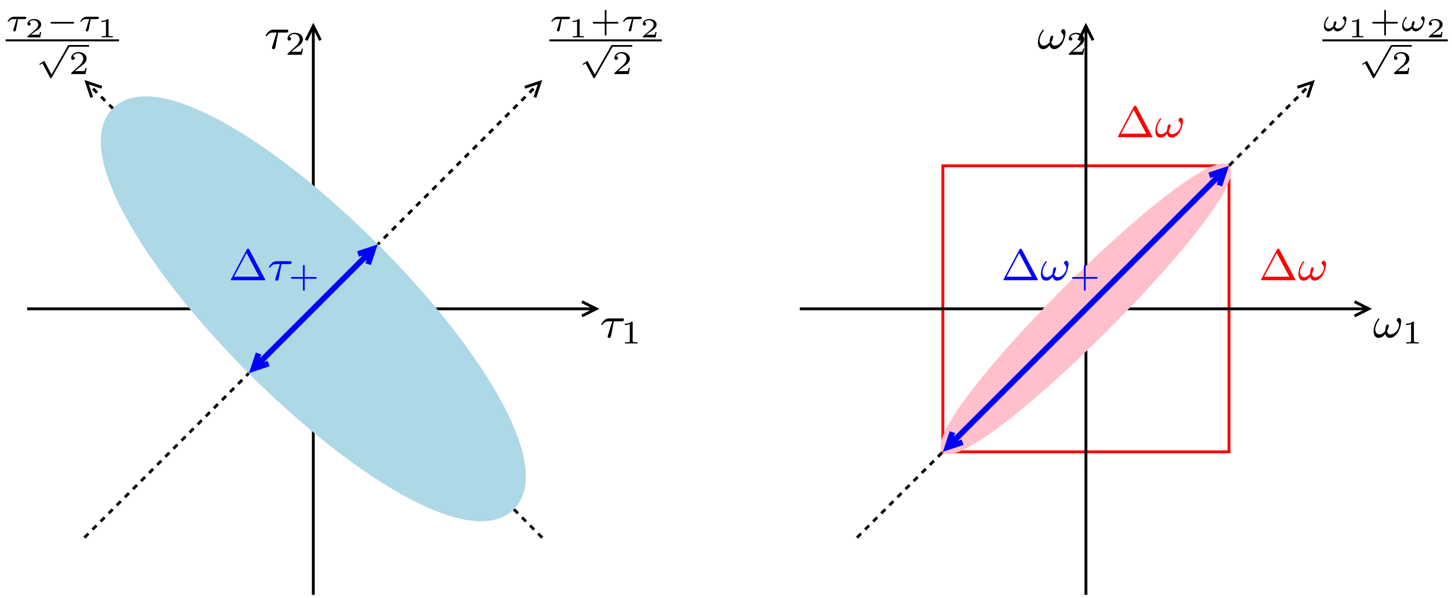
$$\psi(\tau_1,\tau_2) = \langle 0|\hat{A}_1(\tau_1)\hat{A}_2(\tau_2)|\Psi\rangle \tag{2}$$

$$\phi(\omega_1,\omega_2) = \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2\, \psi(\tau_1,\tau_2)\exp(i\omega_1\tau_1 + i\omega_2\tau_2) \tag{3}$$

- $|\psi(\tau_1,\tau_2)|^2$ is the probability distribution of finding photon 1 at τ_1 and finding photon 2 at τ_2.

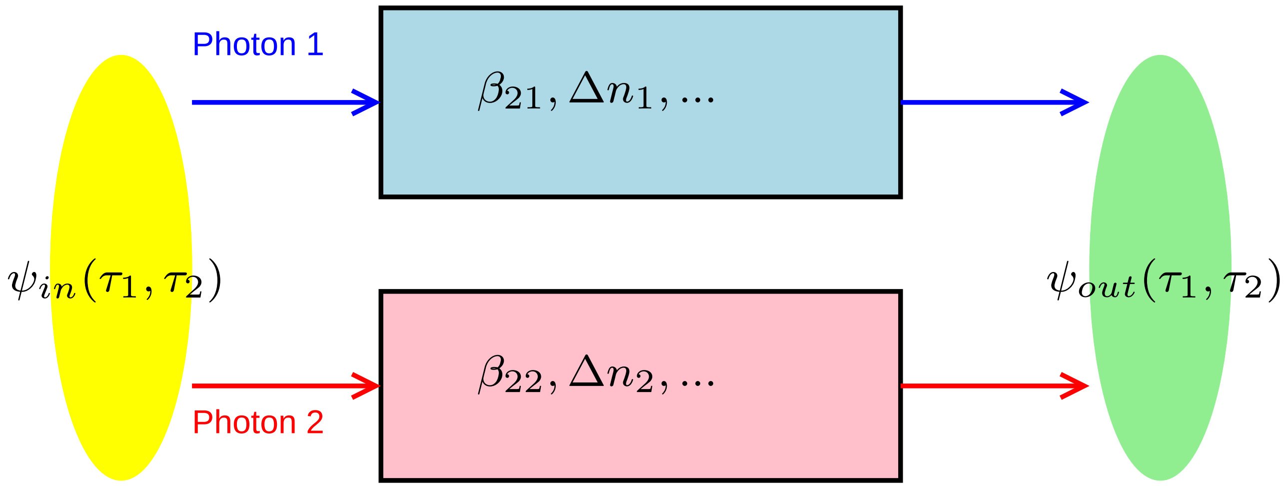


Negative Time Correlation



- Negative time correlation is useful for e.g. one-way autocompensating quantum cryptography (Walton et al. PRA **67**, 062309 (2003)) and quantum enhancement of timing accuracy (Giovannetti et al. Nature **412**, 417 (2001)).
- Analogous to mutual fund, negative-correlated stocks minimize risk of investment.
- Producing 2 negative-time-correlated photons requires an ultrashort pump pulse
- Nobody yet knows how to produce N negative-time-correlated photons.

Quantum Temporal Imaging

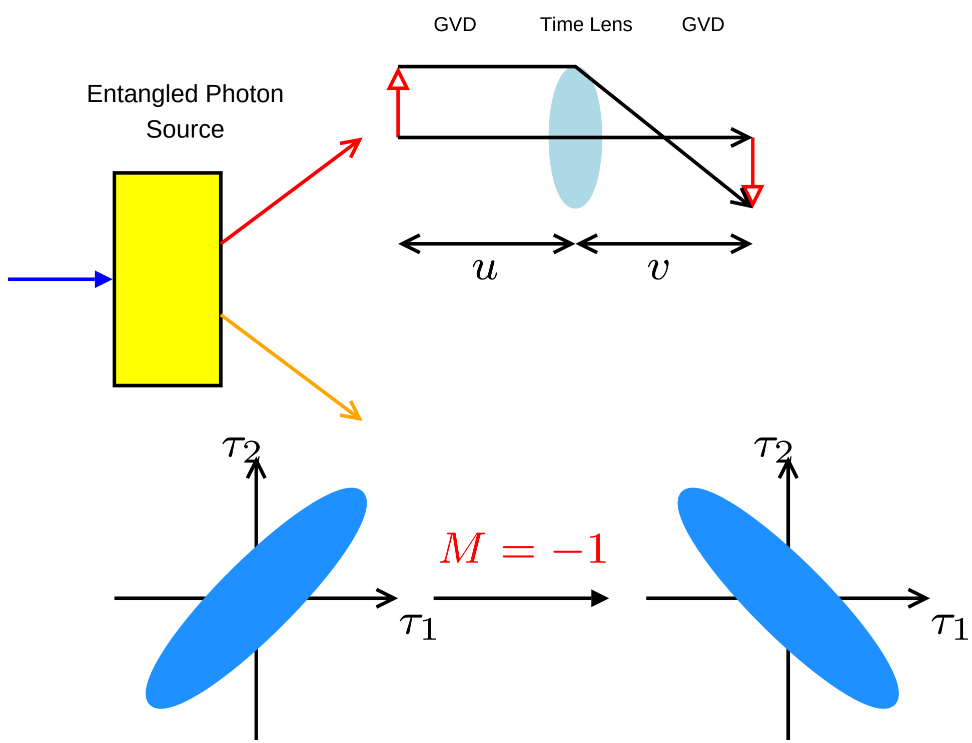


$$\psi(\tau_1, \tau_2) = \langle 0|\hat{A}_1(\tau_1)\hat{A}_2(\tau_2)|\Psi\rangle \tag{4}$$

$$\frac{\partial}{\partial z_1}\psi(z_1, z_2, \tau_1, \tau_2) = \left[\frac{j\beta_{21}(z_1)}{2}\frac{\partial^2}{\partial {\tau_1}^2} + jk_0\Delta n_1(z_1, \tau_1)\right]\psi(z_1, z_2, \tau_1, \tau_2) \tag{5}$$

$$\frac{\partial}{\partial z_2}\psi(z_1, z_2, \tau_1, \tau_2) = \left[\frac{j\beta_{22}(z_2)}{2}\frac{\partial^2}{\partial {\tau_2}^2} + jk_0\Delta n_2(z_2, \tau_2)\right]\psi(z_1, z_2, \tau_1, \tau_2) \tag{6}$$

Time Reversal

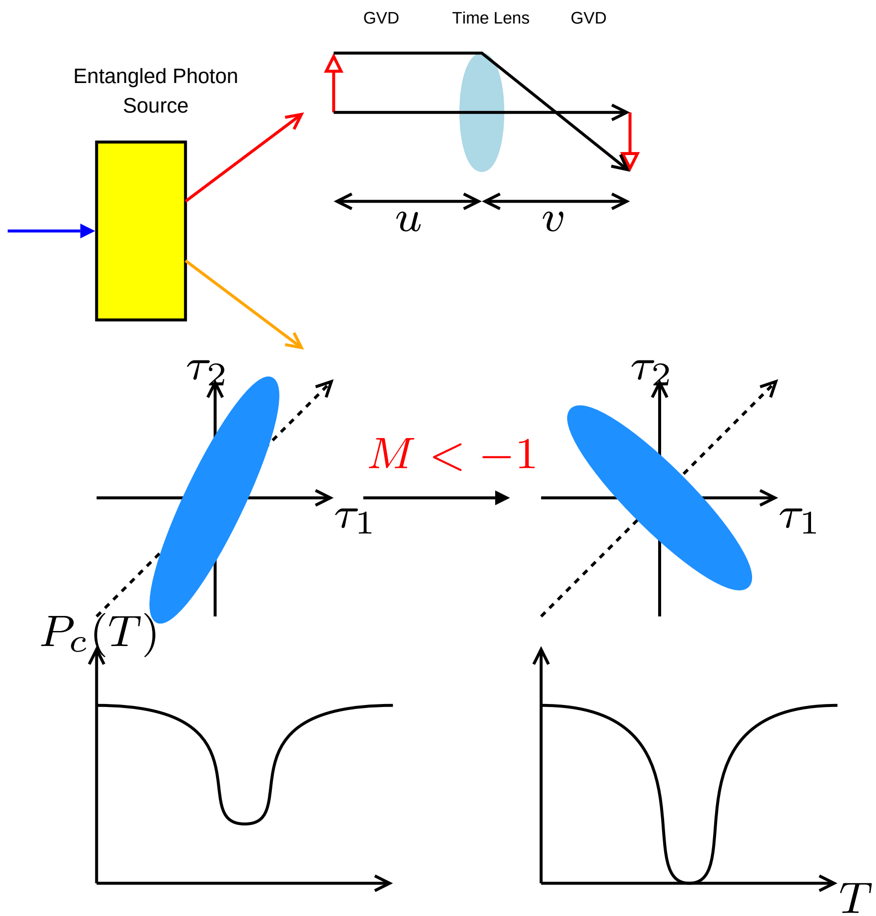


$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}, \quad M = -\frac{v}{u}, \tag{7}$$

$$\psi(\tau_1, \tau_2) \to \psi(\frac{\tau_1}{M}, \tau_2) = \psi(-\tau_1, \tau_2) \tag{8}$$

- create negative-time-correlated photons without an ultrashort pump pulse.

Enhancing Temporal Indistinguishability



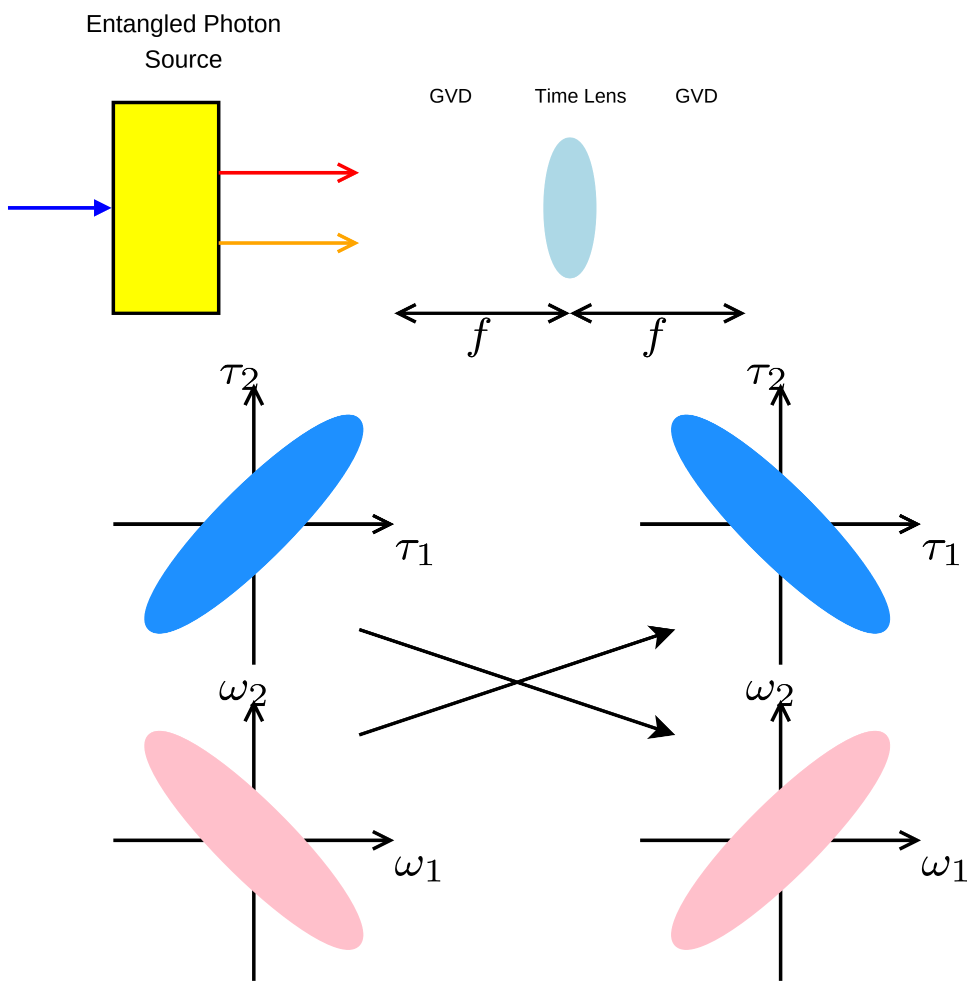
$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}, \qquad M=-\frac{v}{u}, \qquad (9)$$

$$\psi(\tau_1,\tau_2)\rightarrow\psi\left(\frac{\tau_1}{M},\tau_2\right) \qquad (10)$$

$$P_c(0)\propto\int_{-\infty}^{\infty}d\tau_1\int_{-\infty}^{\infty}d\tau_2|\psi(\tau_1,\tau_2)-\psi(\tau_2,\tau_1)|^2$$

- Enhance the temporal indistinguishability of two photons and therefore enhance the visibility of Hong-Ou-Mandel dip

Biphoton Fourier Transform

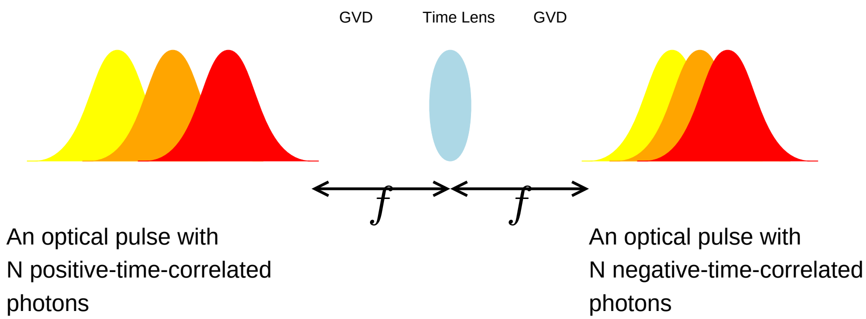


$$u = v = f, \tag{11}$$

$$\psi(\tau_1, \tau_2) \rightarrow \phi\left(\frac{\tau_1}{\beta_2 f}, \frac{\tau_2}{\beta_2 f}\right) \tag{12}$$

- continuous-variable Hadamard gate
- Can also convert between positive time correlation and negative time correlation

Multiphoton Fourier Transform for N photons



$$\psi(\tau_1, \ldots, \tau_N) \rightarrow \phi\left(\frac{\omega_1}{\beta_2 f}, \ldots, \frac{\omega_N}{\beta_2 f}\right) \tag{13}$$

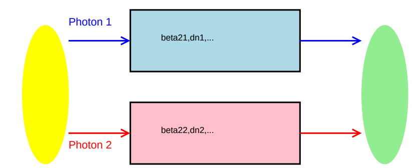
- Positive time correlation can be created by e.g. N-photon down conversion or adiabatic soliton compression (Fini and Hagelstein PRA **66**, 033818 (2002)):

$$\langle \tau_i \tau_j \rangle - \langle \tau_i \rangle \langle \tau_j \rangle = \int_{-\infty}^{\infty} d\tau_1 \ldots \int_{-\infty}^{\infty} d\tau_N \; \tau_i \tau_j |\psi(\tau_1, \ldots, \tau_N)|^2 - \langle \tau_i \rangle \langle \tau_j \rangle > 0 \tag{14}$$

$$\langle \omega_i \omega_j \rangle - \langle \omega_i \rangle \langle \omega_j \rangle = \int_{-\infty}^{\infty} d\omega_1 \ldots \int_{-\infty}^{\infty} d\omega_N \; \omega_i \omega_j |\phi(\omega_1, \ldots, \omega_N)|^2 - \langle \omega_i \rangle \langle \omega_j \rangle < 0 \tag{15}$$

- A Fourier transform time lens can convert $\langle \tau_i \tau_j \rangle - \langle \tau_i \rangle \langle \tau_j \rangle$ to < 0.

Conclusion



- Quantum temporal imaging is a technique of using temporal imaging to manipulate the multiphoton probability amplitude $\psi(\tau_1, \tau_2, ..., \tau_N)$.
- Requires only dispersive medium, e.g. optical fibers, and electro-optic modulators
- Temporal correlations and entanglement among photons
- Temporal Fourier transform
- Requires correlated photon sources