# Quantum transition-edge detectors and other stuff

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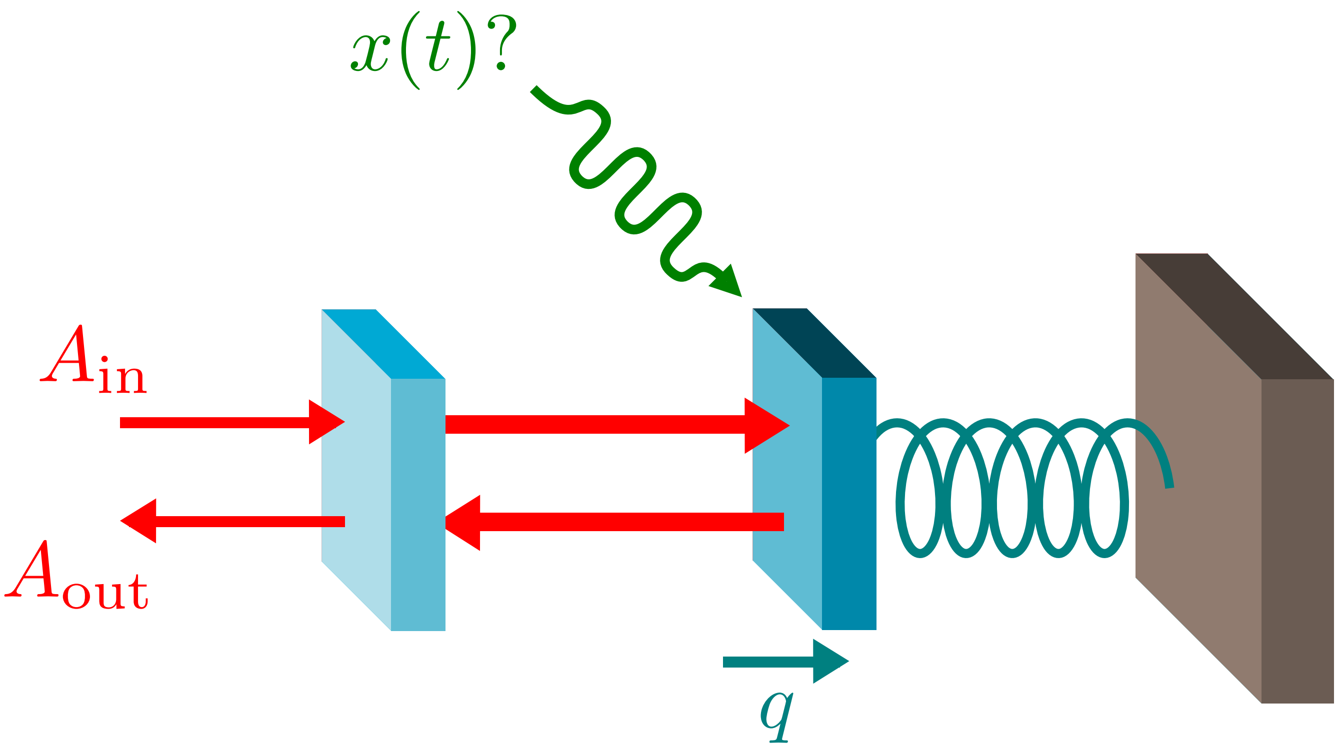
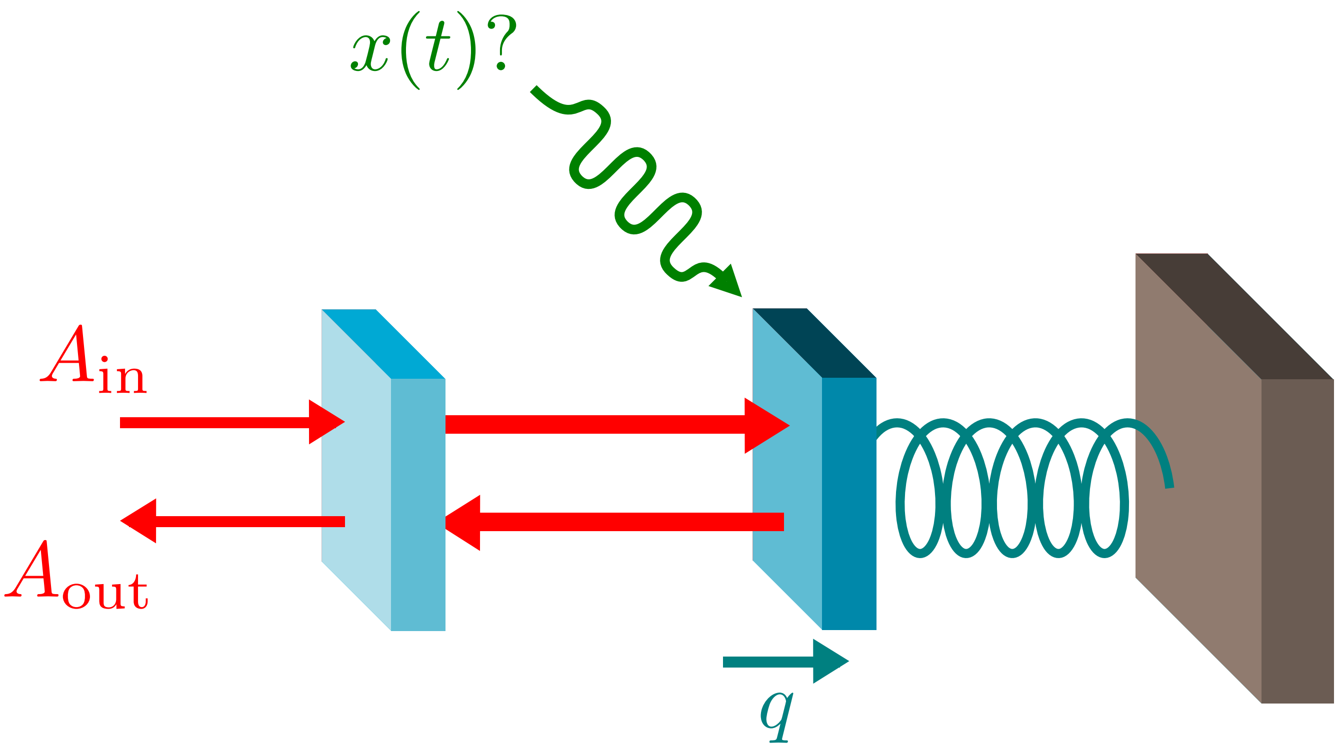


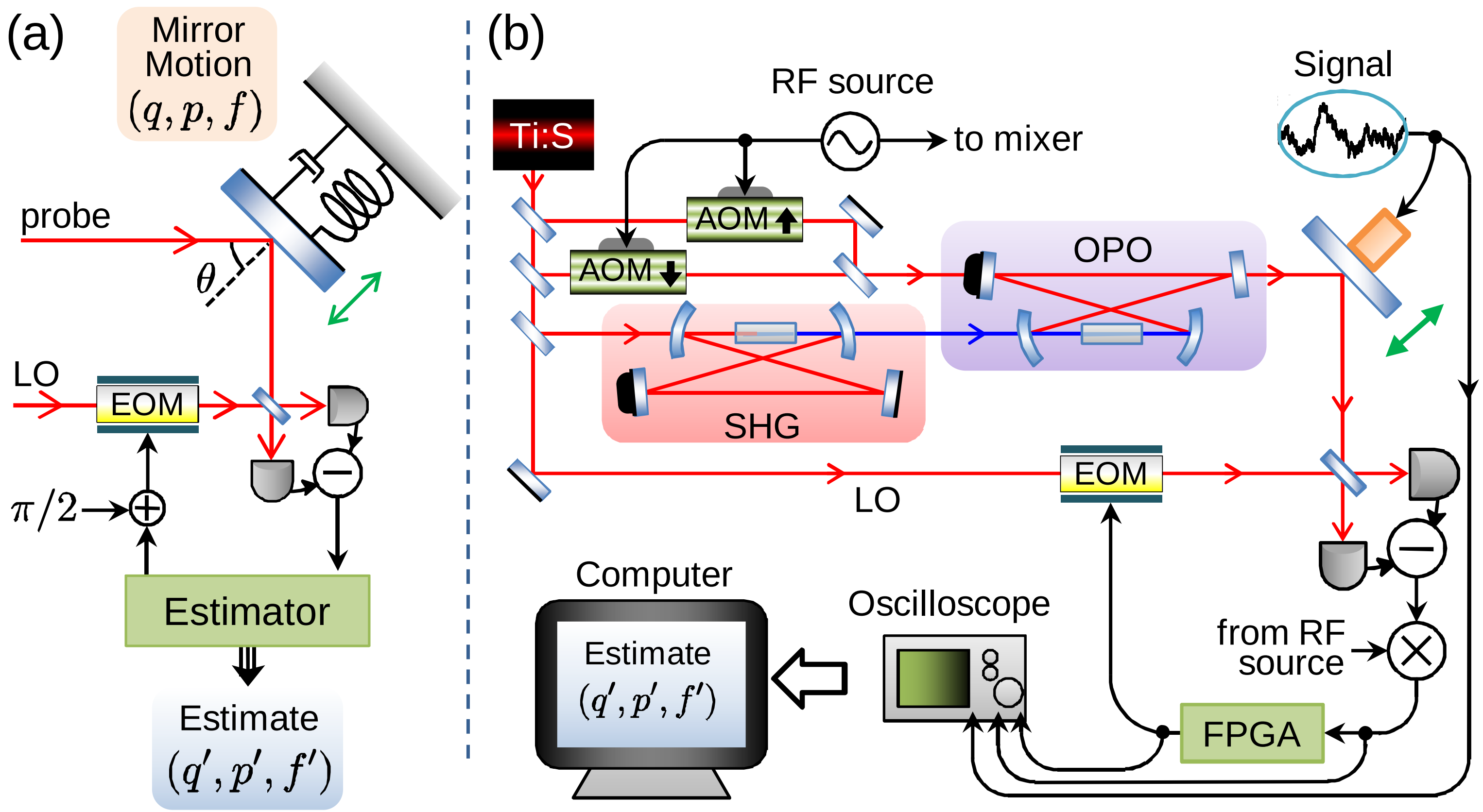

- Braginsky and Khalili, *Quantum Measurement*
- Caves *et al.*, RMP **52**, 341-392 (1980).

# Fundamental quantum limits to optomechanical sensing



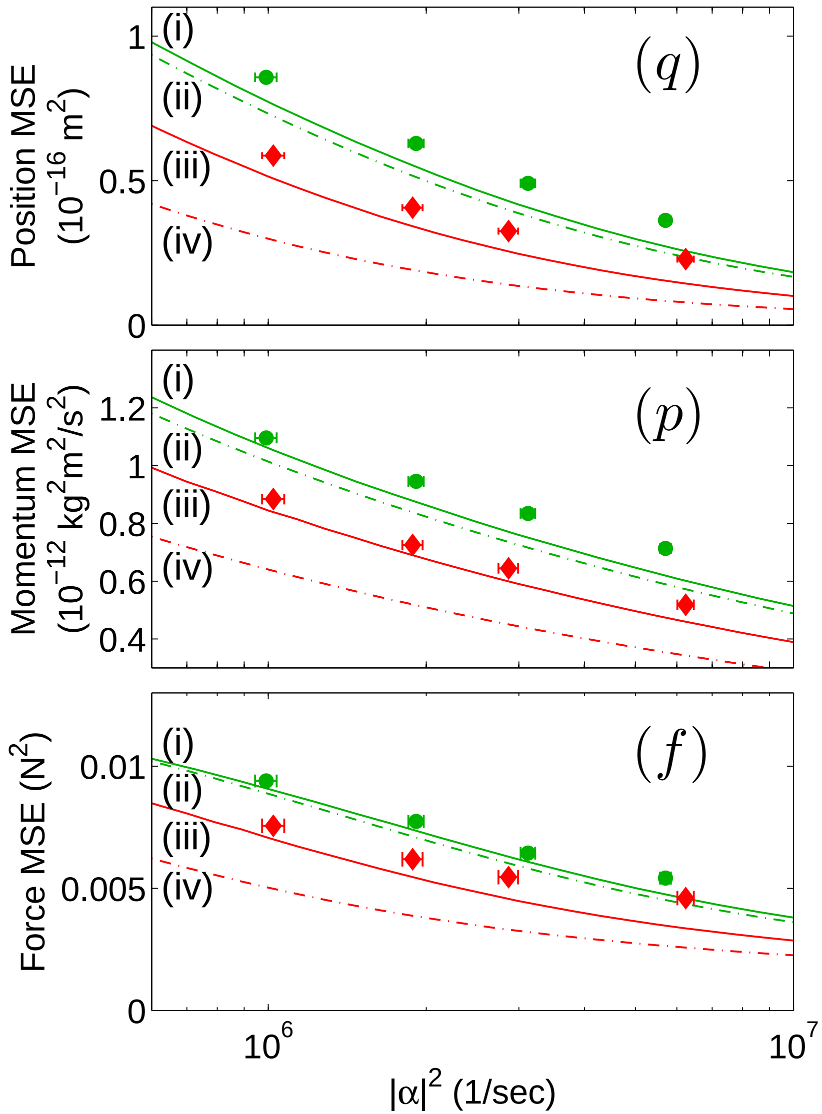

- **Quantum Cramér-Rao Bound (QCRB)**:
  - **No-go theorem**
  - Lower bound on **mean-square estimation error** given **initial state** and **dynamics**
  - Valid for any measurement (POVM) on $A_{\text{out}}$
  - No clue on attainability and best measurement
- M. Tsang, H. M. Wiseman, and C. M. Caves, PRL **106**, 090401 (2011).
- **Detection (Helstrom) bounds**: M. Tsang and R. Nair, PRA **86**, 042115 (2012).
- **Decoherence**: M. Tsang, NJP **15**, 073005 (2013).

# Experiment approaching the quantum limit

- K. Iwasawa, K. Makino, H. Yonezawa, M. Tsang, A. Davidovic, E. Huntington, A. Furusawa, arXiv:1305.0066 (PRL?)
- Coherent state/phase-squeezed light + **classical** PZT mirror + **homodyne phase-locked loop** + **smoothing**
- 

## Experimental results

- Green: Coherent states
- Red: Phase-squeezed light
- Solid line: classical Wiener theory
- Dashed lines: **QCRB**
- This is remarkable because
  - QCRB offers no clue about whether it is **attainable**.
  - QCRB offers no clue about what's the **best measurement** to approach it.
  - Large number of parameters in a **waveform**.
  - Large number of optical modes in **continuous-wave beam**.
  - Homodyne phase-locked loop + smoothing is pretty good.
  - More complicated photonic circuit/quantum computer won't help much.



## Waveform parameter estimation

- System and observation equations:

$$\frac{dz(t)}{dt} = Fz(t) + \xi(t), \tag{1}$$

$$y(t) = Cz(t) + \eta(t), \tag{2}$$

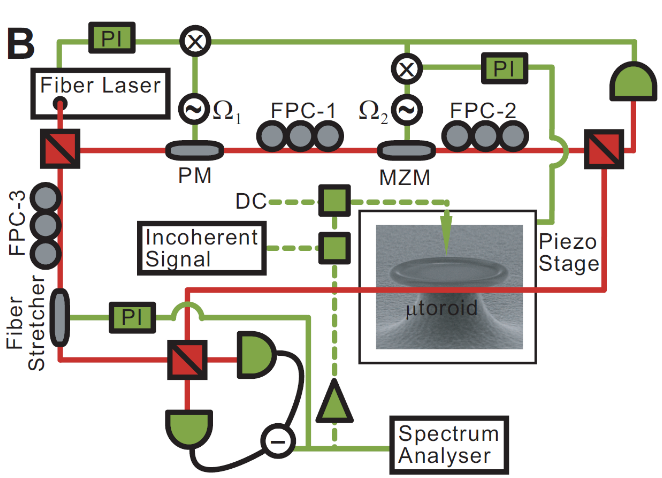
$$\langle \xi(t)\xi(t') \rangle = Q\delta(t - t'), \tag{3}$$

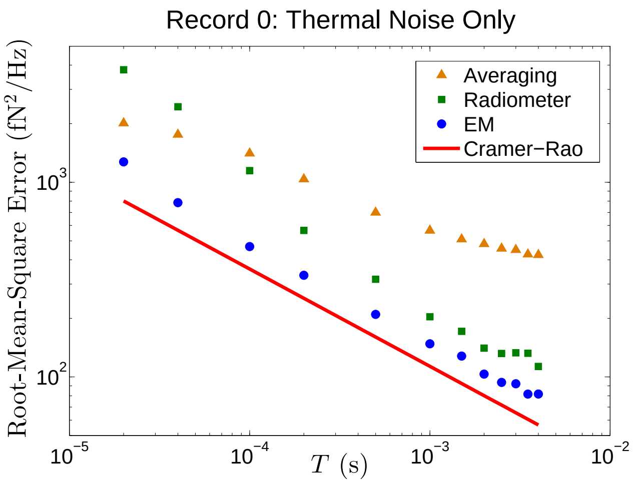
$$\langle \eta(t)\eta(t') \rangle = R\delta(t - t'). \tag{4}$$

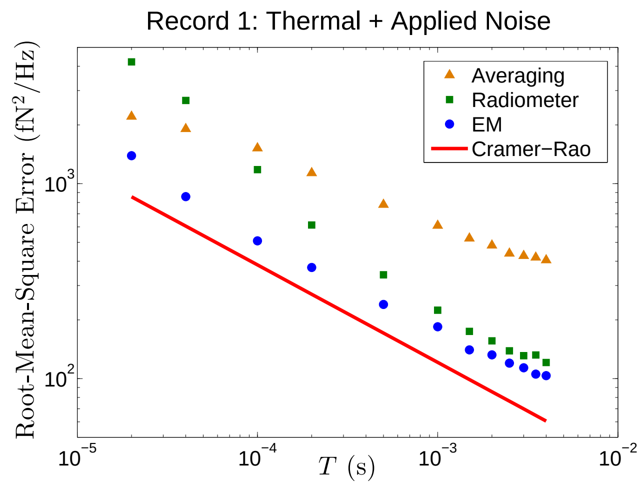
- Our study so far assumes $F$, $C$, $Q$, $R$ are known exactly and we estimate $z(t)$ from $y(t)$ (linear estimation).
- What if we don't know and linearization doesn't work?
  - **System Identification**: How to perform parameter estimation?
  - **Experimental Design**: How to enhance sensitivity to parameters?
  - **Quantum Limits**?
- **Applications**: many sensing applications rely on these parameters:
  - **Optical resonance frequency**: cavity enhanced detection of nanoparticles
  - **Mechanical resonance frequency**: e.g., Albrecht, Rugar *et al.*, JAP **69**, 668 (1991).
  - **Noise power/Damping rate**: Thermometry/bolometry, rheology, etc.
  - **System identification**: Demonstration of optomechanical phenomena, e.g., cooling, ponderomotive squeezing, BAE, etc.

# Optomechanical parameter estimation

- Most optomechanics experiments don't do statistics properly, e.g., Gavartin, Verlot, Kippenberg, Nature Nanotech. **7**, 509 (2012).
- S. Z. Ang, G. I. Harris, W. P. Bowen, M. Tsang, arXiv:1307.3800:
  - Analytic results for classical **Cramér-Rao bounds**
  - **Expectation-Maximization (EM)** algorithm (smoothing + iteration, converges to maximum-likelihood) [Shumway and Stoffer, *Time Series Analysis and its Applications*]
  - EM can also estimate most of the other parameters, e.g., mechanical resonance frequency, damping rate, useful for **system identification**.
  - See also Guta and Yamamoto, arXiv:1303.3771.



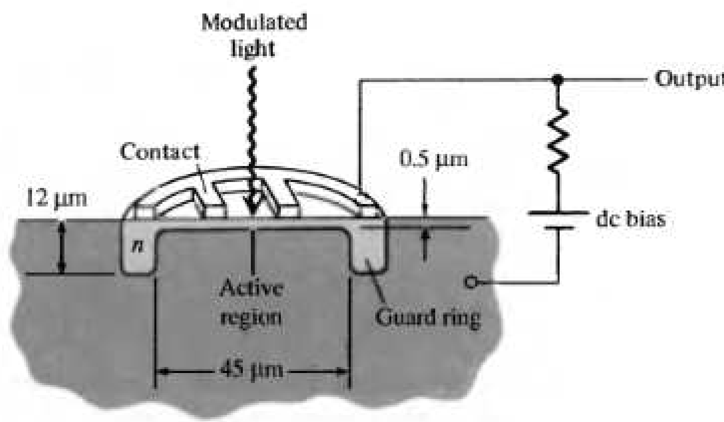



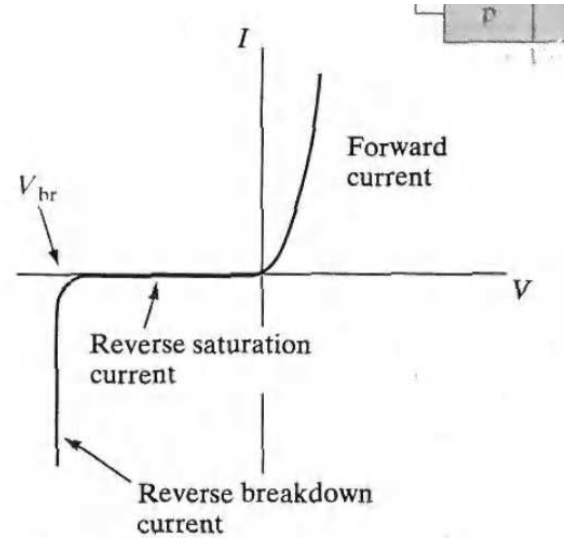




- Next question: How to enhance parameter sensitivity?

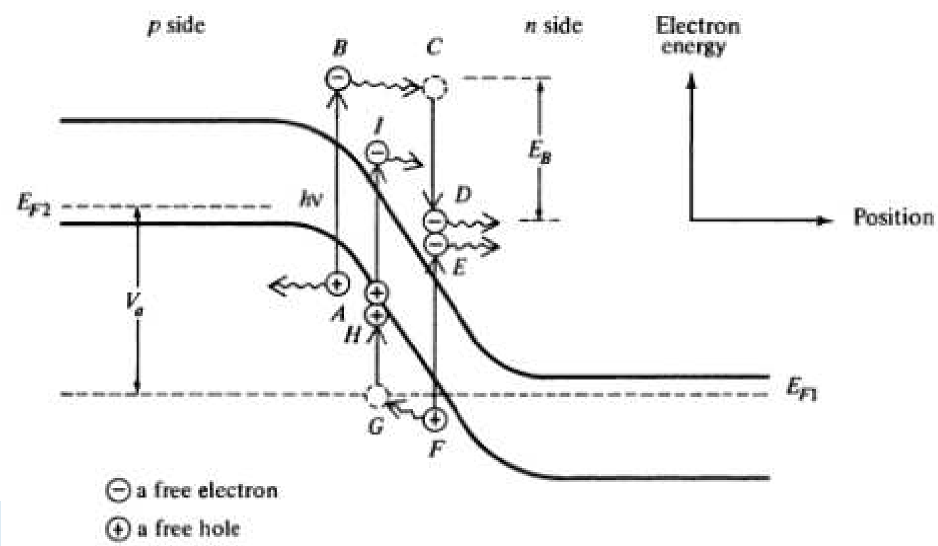
# Example #1: Avalanche photodiode

- Bias diode close to breakdown voltage



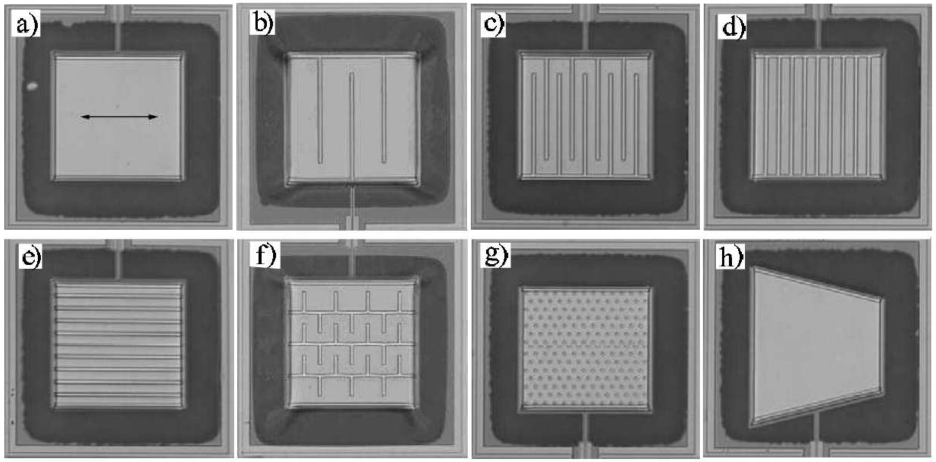

- Photon induces avalanche electron-hole-pair creation

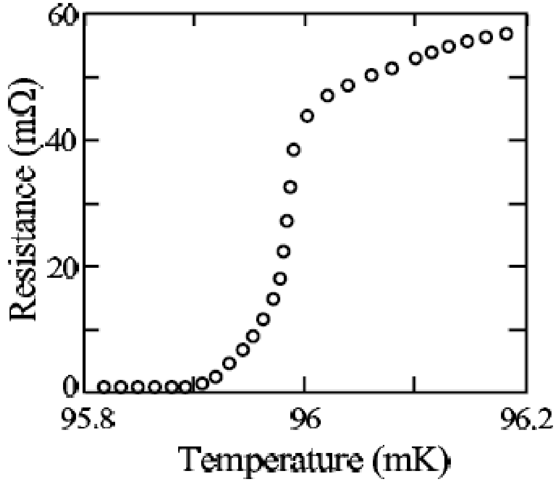



- Yariv and Yeh, *Photonics*

# Example #2: Superconducting transition-edge sensor

- Bias temperature just below critical




- Increase in temperature induces phase transition and gigantic increase in resistance
- Irwin and Hilton, Cryogenic Particle Detection, Topics Appl. Phys. **99**, 63152 (2005).

# Final straw that broke a camel's back

- TES is an example of **classical phase transitions**.
- What if we want to detect **optical phase shifts**/resonance frequency shifts:

$$H_I = \omega a^\dagger a? \tag{5}$$

- use **quantum phase transitions**:

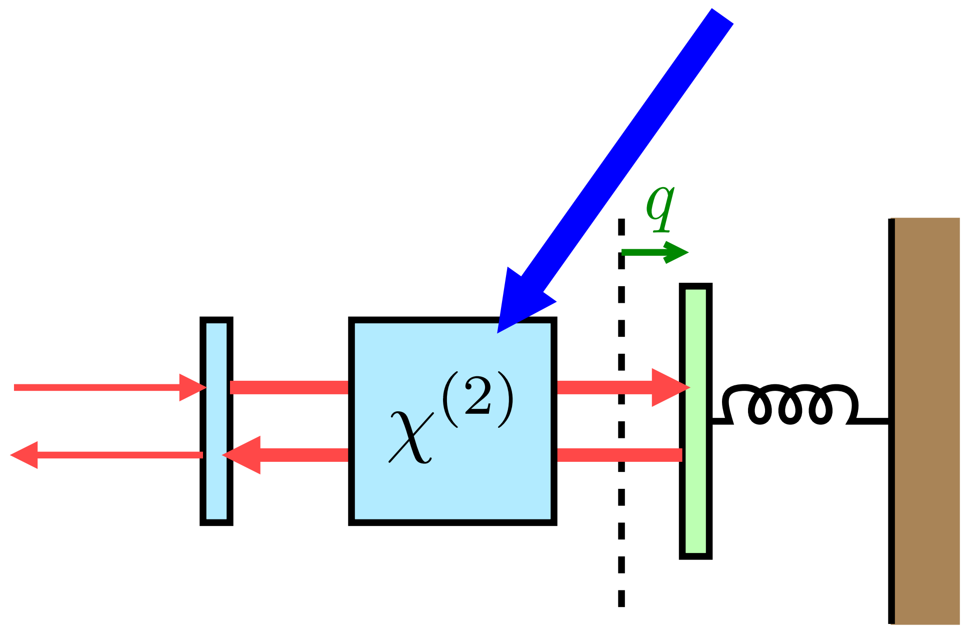
$$H = \omega a^\dagger a + H_C \tag{6}$$

- Think of $H_C$ as a **coherent control Hamiltonian** that increase the system sensitivity (e.g., ground state) to $\omega$.
- Examples:
  - Ising (spin-spin interaction, magnetic field)
  - Dicke (light-atom interaction)
  - Bose-Hubbard (bosons in lattice)
  - Dicke-Ising (Heisenberg-scaling metrology: Gammelmark and Molmer, NJP **13**, 053035 (2011))

Sachev, *Quantum Phase Transitions*

- Laser/OPO near threshold (sensitive to cavity detuning)

$$H = \omega a^\dagger a + \lambda \left( a^2 + a^{\dagger 2} \right) + \dots \tag{7}$$



# Loschmidt echo and Helstrom bound

- Two Hamiltonians:

$$H_0 = \omega_0 a^\dagger a + H_C \tag{8}$$

$$H_1 = (\omega_0 + \delta)\, a^\dagger a + H_C \tag{9}$$

- Let $|\psi\rangle$ be, say, ground state of $H_0$. Define

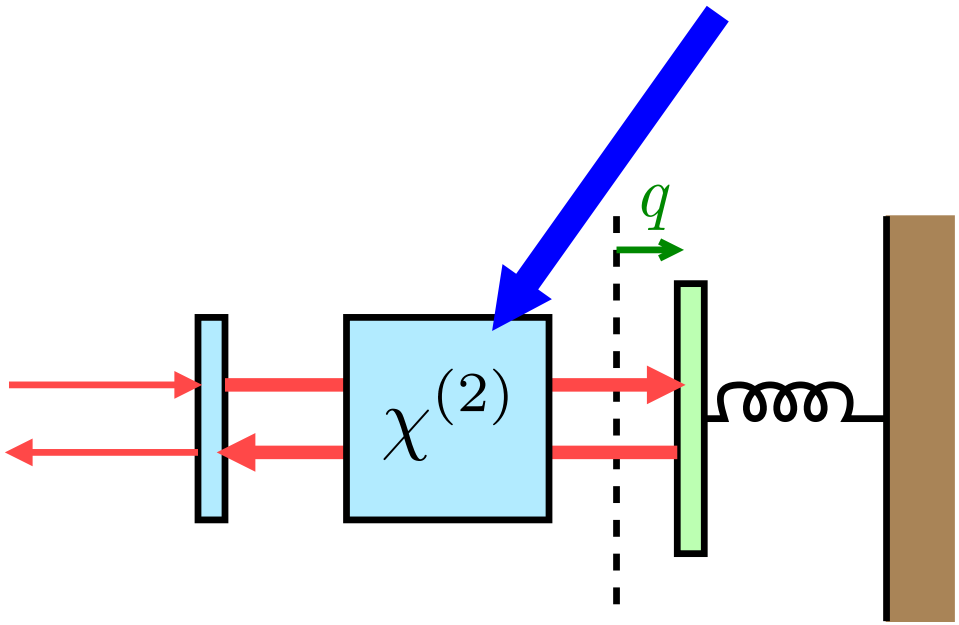
$$F = \left| \langle \psi | U_1^\dagger U_0 | \psi \rangle \right|^2 \tag{10}$$

  called **Loschmidt echo** (Peres, *Quantum Theory*).
- Used to study time reversibility of quantum chaos. Big drop in $F$ for small $\Delta\omega$ implies **time irreversibility**, chaos. [Gorin *et al.*, Phys. Rep. **435**, 33156 (2006)].
- **Quantum metrology**: Suppose $H = H_0$ when no perturbation and $H = H_1$ when there is.

$$\min_{E(Y)} P_e = \frac{1}{2}\left(1 - \sqrt{1 - F}\right). \tag{11}$$

- Small $F$ means small $P_e$ (for the optimal POVM).
- Don't worry about photon-number constraints (photons are cheap)
- Coherent state, no $H_C$: $-\ln P_{e\min} \approx -\ln F \propto \delta^2 t^2$, $\delta \propto 1/t$
- Ground state of $H_0$, parametric $H_C$, $H_1$ is above critical point: $-\ln F \propto \sqrt{\delta} t$, $\delta \propto 1/t^2$,
- M. Tsang, PRA **88**, 021801(R) (2013).

# Continuous measurements




- Continuous homodyne/heterodyne measurements
- Classical fidelity (Bhattacharyya distance) below threshold:

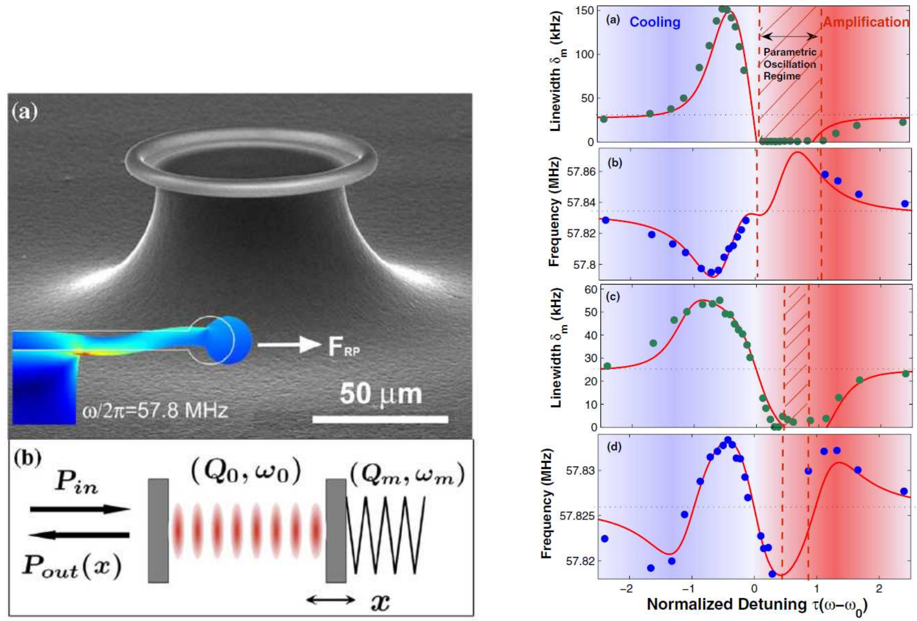
$$-\ln F = \frac{t}{2}\int_{-\infty}^{\infty}\frac{d\Omega}{2\pi}\ln\frac{|S_1(\Omega)+S_0(\Omega)|}{2\sqrt{|S_1(\Omega)||S_0(\Omega)|}} \tag{12}$$

- Fisher information $G$:

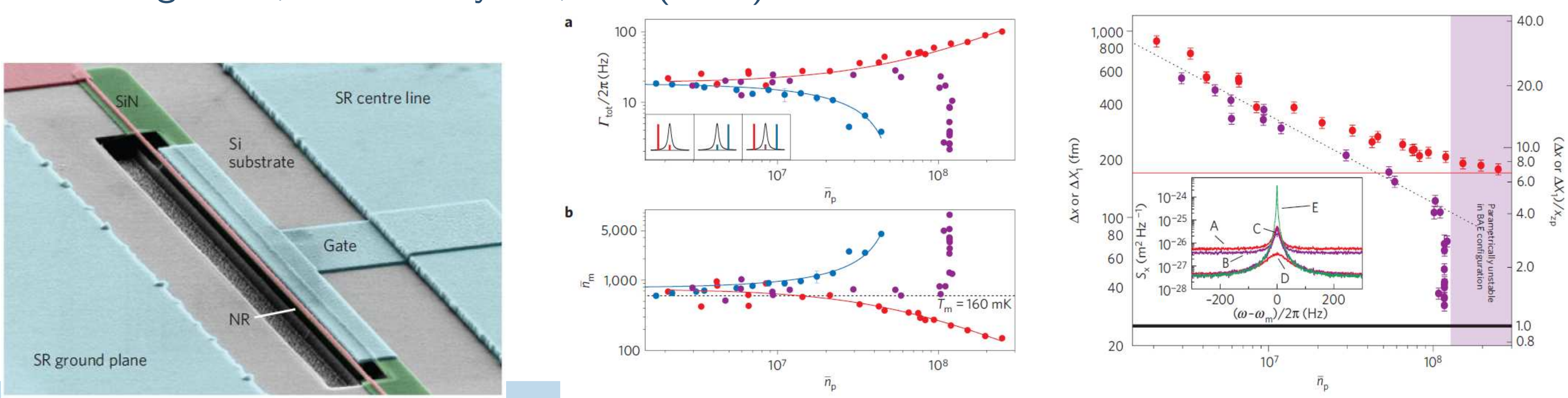
$$F(\omega,\omega+\delta)\approx 1-G(\omega)\delta^2/4 \tag{13}$$

# Optomechanical instabilities

- Schliesser *et al.*, PRL **97**, 243905 (2006):

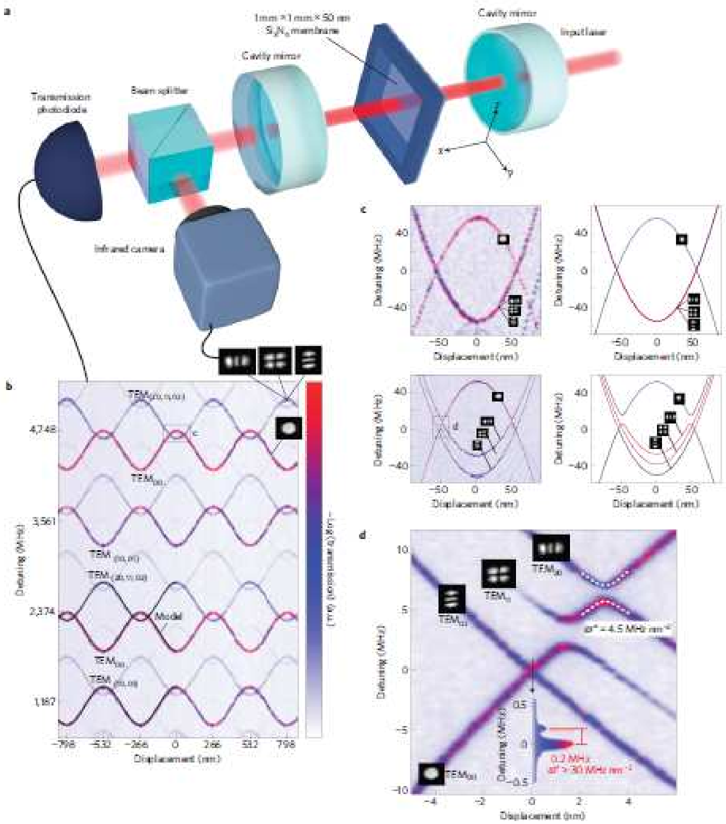



- Hertzberg *et al.*, Nature Phys. **6**, 213 (2010):



# Testing quantized energy

- **Question**: how to demonstrate **quantized** mechanical energy?
- Thompson *et al.*, Nature; Sankey *et al.*, Nature Phys. **6**, 707 (2010)
    - Santamore, *et al.*, PRB **70**, 144301 (2004)
    - Jacobs *et al.*, PRL 98, 147201 (2007)
    - Miao *et al.*, PRL **103**, 100402 (2009)
    - Clerk *et al.*, PRL **104**, 213603 (2010).
- **Begging the question**: prove quantum mechanics by assuming quantum mechanics
- **Expected values** can't be measured in finite time.
- **Asymmetric sidebands with heterodyne**
    - Safavi-Naeini *et al.*, PRL **108**, 033602 (2012).
    - Brahms *et al.*, PRL **108**, 133601 (2012).
    - `http://www.youtube.com/watch?v=pktWhH6m_DM`
- Linear Gaussian model:
    - Khalili *et al.*, PRA **86**, 033840 (2012).
    - Jayich *et al.* NJP **14**, 115018 (2012).
    - Safavi-Naeini *et al.*, NJP **15**, 035007 (2013).
    - M. Tsang, arXiv:1306.2699.



# Quantum hypothesis testing

- Compare quantum model with your **best** classical model
- e.g., under quantum hypothesis $\mathcal{H}_1$:

$$dy_t = n_t dt + dV_t, \tag{14}$$

$n_t$ is energy of quantized harmonic oscillator in thermal bath.

- compare against classical hypothesis $\mathcal{H}_0$:

$$dy_t = \mathcal{E}_t dt + dV_t. \tag{15}$$

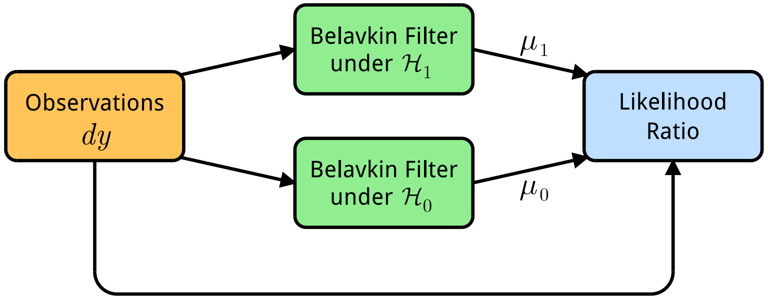
$\mathcal{E}_t$ should be energy of classical harmonic oscillator in thermal bath.

- **Statistical hypothesis testing**:
  - M. Tsang, PRL **108**, 170502 (2012).
  - Do likelihood-ratio test.
  - Nice formula (valid for any $dy_t = X_t dt + dV_t$):

$$\Lambda = \frac{P(Y|\mathcal{H}_1)}{P(Y|\mathcal{H}_0)} = \exp\left[\int \frac{dy}{R}(\mu_1 - \mu_0) - \frac{1}{2}\int \frac{dt}{R}(\mu_1^2 - \mu_0^2)\right] \tag{16}$$

Quantum filtering expectations: $\mu_0 = E[X_t|\mathcal{H}_0]$, $\mu_1 = E[X_t|\mathcal{H}_1]$.

  - Classical case: Duncan, Inform. Control **13**, 62 (1968).



## Expected information

- How well is the testing procedure expected to work?
- Information measure: **relative entropy**

$$\ln \Lambda \to E\left[\ln \Lambda | \mathcal{H}_1\right] \equiv D(P_1 || P_0) \tag{17}$$

- **Increases with more data**
- **Cute formula** valid for $dy_t = X_t dt + dV_t$ and any $X_t$ (**unpublished**, use martingale property of **innovation** $E[dy_t - \mu_1 dt | \mathcal{Y}_t, \mathcal{H}_1] = 0$ and **orthogonality principle** for quantum conditional expectations $E[h(Y)(\mu_1 - X) | \mathcal{Y}_t, \mathcal{H}_1] = 0$):

$$D(P_1 || P_0) = E\left[\int \frac{dy}{R}(\mu_1 - \mu_0) - \frac{1}{2}\int \frac{dt}{R}\left(\mu_1^2 - \mu_0^2\right)\Big|\mathcal{H}_1\right] \tag{18}$$

$$= E\left[\int \frac{\mu_1 dt}{R}(\mu_1 - \mu_0) - \frac{1}{2}\int \frac{dt}{R}\left(\mu_1^2 - \mu_0^2\right)\Big|\mathcal{H}_1\right] \tag{19}$$

$$= \frac{1}{2}\int \frac{dt}{R} E\left[(\mu_1 - \mu_0)^2 \Big|\mathcal{H}_1\right] \tag{20}$$

$$= \frac{1}{2}\int \frac{dt}{R} E\left[(\mu_0 - X)^2 - (\mu_1 - X)^2 \Big|\mathcal{H}_1\right] \tag{21}$$

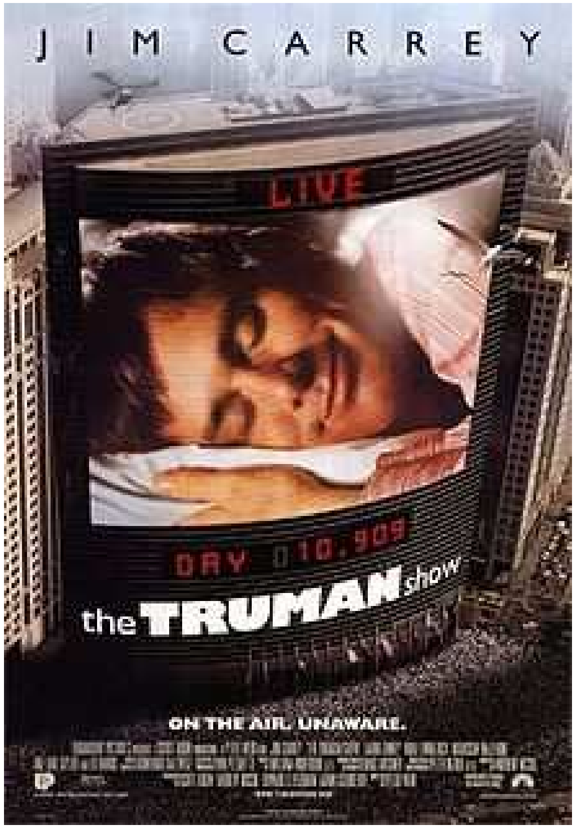
- Relates relative entropy to **mismatched** quantum filtering error.
- Classical case: Weissman, IEEE TIT **56**, 4256 (2010).
- Relates mutual information, classical filtering and smoothing errors: Duncan, SIAM JAM **19**, 215 (1970); Guo, Shamai, and Verdu, IEEE TIT **51**, 1261 (2005), etc.

## Different notions of classicality

- **Quantum-mechanics-free systems**:
  - QND condition in Heisenberg picture

$$\left[O_j(t), O_k(t')\right] = 0. \tag{22}$$

  - Measurement of the commuting observables only
  - **Spectral theorem**: Any classical deterministic/stochastic model can be framed this way.
    - Koopman, PNAS **17**, 315 (1931).
    - Gough and James, IEEE TAC **54**, 2530 (2009).
  - **Backaction-evading**
    - Caves *et al.*, RMP **52**, 341-392 (1980).
    - Tsang and Caves, PRX **2**, 031016 (2012).
    - If one has access to these commuting observables only, he/she would never find out about quantum mechanics.



## Classical simulable systems

- Efficiently simulable by classical stochastic systems
- Examples:
    - linear Gaussian (nonnegative Wigner)
    - Gottesman-Knill theorem
    - Any dynamics with nonnegative quasiprobability representations
- ad-hoc rules on system/observation noise (**epistemic restrictions**)
- This class is incredibly important for QIP as a **no-go**:
    - Quantum computing
        - No need for quantum computer
    - Quantum simulation
        - Good news for quantum chemistry, condensed matter etc.
    - Quantum filtering/estimation
        - Good news because we can use finite-dimensional classical algorithms to avoid **curse of dimensionality**:
            - Kalman filter
            - Monte Carlo/particle filters

## What's truly quantum?

- **Bell-type (Bell, CHSH, Kochen-Specker, etc.) theorems** rule out a wide class of classical models
  - **Contextuality**
  - Nonlocality: related to contextuality. (Mermin RMP)
- **Universal quantum computation**: Single-photon sources, beam splitters, phase shifters, photodetectors (Knill-Laflamme-Milburn)
- **Boson sampling** (Aaronson *et al.*)

# Summary

- **Waveform QCRB**
  - Theory: M. Tsang, H. M. Wiseman, and C. M. Caves, PRL **106**, 090401 (2011); M. Tsang, NJP **15**, 073005 (2013).
  - Experiment: Iwasawa *et al.*, arXiv:1305.0066.
- **Optomechanical parameter estimation**
  - S. Z. Ang *et al.*, arXiv:1307.3800.
- **Quantum transition-edge detectors**
  - M. Tsang, PRA **88**, 021801(R) (2013).
- **Continuous quantum hypothesis testing**
  - M. Tsang, PRL **108**, 170502 (2012).
  - M. Tsang, "Relative entropy and mismatched quantum filtering error," unpublished.
  - M. Tsang, arXiv:1306.2699v2.
- supported by the Singapore National Research Foundation Fellowship (NRF-NRFF2011-07).