Quantum transition-edge detectors and other stuff

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Fundamental quantum limits to optomechanical sensing

- Braginsky and Khalili, *Quantum Measurement*
Quantum Cramér-Rao Bound (QCRB):
- No-go theorem
- Lower bound on mean-square estimation error given initial state and dynamics
- Valid for any measurement (POVM) on $A_{\text{out}}$
- No clue on attainability and best measurement


- Coherent state/phase-squeezed light + \textbf{classical} PZT mirror + \textbf{homodyne phase-locked loop} + smoothing
Experimental results

- **Green**: Coherent states
- **Red**: Phase-squeezed light
- **Solid line**: classical Wiener theory
- **Dashed lines**: QCRB

This is remarkable because

- QCRB offers no clue about whether it is **attainable**.
- QCRB offers no clue about what’s the best **measurement** to approach it.
- Large number of parameters in a **waveform**.
- Large number of optical modes in **continuous-wave beam**.
- Homodyne phase-locked loop + smoothing is pretty good.
- More complicated photonic circuit/quantum computer won’t help much.
Waveform parameter estimation

- System and observation equations:

\[
\frac{dz(t)}{dt} = Fz(t) + \xi(t), \quad (1)
\]

\[
y(t) = Cz(t) + \eta(t), \quad (2)
\]

\[
\langle \xi(t)\xi(t') \rangle = Q\delta(t - t'), \quad (3)
\]

\[
\langle \eta(t)\eta(t') \rangle = R\delta(t - t'). \quad (4)
\]

- Our study so far assumes \( F, C, Q, R \) are known exactly and we estimate \( z(t) \) from \( y(t) \) (linear estimation).

- What if we don’t know and linearization doesn’t work?

  - **System Identification**: How to perform parameter estimation?
  - **Experimental Design**: How to enhance sensitivity to parameters?
  - **Quantum Limits**?

- **Applications**: many sensing applications rely on these parameters:

  - **Optical resonance frequency**: cavity enhanced detection of nanoparticles
  - **Mechanical resonance frequency**: e.g., Albrecht, Rugar et al., JAP 69, 668 (1991).
  - **Noise power/Damping rate**: Thermometry/bolometry, rheology, etc.
  - **System identification**: Demonstration of optomechanical phenomena, e.g., cooling, ponderomotive squeezing, BAE, etc.
Optomechanical parameter estimation

  - Analytic results for classical Cramér-Rao bounds
  - Expectation-Maximization (EM) algorithm (smoothing + iteration, converges to maximum-likelihood) [Shumway and Stoffer, *Time Series Analysis and its Applications*]
  - EM can also estimate most of the other parameters, e.g., mechanical resonance frequency, damping rate, useful for system identification.
  - See also Guta and Yamamoto, arXiv:1303.3771.

Next question: How to enhance parameter sensitivity?
Example #1: Avalanche photodiode

- Bias diode close to breakdown voltage
- Photon induces avalanche electron-hole-pair creation

Yariv and Yeh, *Photonics*
Example #2: Superconducting transition-edge sensor

- Bias temperature just below critical

![Images of superconducting transition-edge sensors]

- Increase in temperature induces phase transition and gigantic increase in resistance
■ TES is an example of **classical phase transitions**.

■ What if we want to detect **optical phase shifts**/resonance frequency shifts:

\[
H_I = \omega a^\dagger a?
\]  

(5)

■ use **quantum phase transitions**:

\[
H = \omega a^\dagger a + H_C
\]  

(6)

■ Think of \(H_C\) as a **coherent control Hamiltonian** that increase the system sensitivity (e.g., ground state) to \(\omega\).

■ Examples:

- Ising (spin-spin interaction, magnetic field)
- Dicke (light-atom interaction)
- Bose-Hubbard (bosons in lattice)
- Dicke-Ising (Heisenberg-scaling metrology: Gammelmark and Molmer, NJP 13, 053035 (2011))

Sachev, *Quantum Phase Transitions*
- Laser/OPO near threshold (sensitive to cavity detuning)

\[ H = \omega a\dagger a + \lambda \left( a^2 + a\dagger^2 \right) + \ldots \] (7)
Two Hamiltonians:

\[ H_0 = \omega_0 a^\dagger a + H_C \]  
\[ H_1 = (\omega_0 + \delta) a^\dagger a + H_C \]

Let \( |\psi\rangle \) be, say, ground state of \( H_0 \). Define

\[ F = \left| \langle \psi | U_1^\dagger U_0 | \psi \rangle \right|^2 \]

called **Loschmidt echo** (Peres, *Quantum Theory*).


Quantum metrology: Suppose \( H = H_0 \) when no perturbation and \( H = H_1 \) when there is.

\[ \min_{E(Y)} P_e = \frac{1}{2} \left( 1 - \sqrt{1 - F} \right) \]

Small \( F \) means small \( P_e \) (for the optimal POVM).

Don’t worry about photon-number constraints (photons are cheap)

Coherent state, no \( H_C \): \(-\ln P_{emin} \approx -\ln F \propto \delta^2 t^2, \delta \propto 1/t\)

Ground state of \( H_0 \), parametric \( H_C, H_1 \) is above critical point: \(-\ln F \propto \sqrt{\delta} t, \delta \propto 1/t^2,\)

Continuous homodyne/heterodyne measurements

Classical fidelity (Bhattacharyya distance) below threshold:

\[
- \ln F = \frac{t}{2} \int_{-\infty}^{\infty} \frac{d\Omega}{2\pi} \ln \frac{|S_1(\Omega) + S_0(\Omega)|}{2\sqrt{|S_1(\Omega)||S_0(\Omega)|}}
\]

(12)

Fisher information \( G \):

\[
F(\omega, \omega + \delta) \approx 1 - G(\omega)\delta^2 / 4
\]

(13)
Schliesser et al., PRL 97, 243905 (2006):

Hertzberg et al., Nature Phys. 6, 213 (2010):
Testing quantized energy

- **Question**: how to demonstrate quantized mechanical energy?
  - Jacobs *et al.*, PRL 98, 147201 (2007)
  - Clerk *et al.*, PRL 104, 213603 (2010).
- **Begging the question**: prove quantum mechanics by assuming quantum mechanics
- **Expected values** can’t be measured in finite time.
- **Asymmetric sidebands with heterodyne**
  - [http://www.youtube.com/watch?v=pktWhH6m_DM](http://www.youtube.com/watch?v=pktWhH6m_DM)
- **Linear Gaussian model**:
  - Jayich *et al.*. NJP 14, 115018 (2012).
Quantum hypothesis testing

- Compare quantum model with your best classical model
- e.g., under quantum hypothesis $\mathcal{H}_1$:
  \[ dy_t = n_t dt + dV_t, \]  
  \[ n_t \text{ is energy of quantized harmonic oscillator in thermal bath.} \]
- compare against classical hypothesis $\mathcal{H}_0$:
  \[ dy_t = \mathcal{E}_t dt + dV_t. \]  
  \[ \mathcal{E}_t \text{ should be energy of classical harmonic oscillator in thermal bath.} \]

- Statistical hypothesis testing:
  - Do likelihood-ratio test.
  - Nice formula (valid for any $dy_t = X_t dt + dV_t$):
    \[ \Lambda = \frac{P(Y|\mathcal{H}_1)}{P(Y|\mathcal{H}_0)} = \exp \left[ \int \frac{dy}{R} (\mu_1 - \mu_0) \right. \]
    \[ - \frac{1}{2} \int \frac{dt}{R} \left( \mu_1^2 - \mu_0^2 \right) \]  
    \[ (16) \]

    Quantum filtering expectations: $\mu_0 = E[X_t|\mathcal{H}_0]$, $\mu_1 = E[X_t|\mathcal{H}_1]$.
Expected information

- How well is the testing procedure expected to work?
- Information measure: relative entropy

\[ \ln \Lambda \rightarrow E \left[ \ln \Lambda \mid \mathcal{H}_1 \right] \equiv D(P_1 \mid \mid P_0) \]  \hspace{1cm} (17)

- Increases with more data
- Cute formula valid for \( dy_t = X_t dt + dV_t \) and any \( X_t \) (unpublished, use martingale property of innovation \( E[dy_t - \mu_1 dt \mid \mathcal{Y}_t, \mathcal{H}_1] = 0 \) and orthogonality principle for quantum conditional expectations \( E[h(Y)(\mu_1 - X) \mid \mathcal{Y}_t, \mathcal{H}_1] = 0 \):

\[
D(P_1 \mid \mid P_0) = E \left[ \int \frac{dy}{R} (\mu_1 - \mu_0) - \frac{1}{2} \int \frac{dt}{R} (\mu_1^2 - \mu_0^2) \mid \mathcal{H}_1 \right] 
\]

\[
= E \left[ \int \frac{\mu_1 dt}{R} (\mu_1 - \mu_0) - \frac{1}{2} \int \frac{dt}{R} (\mu_1^2 - \mu_0^2) \mid \mathcal{H}_1 \right] 
\]

\[
= \frac{1}{2} \int \frac{dt}{R} E \left[ (\mu_1 - \mu_0)^2 \mid \mathcal{H}_1 \right] 
\]

\[
= \frac{1}{2} \int \frac{dt}{R} E \left[ (\mu_0 - X)^2 - (\mu_1 - X)^2 \mid \mathcal{H}_1 \right] 
\]  \hspace{1cm} (18)\hspace{1cm} (19)\hspace{1cm} (20)\hspace{1cm} (21)

- Relates relative entropy to mismatched quantum filtering error.
- Relates mutual information, classical filtering and smoothing errors: Duncan, SIAM JAM 19, 215 (1970); Guo, Shamai, and Verdu, IEEE TIT 51, 1261 (2005), etc.
Different notions of classicality

- **Quantum-mechanics-free systems:**
  - QND condition in Heisenberg picture
    \[
    [O_j(t), O_k(t')] = 0.
    \]  
    (22)
  - Measurement of the commuting observables only
  - **Spectral theorem:** Any classical deterministic/stochastic model can be framed this way.
    - Koopman, PNAS 17, 315 (1931).
    - Gough and James, IEEE TAC 54, 2530 (2009).
  - **Backaction-evading**
    - Caves et al., RMP 52, 341-392 (1980).
    - Tsang and Caves, PRX 2, 031016 (2012).
    - If one has access to these commuting observables only, he/she would never find out about quantum mechanics.
Classical simulable systems

- Efficiently simulable by classical stochastic systems
- Examples:
  - linear Gaussian (nonnegative Wigner)
  - Gottesman-Knill theorem
  - Any dynamics with nonnegative quasiprobability representations
- ad-hoc rules on system/observation noise (epistemic restrictions)
- This class is incredibly important for QIP as a no-go:
  - Quantum computing
    - No need for quantum computer
  - Quantum simulation
    - Good news for quantum chemistry, condensed matter etc.
  - Quantum filtering/estimation
    - Good news because we can use finite-dimensional classical algorithms to avoid curse of dimensionality:
      - Kalman filter
      - Monte Carlo/particle filters
What's truly quantum?

- **Bell-type (Bell, CHSH, Kochen-Specker, etc.) theorems** rule out a wide class of classical models
  - **Contextuality**
  - **Nonlocality**: related to contextuality. (Mermin RMP)

- **Universal quantum computation**: Single-photon sources, beam splitters, phase shifters, photodetectors (Knill-Laflamme-Milburn)

- **Boson sampling** (Aaronson et al.)
Summary

- **Waveform QCRB**

- **Optomechanical parameter estimation**

- **Quantum transition-edge detectors**

- **Continuous quantum hypothesis testing**

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