Evading Quantum Mechanics *

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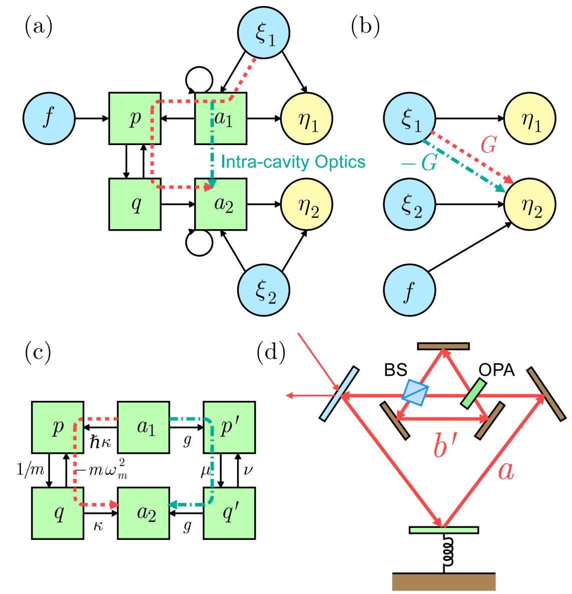
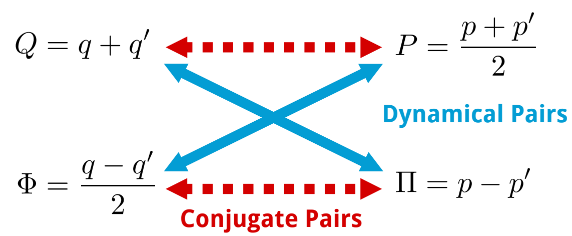
Outline

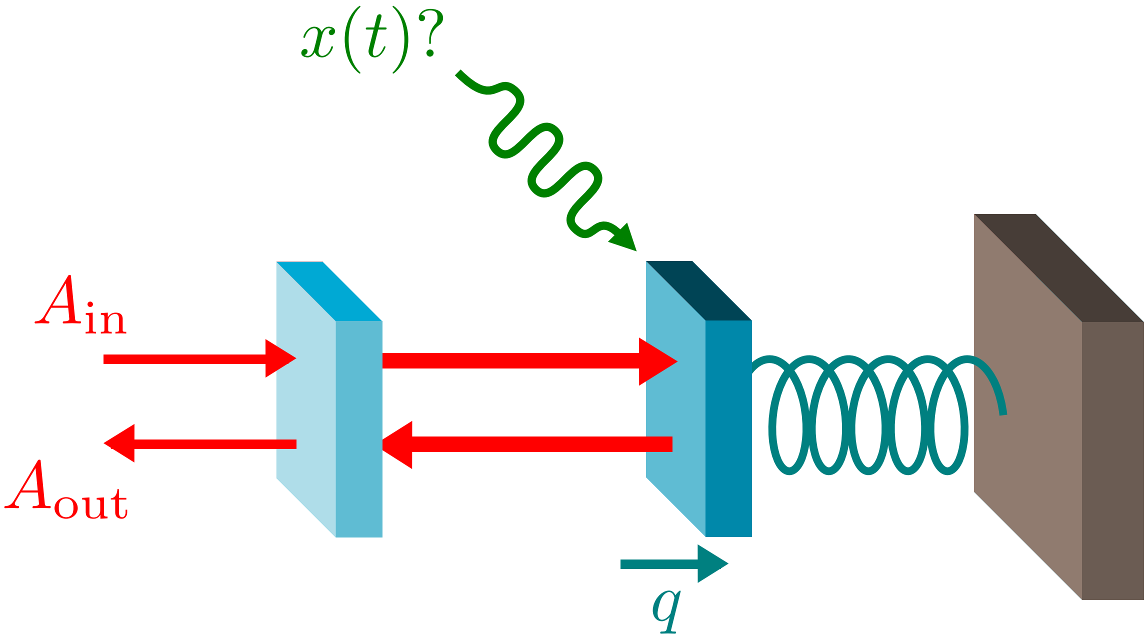
1. Quantum Noise Cancellation
   - Tsang and Caves, PRL **105**, 123601 (2010).
2. Quantum Metrology
   - Tsang, Wiseman, and Caves, PRL **106**, 090401 (2011).
   - Tsang and Nair, PRA **86**, 042115 (2012).
   - Iwasawa *et al.*, PRL **111**, 163602 (2013).
3. Quantum-Mechanics-Free Subsystems
   - Tsang and Caves, PRX **2**, 031016 (2012).
4. Plans for Global Domination



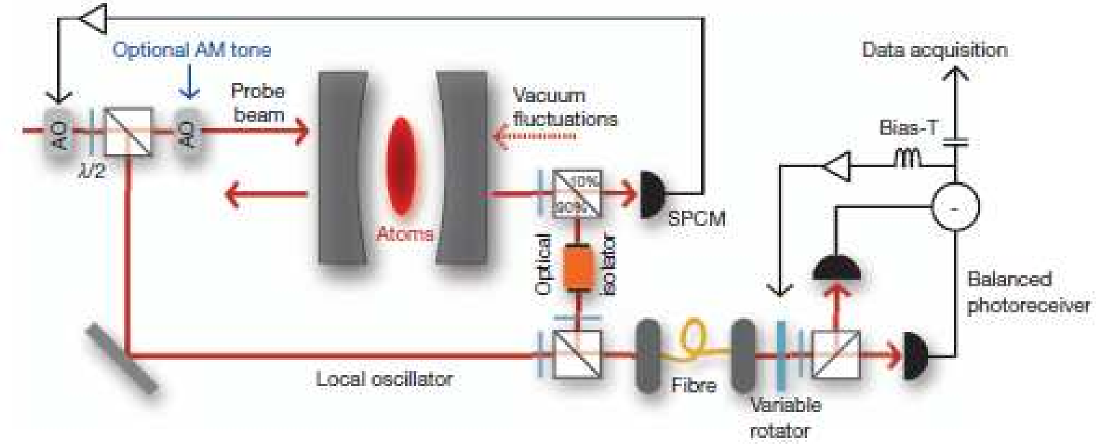
Positive-mass oscillator $\{q,p\}$
and negative-mass oscillator $\{q',p'\}$



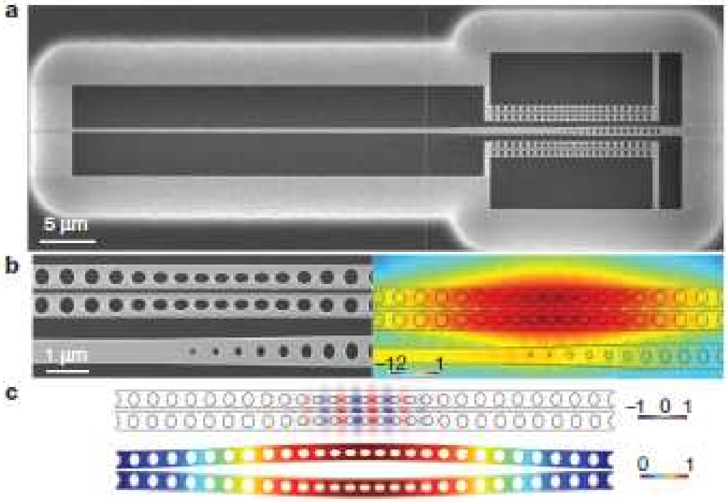
Optomechanical Force Sensing



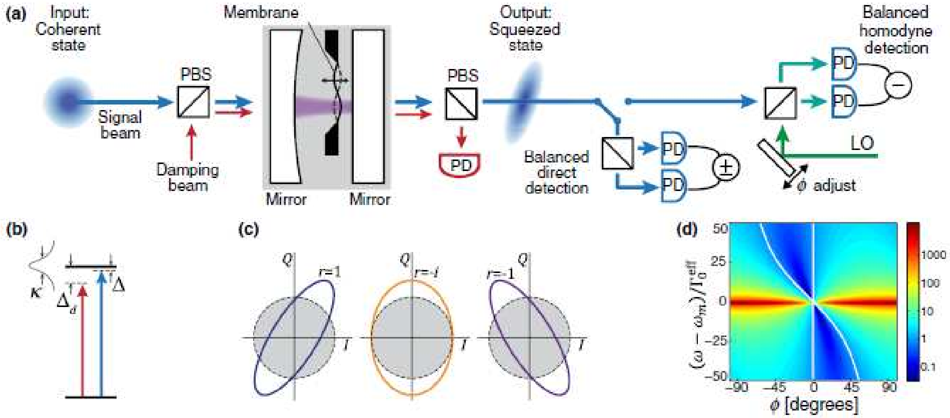
- Caves *et al.*, “On the measurement of a weak classical force coupled to a quantum-mechanical oscillator. I. Issues of principle,” Rev. Mod. Phys. **52**, 341 (1980).
- Yuen, “Contractive States and the Standard Quantum Limit for Monitoring Free-Mass Positions,” Phys. Rev. Lett., **51**, 719-722 (1983).
- Caves, “Defense of the standard quantum limit for free-mass position,” Phys. Rev. Lett., **54**, 2465-2468 (1985).
- Ozawa, “Measurement breaking the standard quantum limit for free-mass position,” Phys. Rev. Lett., **60**, 385-388 (1988).
- Braginsky and Khalili, *Quantum Measurement* (Cambridge University Press, Cambridge, 1992).



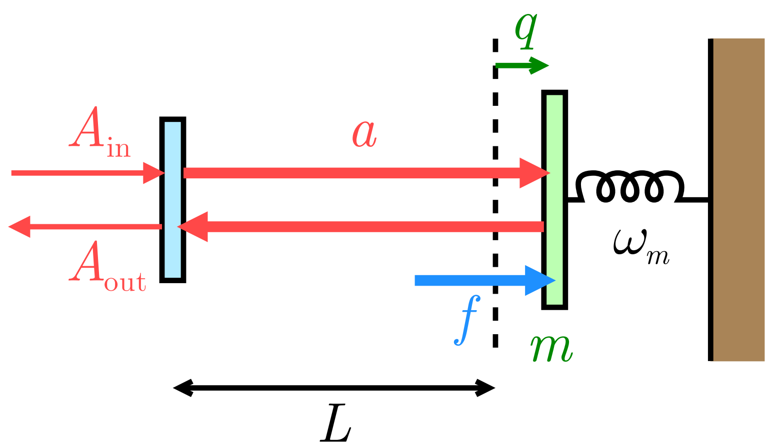
Brooks *et al.*, Nature **488**, 476 (2012)



Safavi-Naeini *et al.*, Nature **500**, 185 (2013)



Purdy *et al.*, PRX **3**, 031012 (2013)



- **Linearized**:

$$\frac{dq(t)}{dt} = \frac{p(t)}{m}, \qquad \frac{dp(t)}{dt} = -m\omega_m^2 q(t) + \hbar\kappa a_1(t) + f(t), \tag{1}$$

$$\frac{da_1(t)}{dt} = -\frac{\gamma}{2}a_1(t) + \sqrt{\gamma}\xi_1(t), \qquad \frac{da_2(t)}{dt} = \kappa q(t) - \frac{\gamma}{2}a_2(t) + \sqrt{\gamma}\xi_2(t), \tag{2}$$

$$\eta_1(t) = \sqrt{\gamma}a_1(t) - \xi_1(t), \qquad \eta_2(t) = \sqrt{\gamma}a_2(t) - \xi_2(t). \tag{3}$$

- Let $\boldsymbol{x}(t) = (q, p, a_1, a_2)^\top$, $\boldsymbol{w} = (f, \xi_1, \xi_2)^\top$, $\boldsymbol{y} = (\eta_1, \eta_2)^\top$,

$$\frac{d\boldsymbol{x}(t)}{dt} = \boldsymbol{F}\boldsymbol{x}(t) + \boldsymbol{G}\boldsymbol{w}(t), \qquad \boldsymbol{y}(t) = \boldsymbol{H}\boldsymbol{x}(t) + \boldsymbol{J}\boldsymbol{w}(t). \tag{4}$$

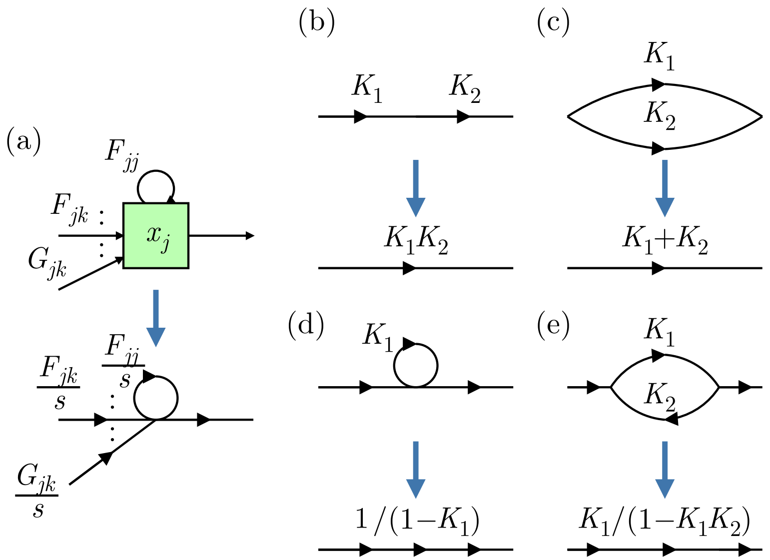
Flowcharts/Block Diagrams

- Laplace transform, ignore initial conditions:

$$\tilde{\boldsymbol{x}}(s) = \frac{1}{s}\left[\boldsymbol{F}\tilde{\boldsymbol{x}}(s) + \boldsymbol{G}\tilde{\boldsymbol{w}}(s)\right], \tag{5}$$

$$\tilde{\boldsymbol{y}}(s) = \left(\sum_{n=1}^{\infty}\frac{1}{s^n}\boldsymbol{H}\boldsymbol{F}^{n-1}\boldsymbol{G} + \boldsymbol{J}\right)\tilde{\boldsymbol{w}}(s), \tag{6}$$

$$\tilde{y}_j(s) = \left(\sum_{n=1}^{\infty}\frac{1}{s^n}H_{jk_n}F_{k_nk_{n-1}}\dots F_{k_3k_2}F_{k_2k_1}G_{k_1l} + J_{jl}\right)\tilde{w}_l(s), \quad \text{(Einstein summation)} \tag{7}$$



- Fourier: $s = -i\omega$.
- Popular in control engineering, similar to Feynman diagrams.

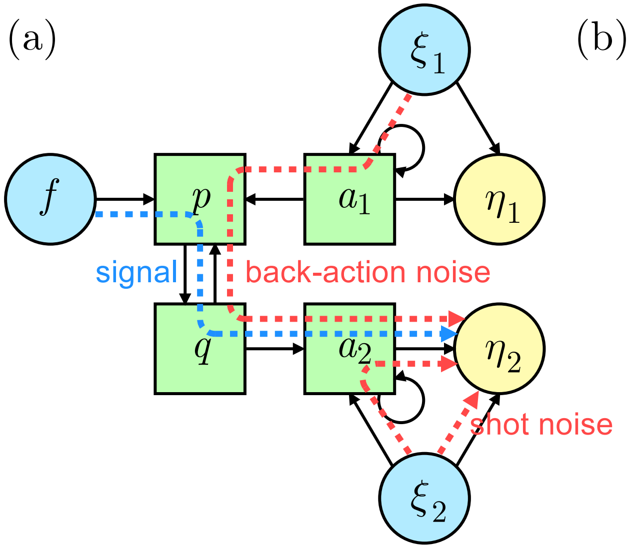
Optomechanics Flowcharts

$$\frac{dq(t)}{dt} = \frac{p(t)}{m}, \qquad \frac{dp(t)}{dt} = -m\omega_m^2 q(t) + \hbar\kappa a_1(t) + f(t), \tag{8}$$

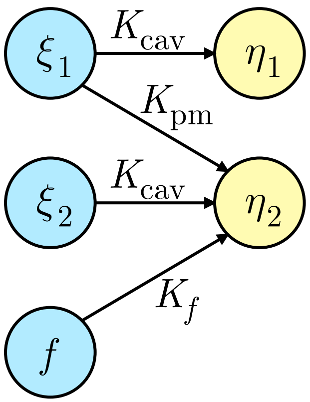
$$\frac{da_1(t)}{dt} = -\frac{\gamma}{2}a_1(t) + \sqrt{\gamma}\xi_1(t), \qquad \frac{da_2(t)}{dt} = \kappa q(t) - \frac{\gamma}{2}a_2(t) + \sqrt{\gamma}\xi_2(t), \tag{9}$$

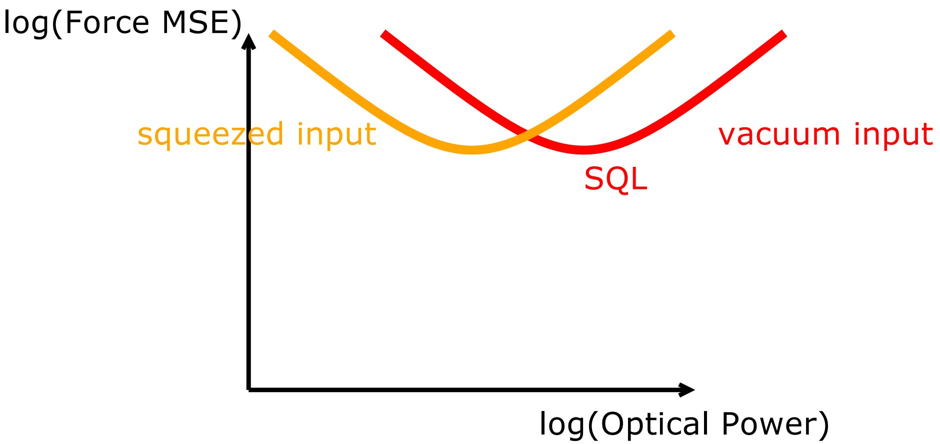
$$\eta_1(t) = \sqrt{\gamma}a_1(t) - \xi_1(t), \qquad \eta_2(t) = \sqrt{\gamma}a_2(t) - \xi_2(t). \tag{10}$$

(a)



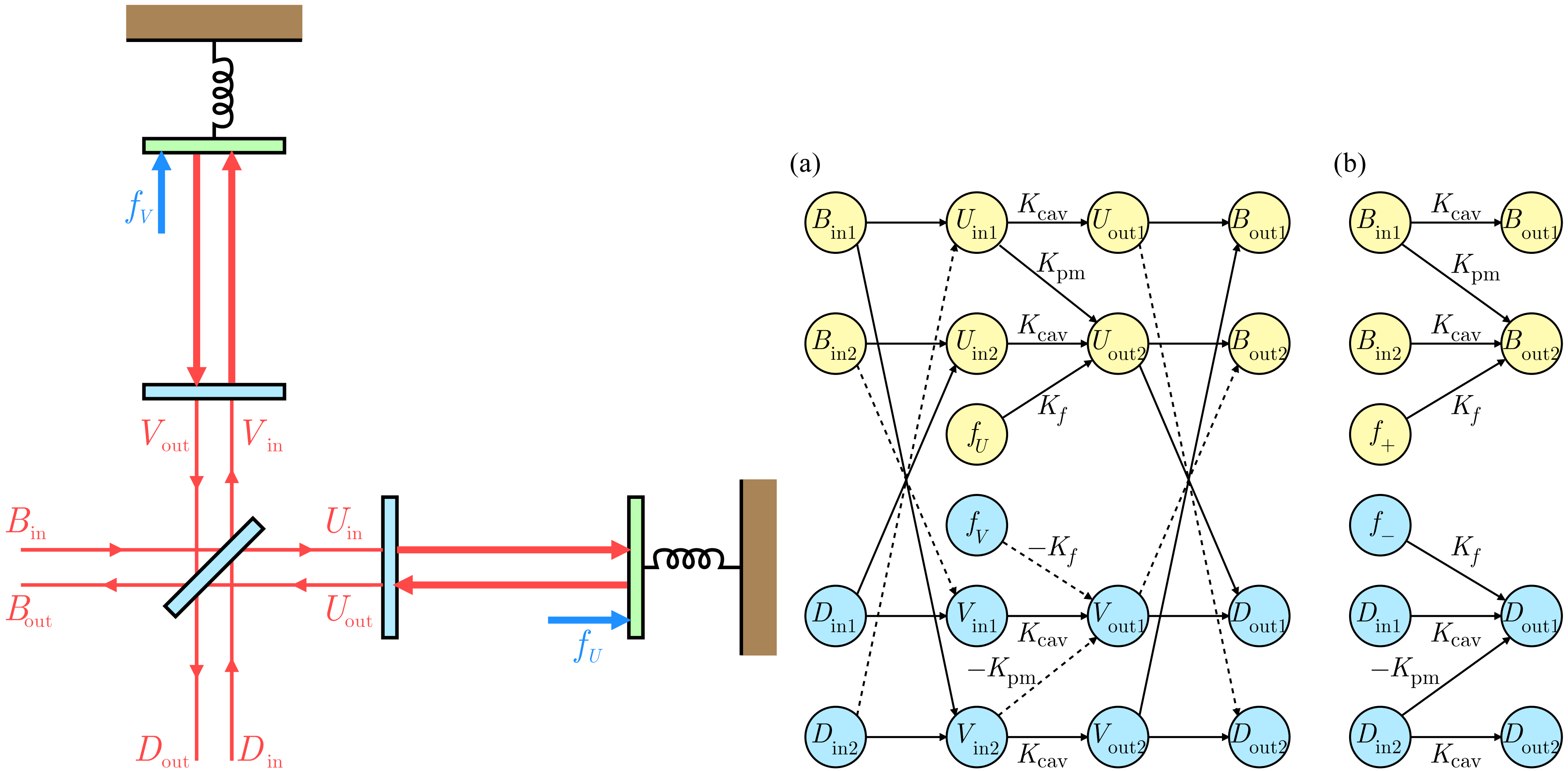
(b)





- $K_{\rm pm}(s)$: **ponderomotive squeezing**, couples input amplitude quadrature ξ_1 to output phase quadrature η_2.

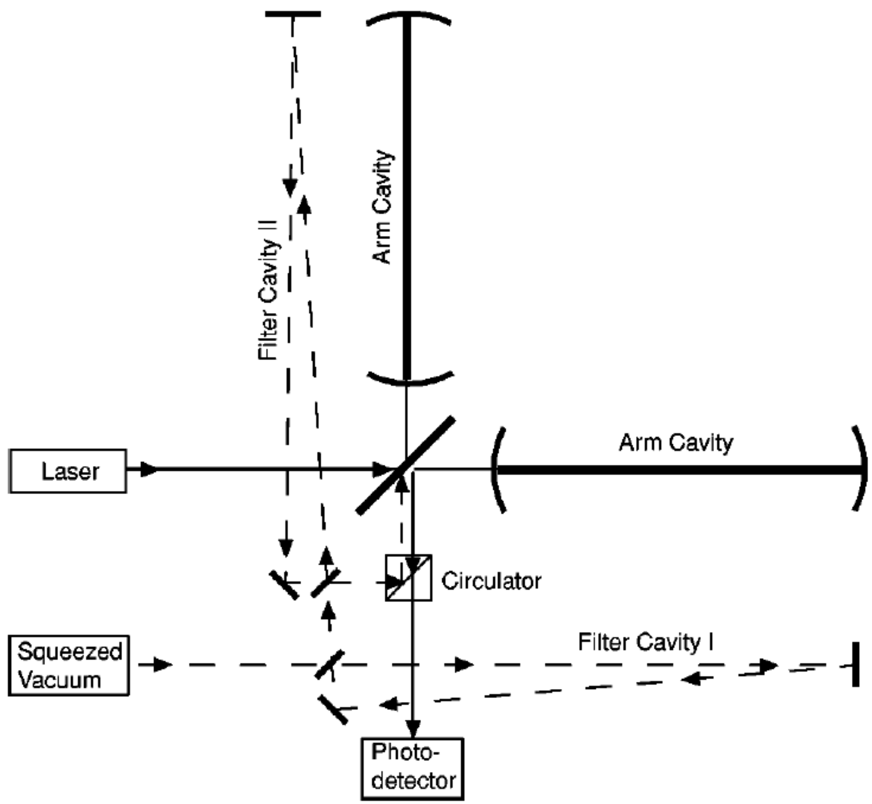
Gravitational-Wave Detector

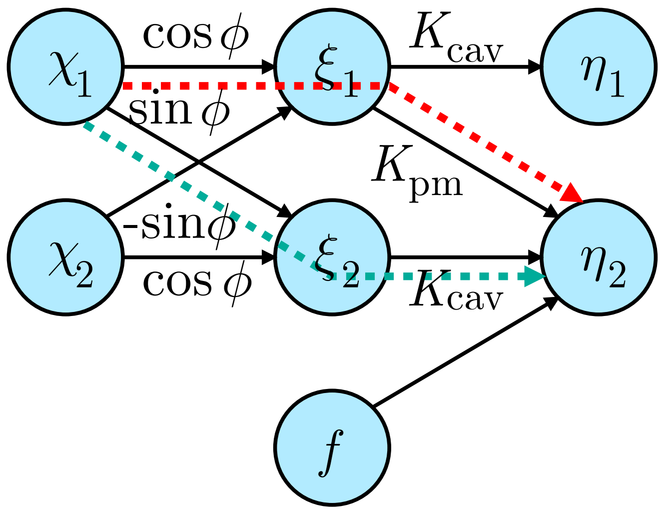


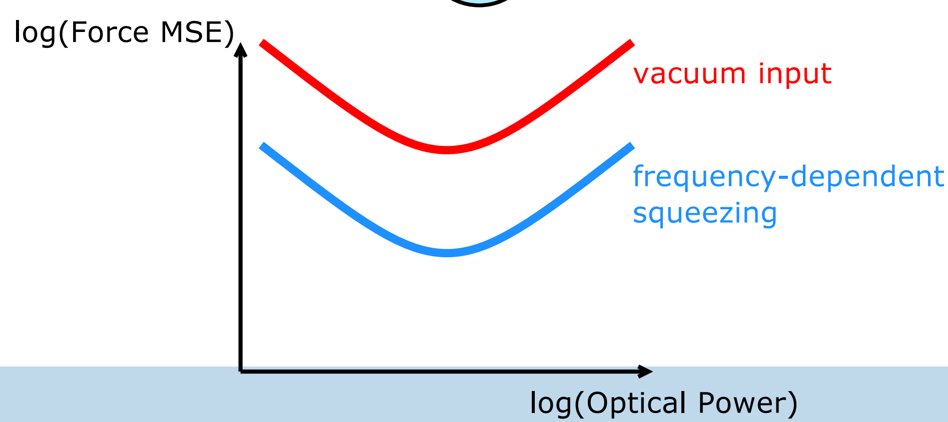
- Pictorial explanation of Caves, PRD **23**, 1693 (1981).

Unruh's Frequency-Dependent Squeezing

- Unruh, in *Quantum Optics, Experimental Gravitation, and Measurement Theory*, edited by P. Meystre and M. O. Scully (Plenum, New York, 1982), p. 647.
    - Squeeze the $K_{\rm pm}\xi_1 + K_{\rm cav}\xi_2$ quadrature.
- Kimble *et al.*, PRD **65**, 022002 (2001).

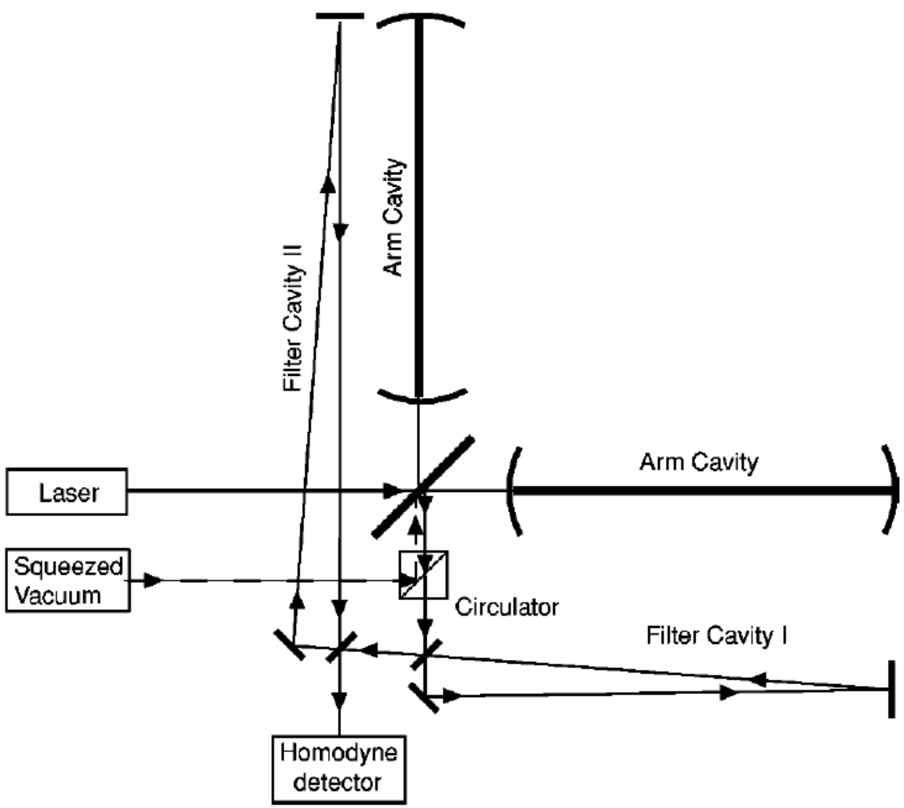


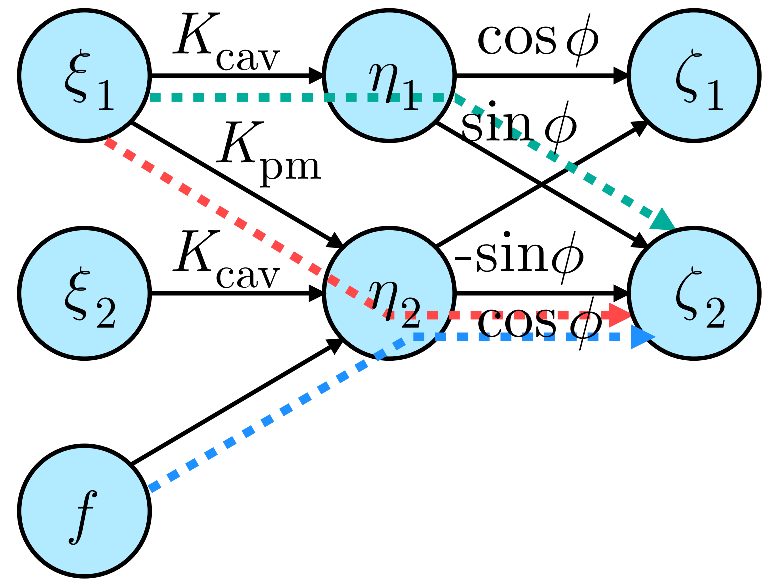


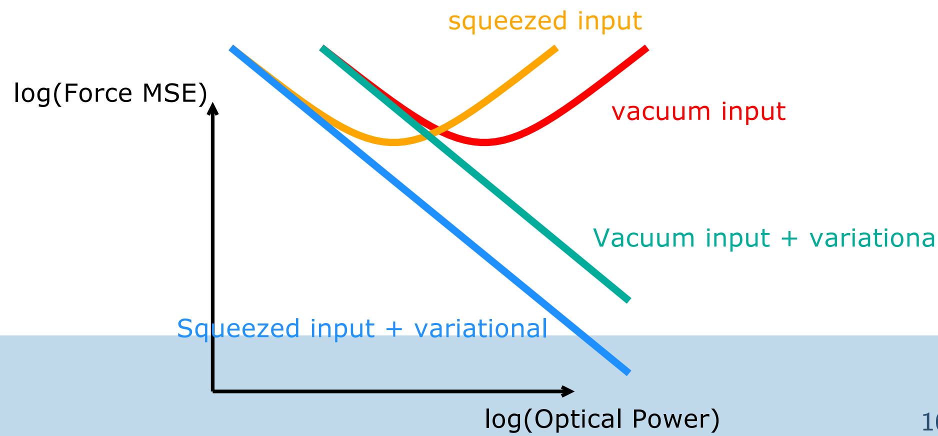


Vyatchanin et al.'s Variational Measurement

- Vyatchanin and Matsko, JETP 77, 218 (1993), etc.
- Kimble *et al.*, PRD **65**, 022002 (2001).
- More sensitive to loss because signal is attenuated by **output** cavities [Khalili, PRD **81**, 122002 (2010)]

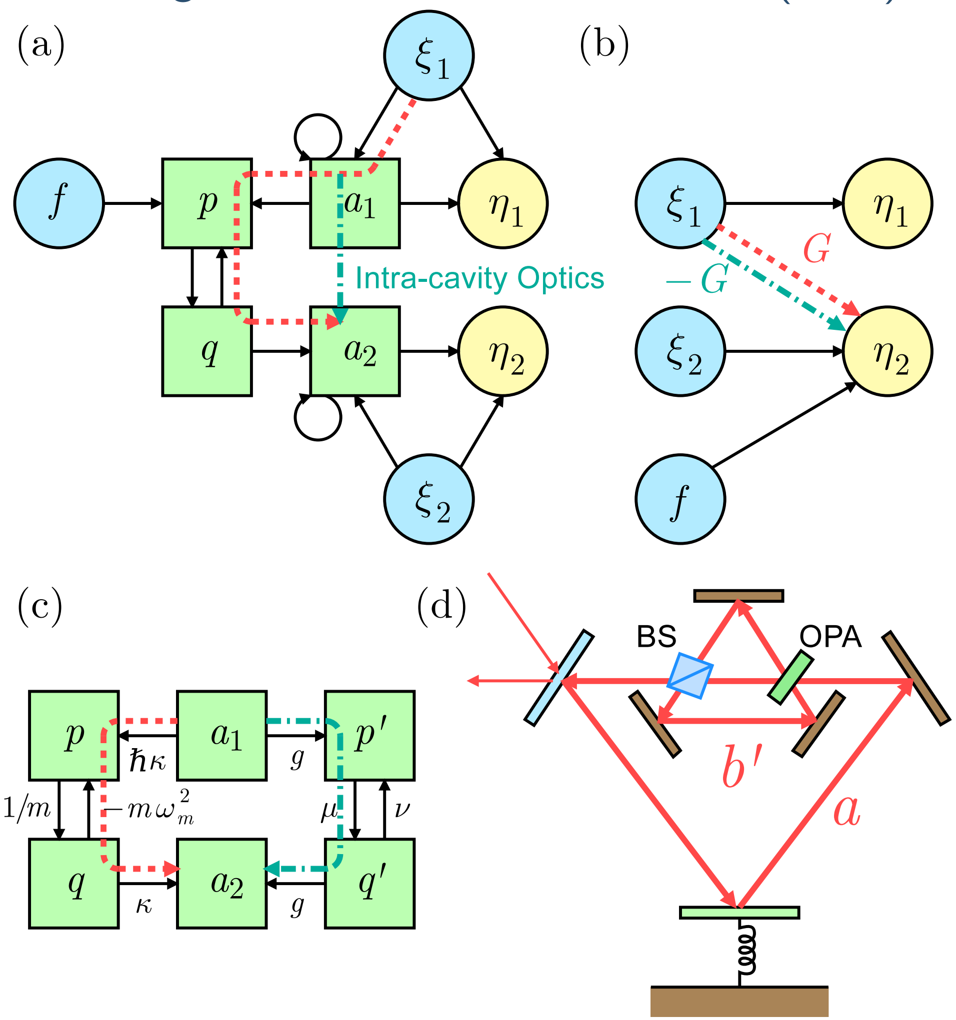


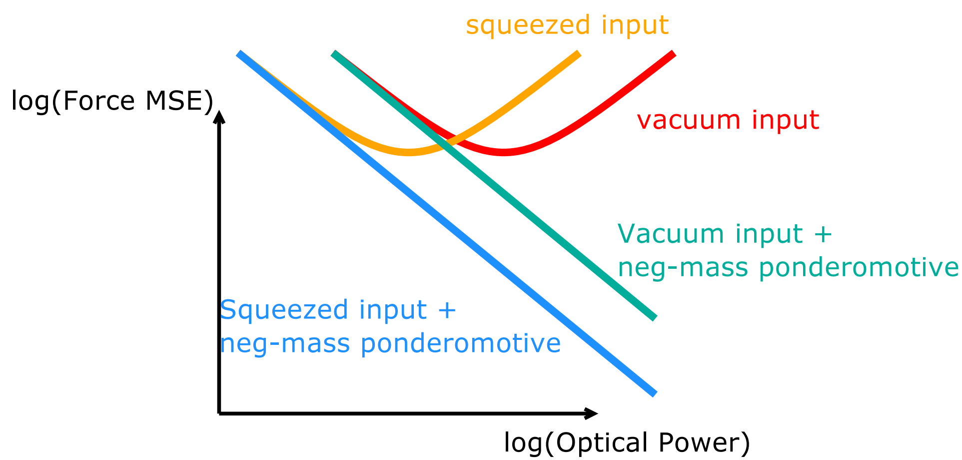




Negative-Mass Ponderomotive Squeezing

- Tsang and Caves, PRL **105**, 123601 (2010).

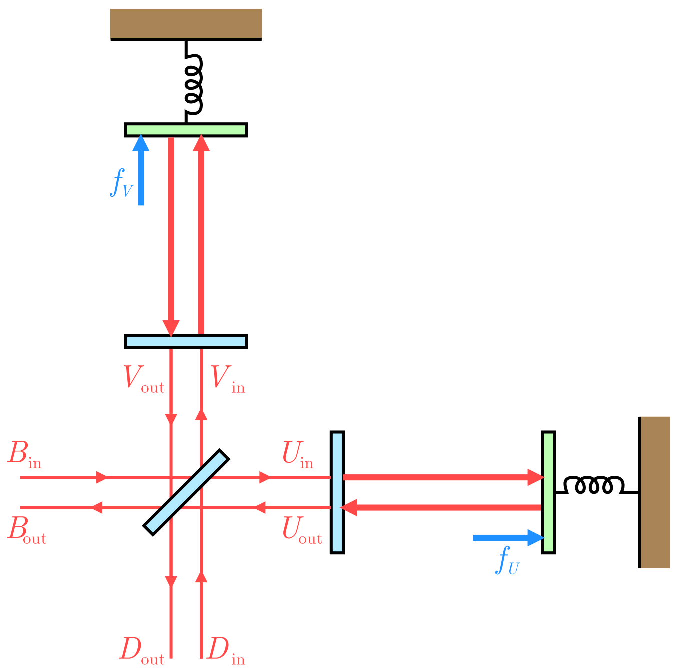


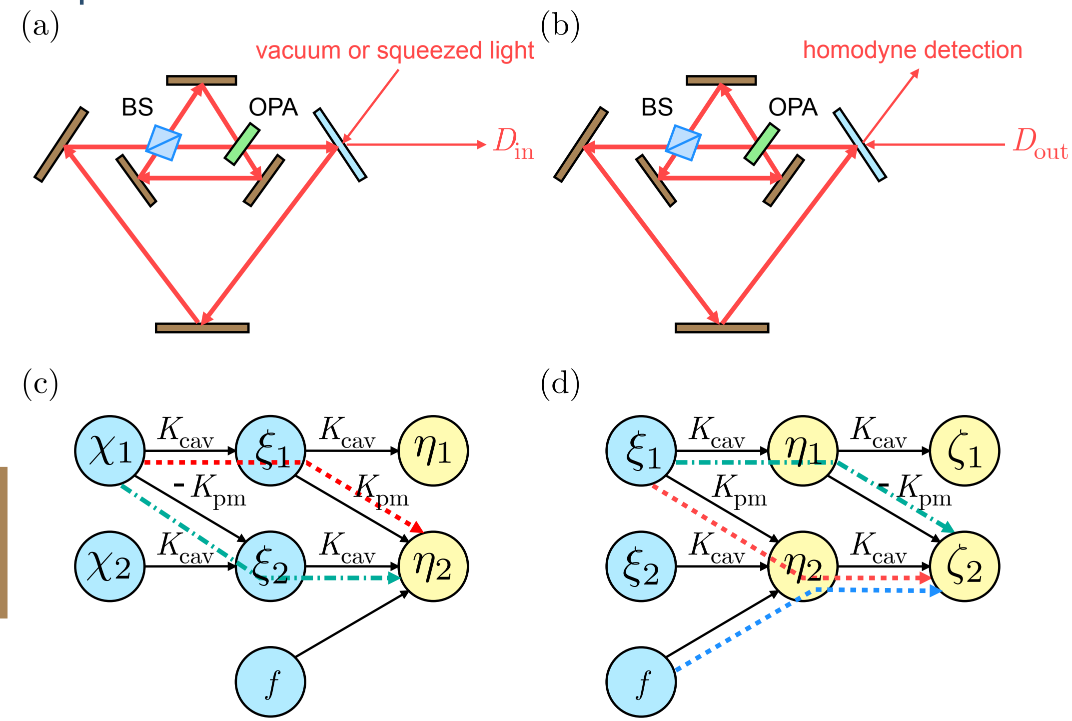


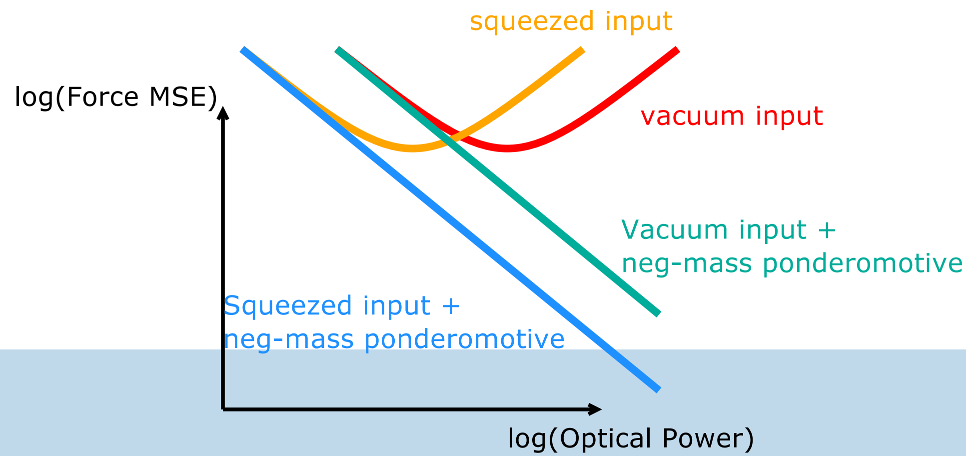
- Experiment? Wimmer, Steinmeyer, Hammerer, Heurs, PRA **89**, 053836 (2014).
- Other possibilities: Zhang, Meystre PRA 88, 043632 (2013); Woolley and Clerk, PRA **87**, 063846 (2013).

Input/Output Optics Modification

- Tsang and Caves, PRL **105**, 123601 (2010): achieve the same behavior as variational measurement by modifying **input** or output.

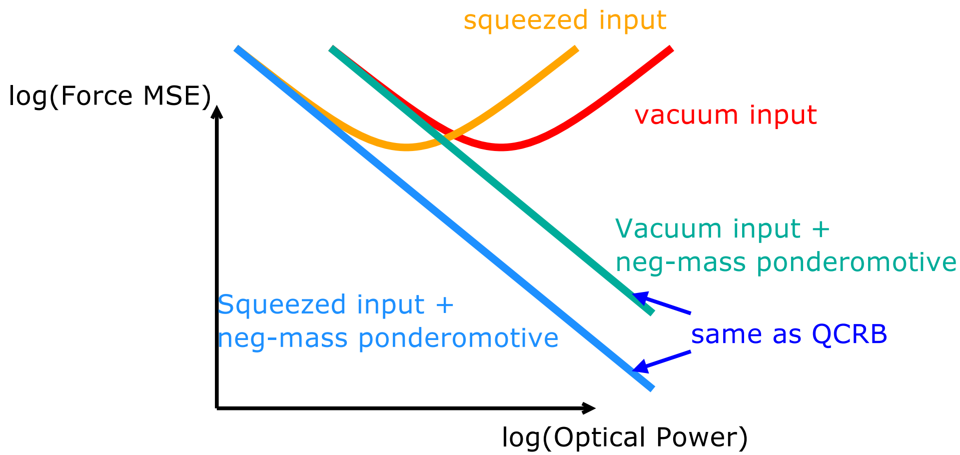




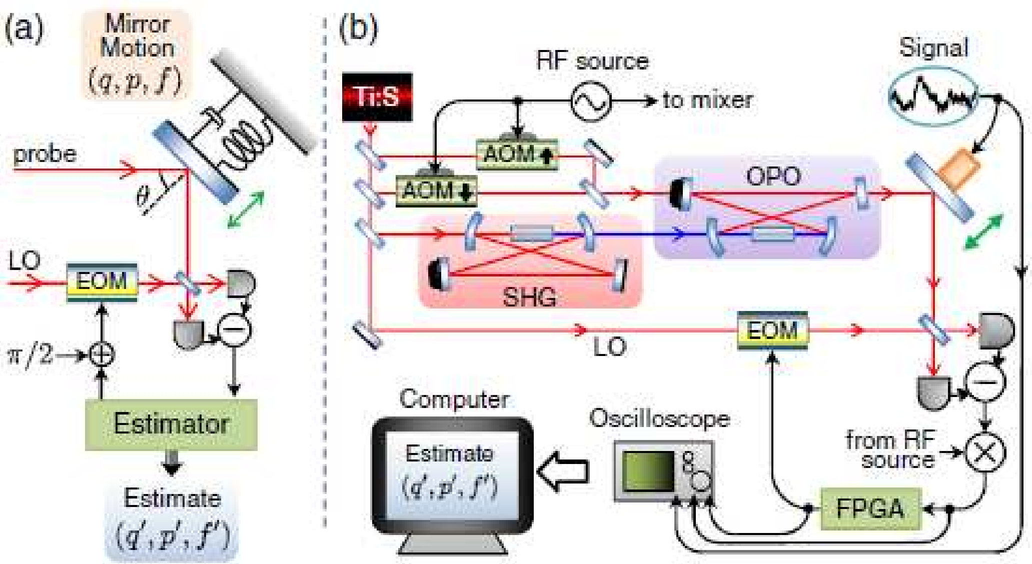


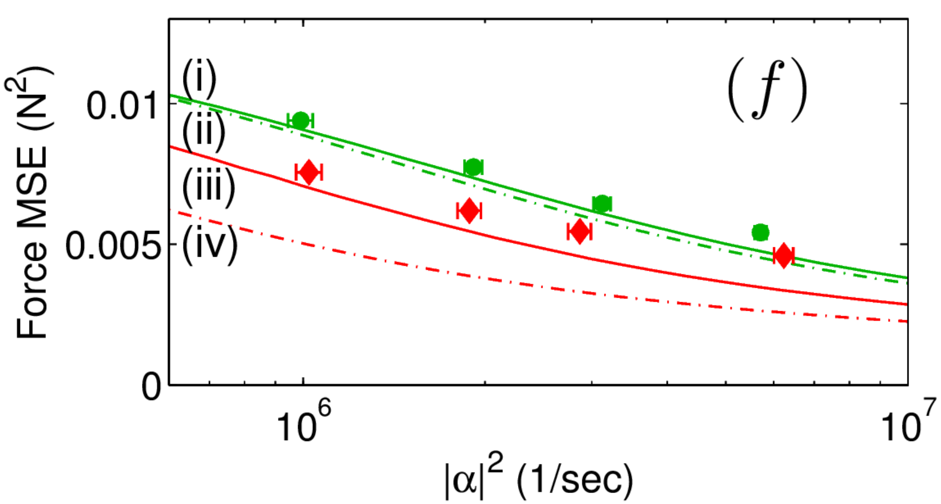
Quantum Cramér-Rao Bound

- Tsang, Wiseman, Caves, PRL 106, 090401 (2011)



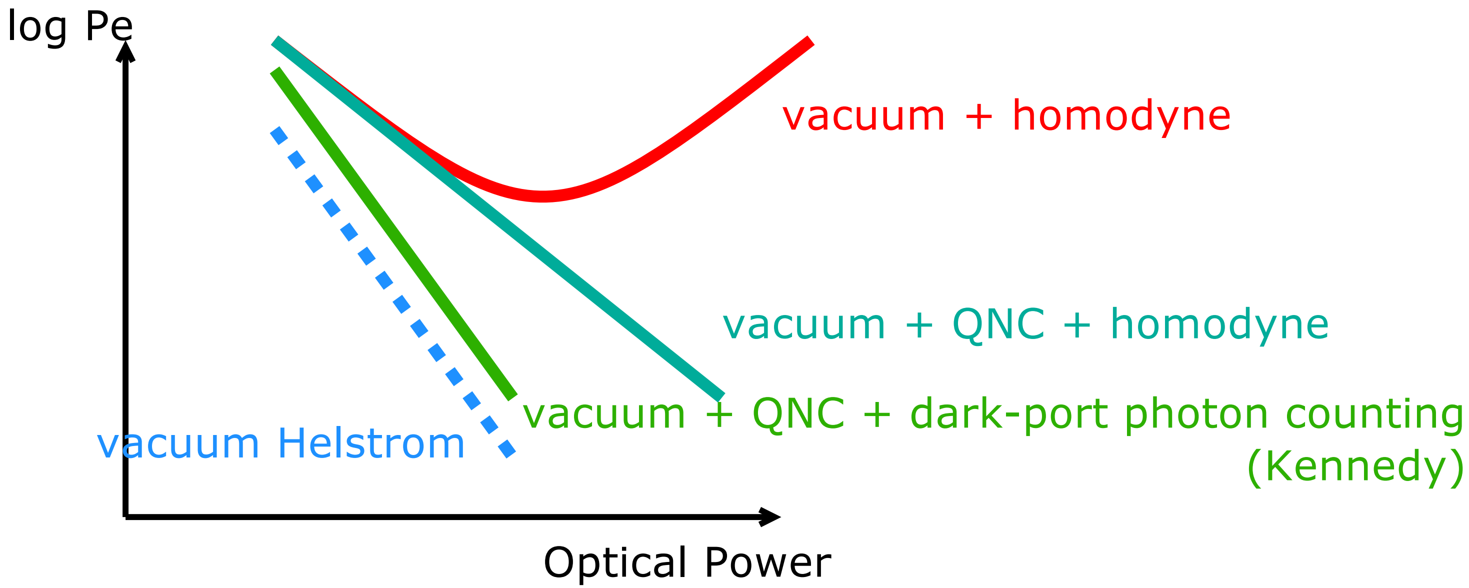
- Negligible backaction noise: Iwasawa *et al.*, PRL **111**, 163602 (2013).

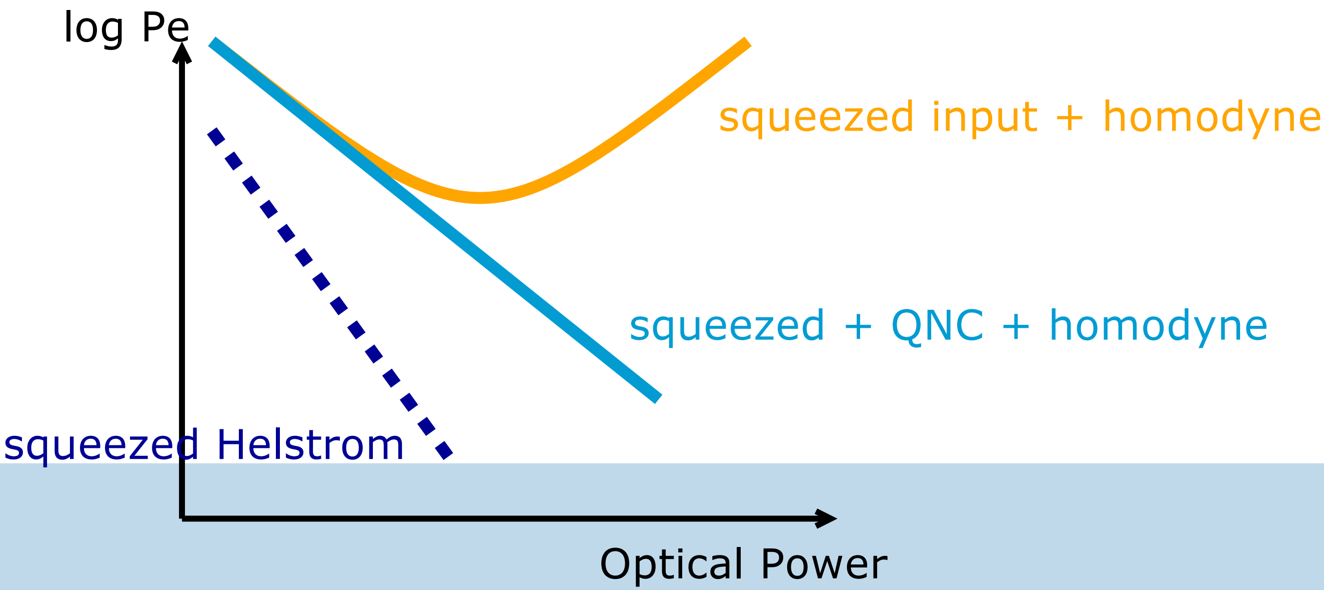




Helstrom Detection Bound

- Gravitational-wave **detection** is binary hypothesis testing.
- Helstrom detection bound: Tsang and Nair, PRA **86**, 042115 (2012).





Backaction Evasion

- Combined equations of motion for intracavity scheme:

$$\frac{d(q+q')}{dt} = \frac{p-p'}{m}, \quad \frac{d(p-p')}{dt} = -m\omega_m^2(q+q') + \hbar\kappa(a_2 - a_2) + f, \tag{11}$$

$$\frac{da_2}{dt} = -\frac{\gamma}{2}a_2 + \kappa(q+q') + \sqrt{\gamma}\xi_2. \tag{12}$$

- Equal backaction on dp/dt and dp'/dt, cancels in $d(p-p')/dt$.
- **QND observables**:

$$Q(t) \equiv q(t) + q'(t), \qquad \Pi(t) \equiv p(t) - p'(t), \tag{13}$$

$$\frac{dQ}{dt} = \frac{\Pi}{m}, \qquad \frac{d\Pi}{dt} = -m\omega_m^2 Q, \tag{14}$$

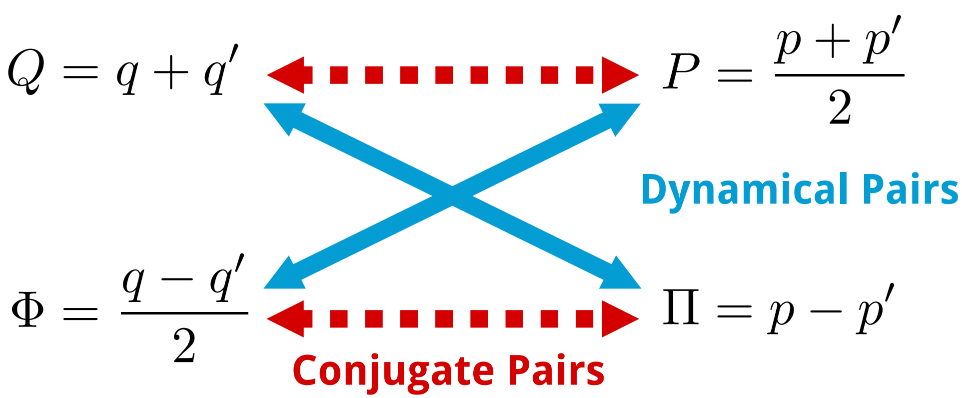
$$\left[Q(t), Q(t')\right] = 0, \qquad \left[\Pi(t), \Pi(t')\right] = 0, \qquad \left[Q(t), \Pi(t')\right] = 0. \tag{15}$$

- **Spectral theorem**: Commuting observables are **simultaneously diagonalizable**, equivalent to classical probability theory, can be measured simultaneously and repeatedly (von Neumann).

$$\tag{16}$$

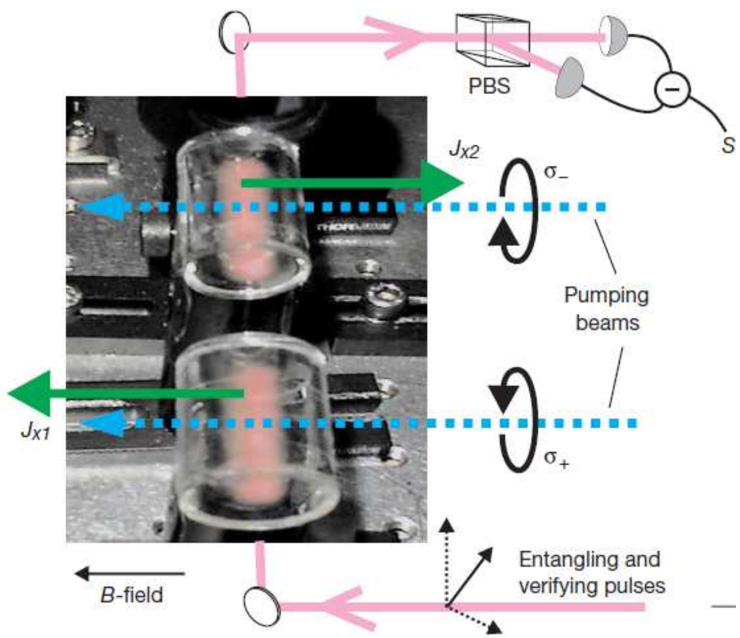
- $$\frac{dQ}{dt} = \frac{\Pi}{m}, \quad \frac{d\Pi}{dt} = -m\omega_m^2 Q, \quad [Q(t), Q(t')] = 0, \quad [\Pi(t), \Pi(t')] = 0, \quad [Q(t), \Pi(t')] = 0. \tag{17}$$

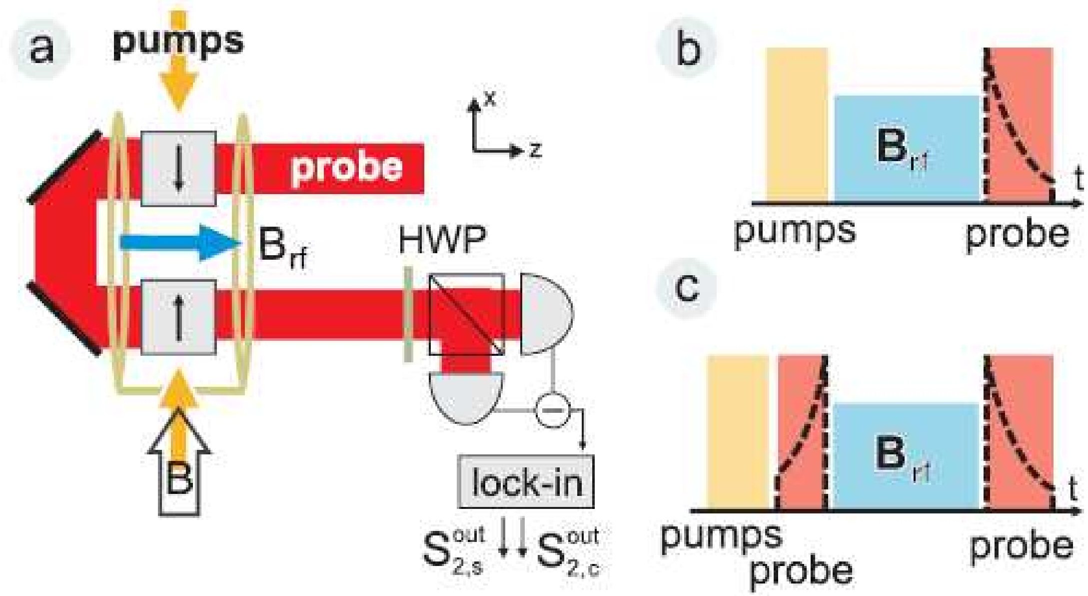
Positive-mass oscillator $\{q, p\}$
and negative-mass oscillator $\{q', p'\}$



Atomic Spin Ensembles

- Julsgaard, Kozhekin, and Polzik, Nature **413**, 400 (2001): $\langle\Delta(q+q')^2\rangle\,\langle\Delta(p-p')^2\rangle \leq 1/4$
- Magnetometry: Wasilewski *et al.*, PRL **104**, 133601 (2010).





- Atomic spin + mechanics: Hammerer, Aspelmeyer, Polzik, Zoller, PRL **102**, 020501 (2009).

Generalization to Arbitrary Dynamics

- Tsang and Caves, PRX **2**, 031016 (2012).
- Define two sets of conjugate variables $\{Q, P\}$ and $\{\Phi, \Pi\}$,

$$[Q_j, P_k] = [\Phi_j, \Pi_k] = i\delta_{jk}, \tag{18}$$

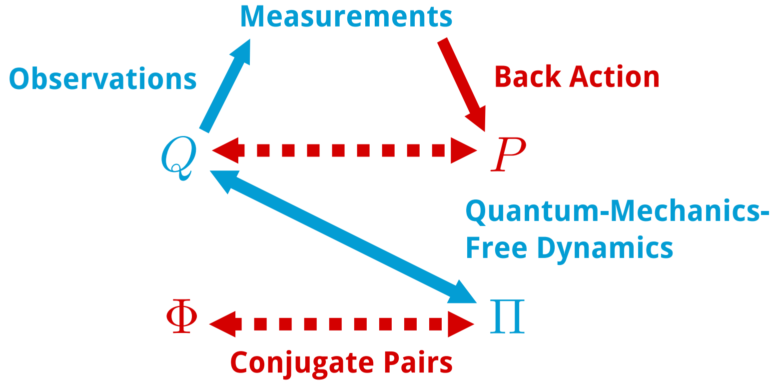
  but otherwise commute, e.g., $[Q, \Pi] = 0$.
- Define Hamiltonian

$$H = \frac{1}{2}\left[P_j f_j(Q,\Pi,t) + \Phi_j g_j(Q,\Pi,t) + \text{H.c.}\right] + h(Q,\Pi,t), \tag{19}$$

- Equations of motion for Q and Π:

$$\frac{dQ}{dt} = f(Q,\Pi,t), \qquad \frac{d\Pi}{dt} = -g(Q,\Pi,t). \tag{20}$$

- **Similar math**: Koopman, PNAS **17**, 315 (1931); Appendix D, Gough and James, IEEE TAC **54**, 2530 (2009).



Terminology

- Definition of QND observables:

$$[Q(t), Q(t')] = 0. \tag{21}$$

- Misconception: this is equivalent to

$$\frac{dQ(t)}{dt} = 0. \tag{22}$$

(e.g., Monroe, Physics Today 64, 8 (2011)) this is sufficient, but not necessary.

- Quantum nondemolition?
  - may be confused with the $dQ/dt = 0$ condition.
  - Is nondemolition even a word?
- Classical subsystem?
  - Many classicality definitions: positive quasiprobability, classical simulability, etc.
  - Positive Wigner can still model quantum measurement backaction (ad-hoc episdemic restrictions, e.g., Bartlett, Rudolph, Spekkens, PRA **86**, 012103 (2012)).
- Backaction-free subsystem?
  - The subset of observables have **no quantum feature at all** owing to spectral theorem, not just backaction.
- **Quantum-mechanics-free subsystem**:
  - Similar flavor to “decoherence-free subsystem.”

Discrete Variables

- A discrete observable $Z(t)$ cannot commute with itself at all times unless $dZ/dt = 0$ [Unruh PRD **19**, 2888 (1979)].
- Stroboscopic QND:

$$[Z(t_j), Z(t_k)] = 0 \text{ for discrete times } t_0, t_1, t_2, \ldots \tag{23}$$

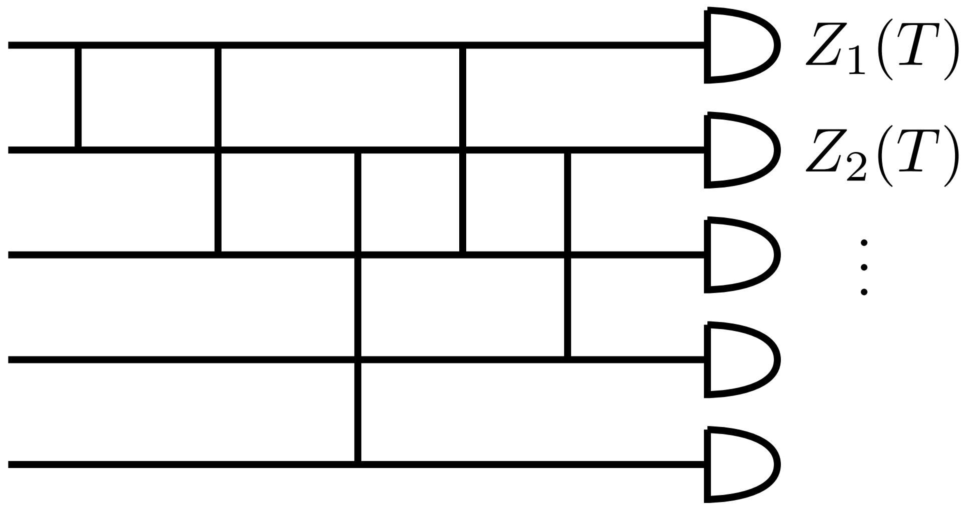
- Arbitrary dynamics can be implemented by considering quantum Toffoli gate, which is a stroboscopic QMFS:

$$Z_1' = Z_1, \qquad Z_2' = Z_2, \qquad Z_3' = \left[I - \frac{1}{2}(I - Z_1)(I - Z_2)\right] Z_3. \tag{24}$$

- Toffoli gates can perform universal classical computation (Feynman's quantum model of classical computer), so stroboscopic QMFS can follow arbitrary discrete-variable discrete-time dynamics.
- Measure Z_j at any discrete time without backaction.

QND Monitoring of Quantum Computers

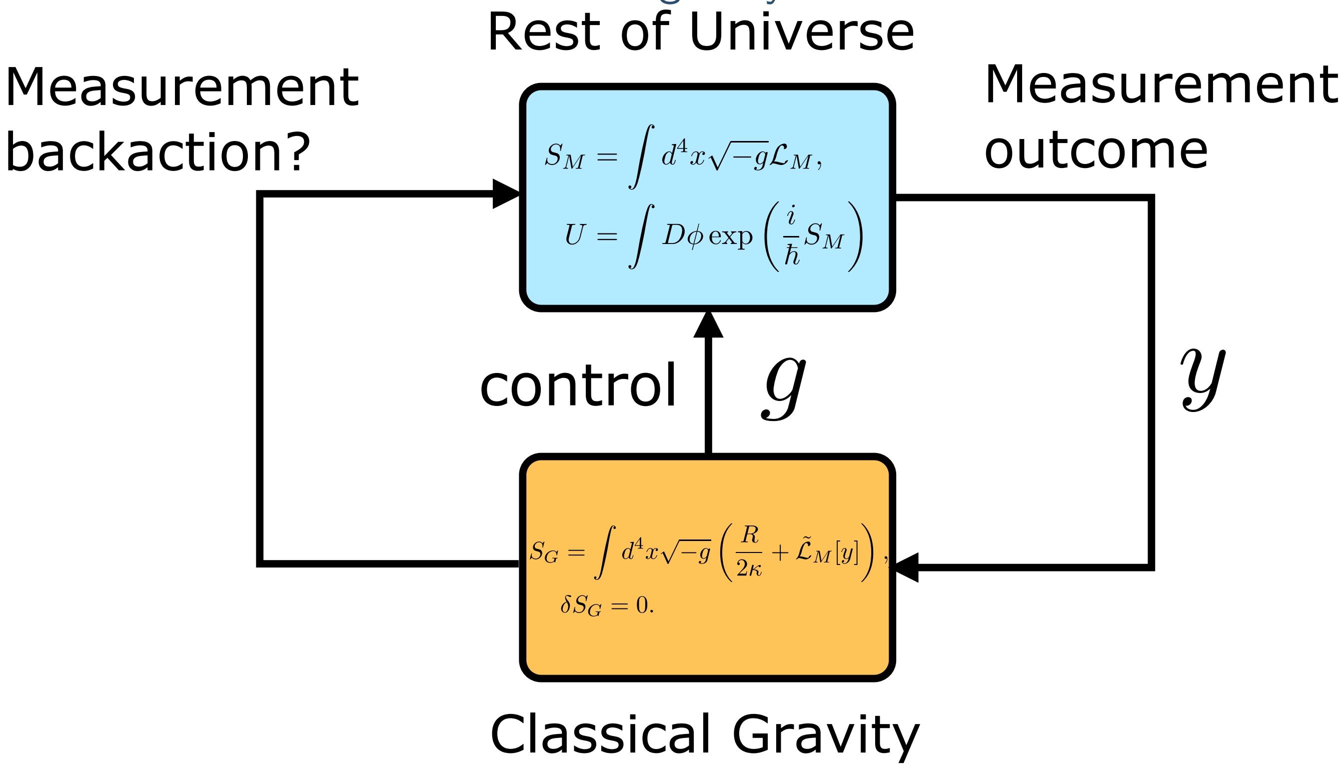
- Quantum computer:



- Any observable that commutes with the final measured Heisenberg-picture observables $Z_j(T)$ can also be measured without affecting the original quantum dynamics.
- Ozawa, PRL **80**, 631 (1998).
- connection with "Heisenberg representation" of quantum error correction?

Quantum Mechanics + Classical Gravity

- Quantum measurement and control model of gravity:



- Diosi, Penrose *et al.*: gravitation-induced decoherence
- If gravity measures QND observables, there's no quantum backaction.
- QMFS from particle + anti-particle?