Quantum Enhancement of Optical Beam Position Accuracy by Self-Focusing

Mankei Tsang

mankei@mit.edu

Center for Extreme Quantum Information Theory (xQIT),
Research Laboratory of Electronics, MIT
May 7, 2008
Outline

- Standard and Heisenberg quantum limits of optical beam position accuracy

- Quantum theory of self-focusing

- Beating the standard quantum limit by self-focusing
Optical Beam Position and Momentum

Atomic Force Microscope: Gravitational Wave Detector:

Meyer and Amer, APL 53, 1045 (1988)
Waldman Class. Quantum Grav. 23, S653 (2006)

Quantum Enhancement of Optical Beam Position Accuracy by Self-Focusing – p.3/10
Beam Position and Momentum Operators

\[ \hat{X} = \frac{1}{N} \int x \hat{A}^\dagger(x) \hat{A}(x) \, dx, \]

\[ W = \left\langle \frac{1}{N} \int_{-\infty}^{\infty} x^2 \hat{A}^\dagger(x) \hat{A}(x) \, dx \right\rangle^{1/2}, \]

\[ \hat{K} = \frac{1}{N} \int k \hat{a}^\dagger(k) \hat{a}(k) \, dk, \]

\[ B = \left\langle \frac{1}{N} \int_{-\infty}^{\infty} k^2 \hat{a}^\dagger(k) \hat{a}(k) \, dk \right\rangle^{1/2}. \]

(Assuming \( \langle \hat{X} \rangle = 0, \langle \hat{K} \rangle = 0 \))
Quantum Limits

Let $\Delta X = \langle \hat{X}^2 \rangle^{1/2}$, $\Delta K = \langle \hat{K}^2 \rangle^{1/2}$,

Standard Quantum Limit (coherent fields):

$$
\Delta X_{\text{SQL}} = \frac{W}{\sqrt{N}} = \frac{1}{2\sqrt{NB}}, \quad \Delta K_{\text{SQL}} = \frac{B}{\sqrt{N}} = \frac{1}{2\sqrt{NW}} \tag{1}
$$

Heisenberg Limit:

$$
\Delta X_{\text{HL}} = \frac{1}{2NB}, \quad \Delta K_{\text{HL}} = \frac{1}{2NW} \tag{2}
$$

Multi-Spatial-Mode Squeezing

Self-Focusing

Chiao, Garmire, and Townes, PRL 13, 479 (1964)

$x$

$W(0)$

$W(z)$

$\chi^{(3)}$

Martin Centurion, femtosecond pulses in KTP
Quantum Theory of Self-Focusing

\[ i \frac{\partial \hat{A}}{\partial z} = -\frac{\beta}{2} \nabla^2 \perp \hat{A} - \gamma \hat{A}^\dagger \hat{A} \hat{A}, \quad \frac{\partial \hat{X}}{\partial z} = \beta \hat{K}, \quad \frac{\partial \hat{K}}{\partial z} = 0. \]  

(3)

\[ \Delta K(z) = \Delta K(0) < \frac{1}{2\sqrt{NW(z)}} < \frac{B(z)}{\sqrt{N}} \]  

(4)

\( \Delta K \) is constant
**Beam Position Accuracy Enhancement**

Ideal case:

\[
\frac{\Delta K}{\Delta K_{\text{SQL}}} = \frac{\Delta X}{\Delta X_{\text{SQL}}} = \frac{W(z)}{W(0)} \quad (5)
\]

- **Loss** and other parasitic effects (e.g. multiphoton absorption) limits maximum achievable enhancement.
Conclusion

- Beam position and momentum are important parameters
- Standard quantum limits and Heisenberg limits exist
- Self-focusing is a simple technique of beating the SQL


http://mankei.tsang.googlepages.com/

Other talks:
- “Resonantly Enhanced Near-Field Lithography,” QTuG1 Tuesday 2:30pm
- “Magnifying Metamaterial Lens Design by Coordinate Transformation,” QFL5 Friday 4:45pm