Quantum Enhancement of Optical Beam Position Accuracy by Self-Focusing

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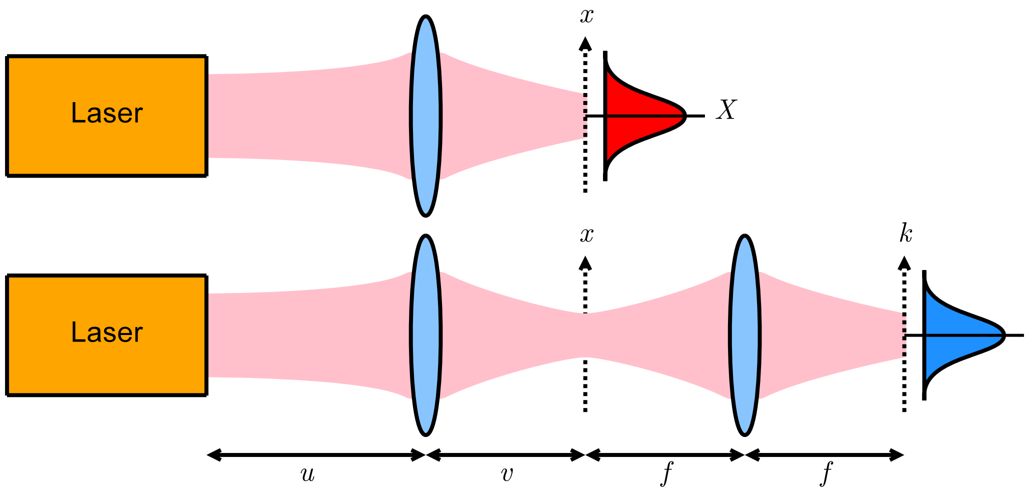
May 7, 2008

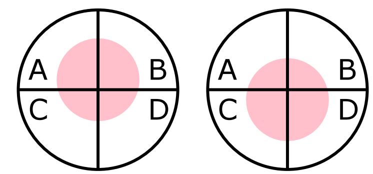
Outline

- Standard and Heisenberg quantum limits of optical beam position accuracy
  - Tsang, Phys. Rev. A **75**, 063809 (2007)
- Quantum theory of self-focusing
- Beating the standard quantum limit by self-focusing

Optical Beam Position and Momentum



Quadrant Photodetector



$X \sim A + B - C - D$

Atomic Force Microscope:

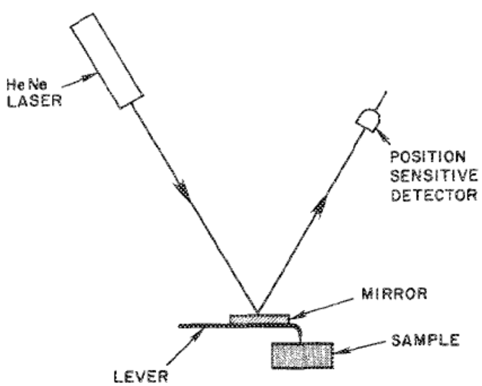
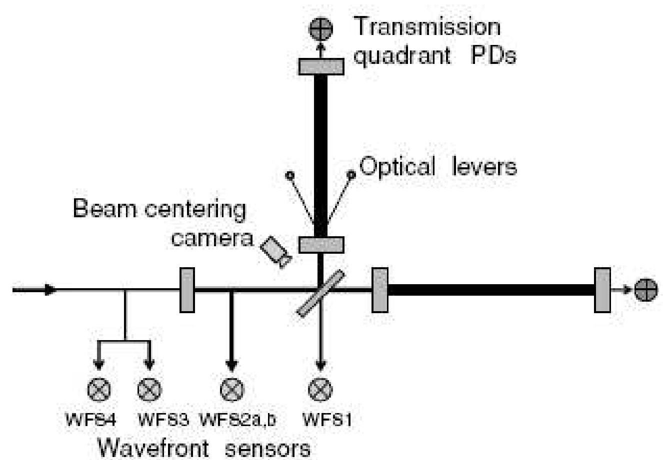


FIG. 1. Cantilever deflection detection scheme.

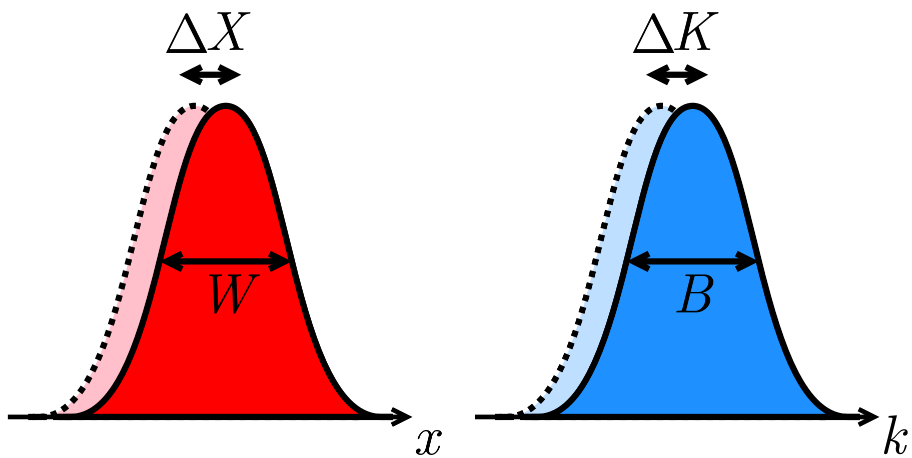
Gravitational Wave Detector:



Meyer and Amer, APL 53, 1045 (1988)

Waldman Class. Quantum Grav. **23**, S653 (2006)

Beam Position and Momentum Operators



$$\hat{A}(x) = \frac{1}{\sqrt{2\pi}} \int \hat{a}(k) \exp(ikx) dk,$$

$$[\hat{A}(x), \hat{A}^\dagger(x')] = \delta(x - x'),$$

$$\hat{X} = \frac{1}{N} \int x \hat{A}^\dagger(x) \hat{A}(x) dx,$$

$$W = \left\langle \frac{1}{N} \int_{-\infty}^{\infty} x^2 \hat{A}^\dagger(x) \hat{A}(x) dx \right\rangle^{1/2},$$

$$N = \left\langle \int \hat{A}^\dagger(x) \hat{A}(x) dx \right\rangle,$$

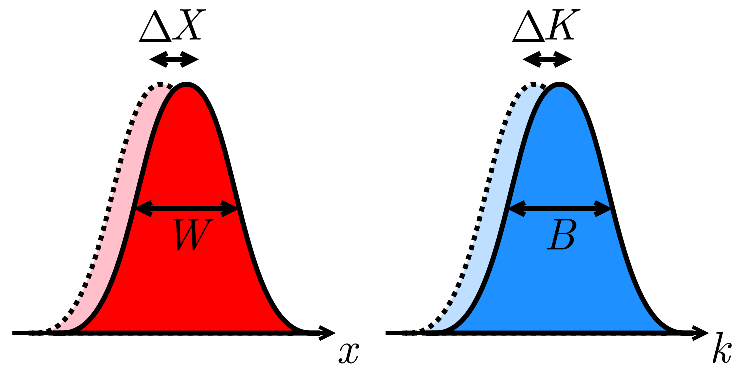
$$[\hat{a}(k), \hat{a}^\dagger(k')] = \delta(k - k'),$$

$$\hat{K} = \frac{1}{N} \int k \hat{a}^\dagger(k) \hat{a}(k) dk,$$

$$B = \left\langle \frac{1}{N} \int_{-\infty}^{\infty} k^2 \hat{a}^\dagger(k) \hat{a}(k) dk \right\rangle^{1/2}$$

- (Assuming $\langle \hat{X} \rangle = 0, \langle \hat{K} \rangle = 0$)

Quantum Limits



- Let $\Delta X = \langle \hat{X}^2 \rangle^{1/2}$, $\Delta K = \langle \hat{K}^2 \rangle^{1/2}$,
- Standard Quantum Limit (coherent fields):

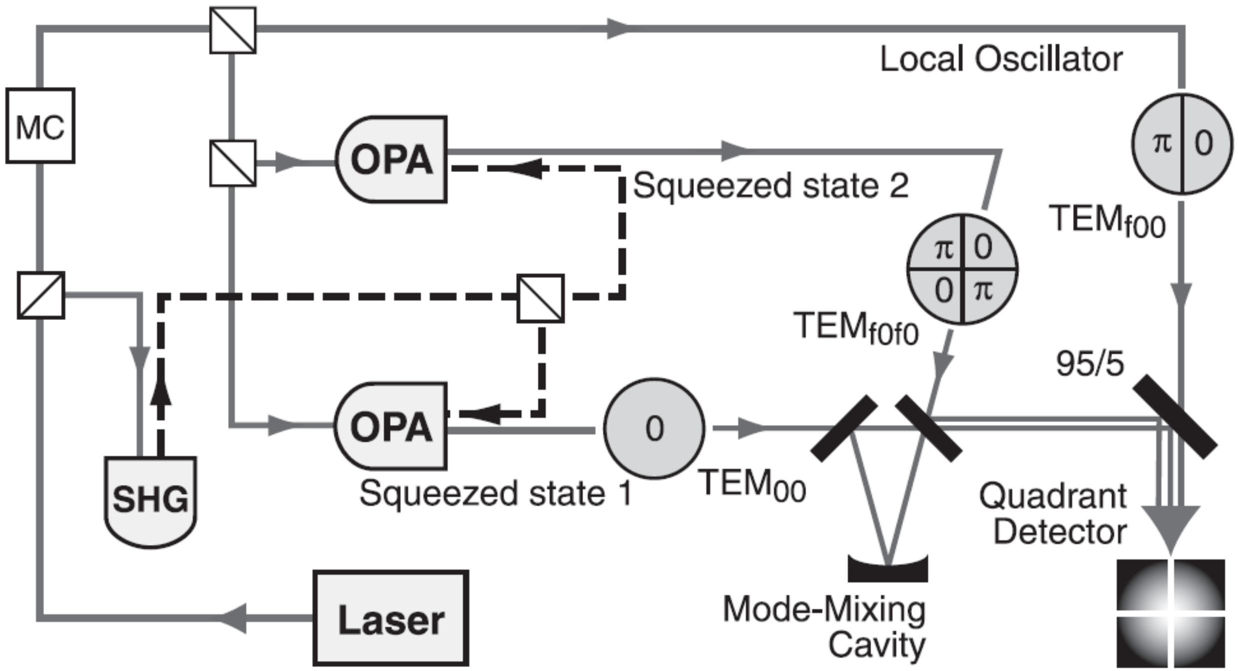
$$\Delta X_{\text{SQL}} = \frac{W}{\sqrt{N}} = \frac{1}{2\sqrt{N}B}, \quad \Delta K_{\text{SQL}} = \frac{B}{\sqrt{N}} = \frac{1}{2\sqrt{N}W} \tag{1}$$

- Heisenberg Limit:

$$\Delta X_{\text{HL}} = \frac{1}{2NB}, \quad \Delta K_{\text{HL}} = \frac{1}{2NW} \tag{2}$$

Fabre *et al.*, Opt. Lett. **25**, 76 (2000); Barnett *et al.*, Eur. Phys. J. D. **22**, 513 (2003); Tsang, Phys. Rev. A **75**, 063809 (2007).

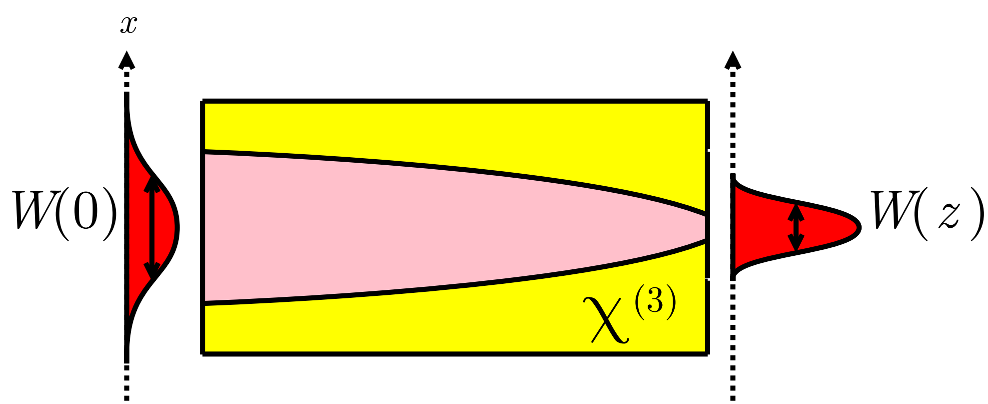
Multi-Spatial-Mode Squeezing

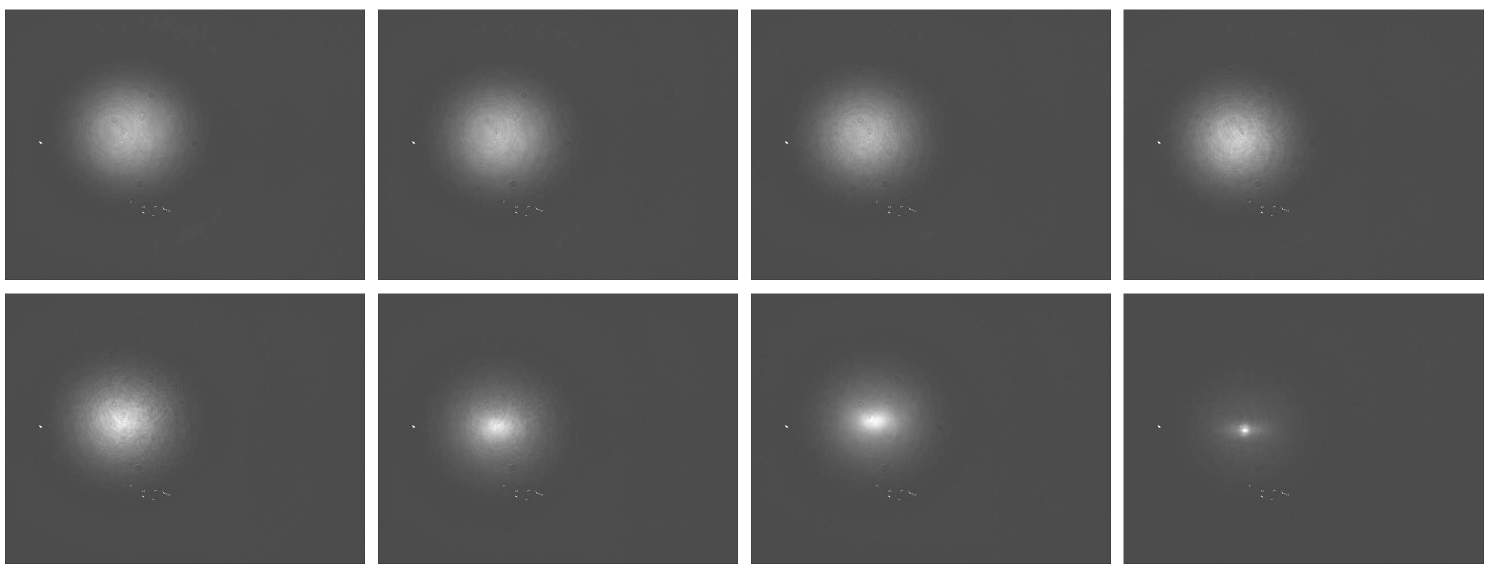


Treps *et al.*, “Quantum Laser Pointer,” Science **301**, 940 (2003).

Self-Focusing

- Chiao, Garmire, and Townes, PRL **13**, 479 (1964)

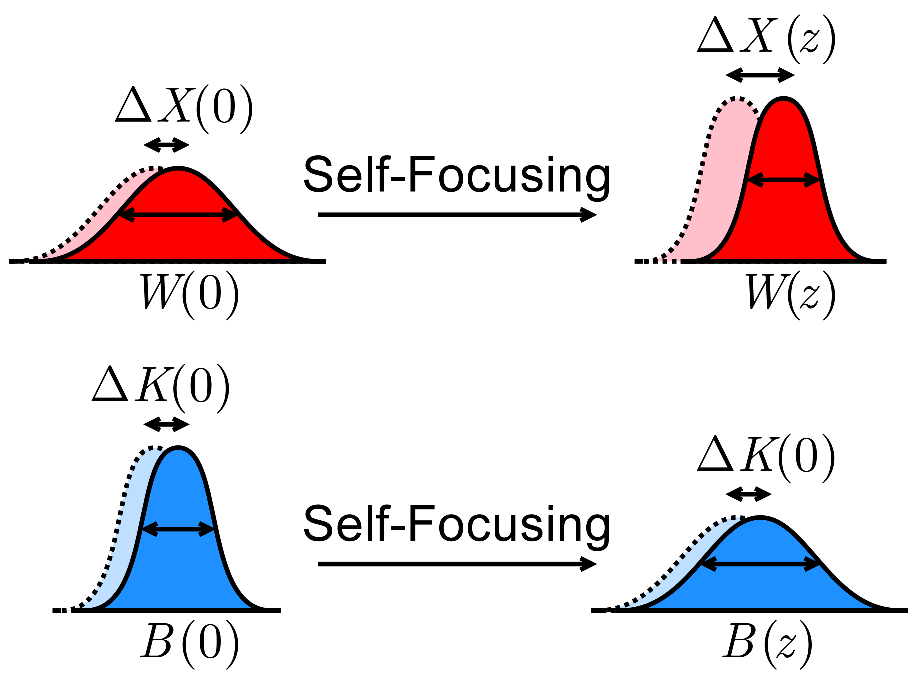




Martin Centurion, femtosecond pulses in KTP

Quantum Theory of Self-Focusing

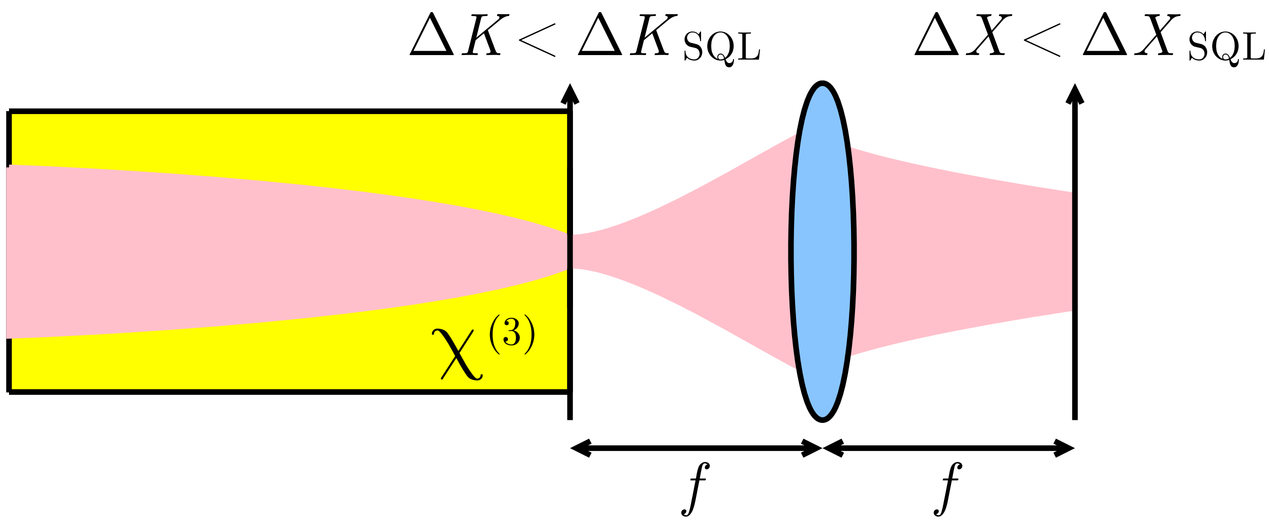
$$i\frac{\partial \hat{A}}{\partial z} = -\frac{\beta}{2}\nabla_\perp^2 \hat{A} - \gamma \hat{A}^\dagger \hat{A}\hat{A}, \quad \frac{\partial \hat{X}}{\partial z} = \beta \hat{K}, \quad \frac{\partial \hat{K}}{\partial z} = 0. \tag{3}$$



- ΔK is constant

$$\Delta K(z) = \Delta K(0) < \frac{1}{2\sqrt{N}W(z)} < \frac{B(z)}{\sqrt{N}} \tag{4}$$

Beam Position Accuracy Enhancement



- Ideal case:

$$\frac{\Delta K}{\Delta K_{\mathrm{SQL}}} = \frac{\Delta X}{\Delta X_{\mathrm{SQL}}} = \frac{W(z)}{W(0)} \tag{5}$$

- Loss and other parasitic effects (e.g. multiphoton absorption) limits maximum achievable enhancement

Conclusion

- Beam position and momentum are important parameters
- Standard quantum limits and Heisenberg limits exist
- Self-focusing is a simple technique of beating the SQL
- Tsang, "Decoherence of quantum-enhanced timing accuracy," Phys. Rev. A **75**, 063809 (2007).
- `http://mankei.tsang.googlepages.com/`
- Other talks:
  - "Resonantly Enhanced Near-Field Lithography,"
QTuG1 Tuesday 2:30pm
  - "Magnifying Metamaterial Lens Design by Coordinate Transformation,"
QFL5 Friday 4:45pm