Quantum Theory of Superresolution for Incoherent Optical Imaging *

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March 2017

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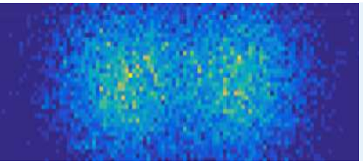
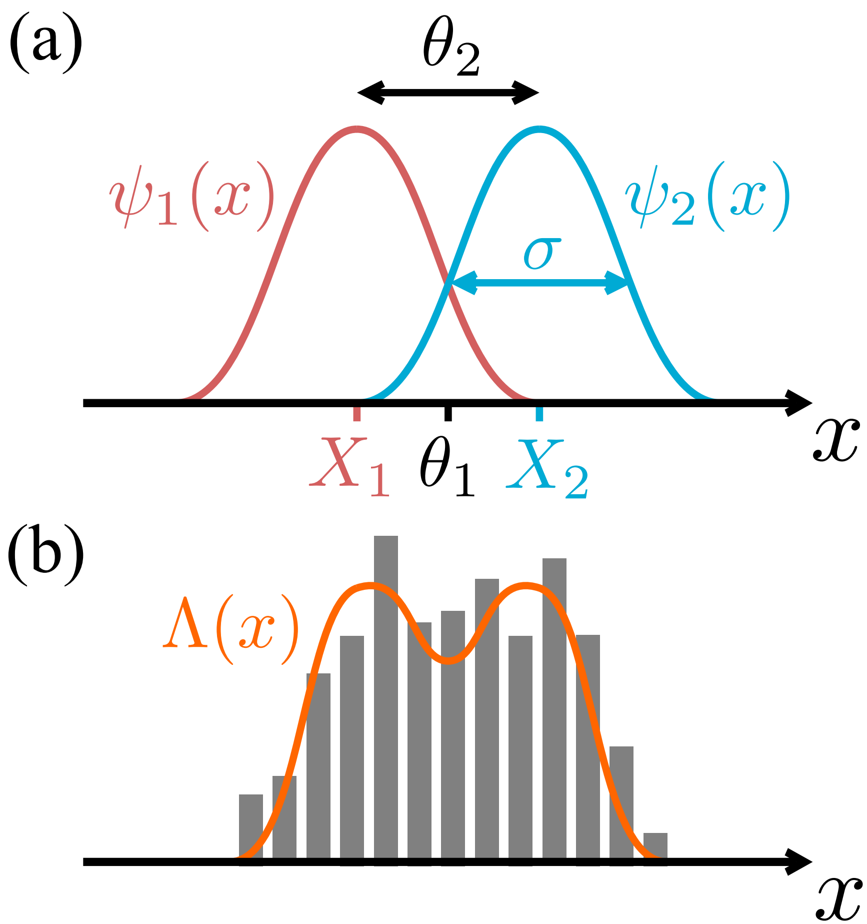
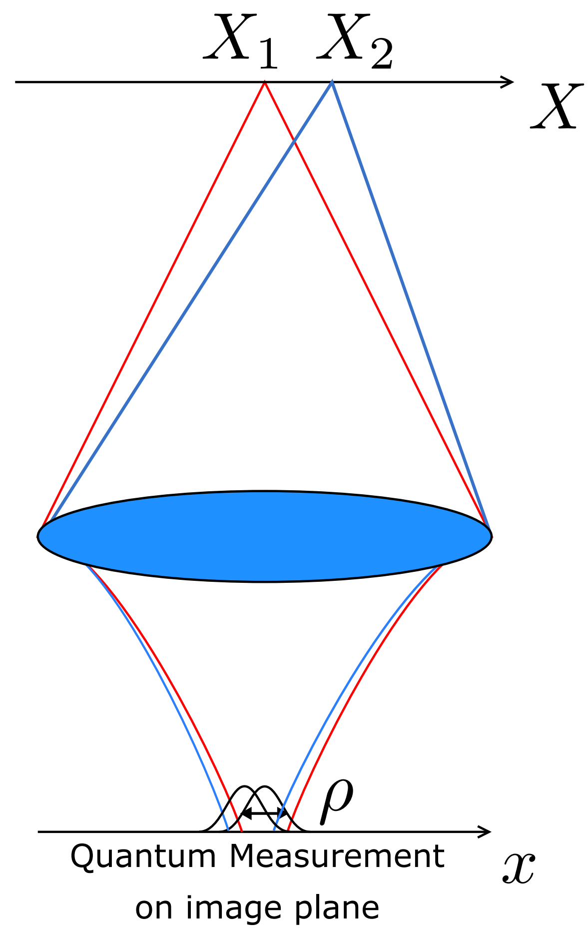


Superresolution Incoherent Imaging





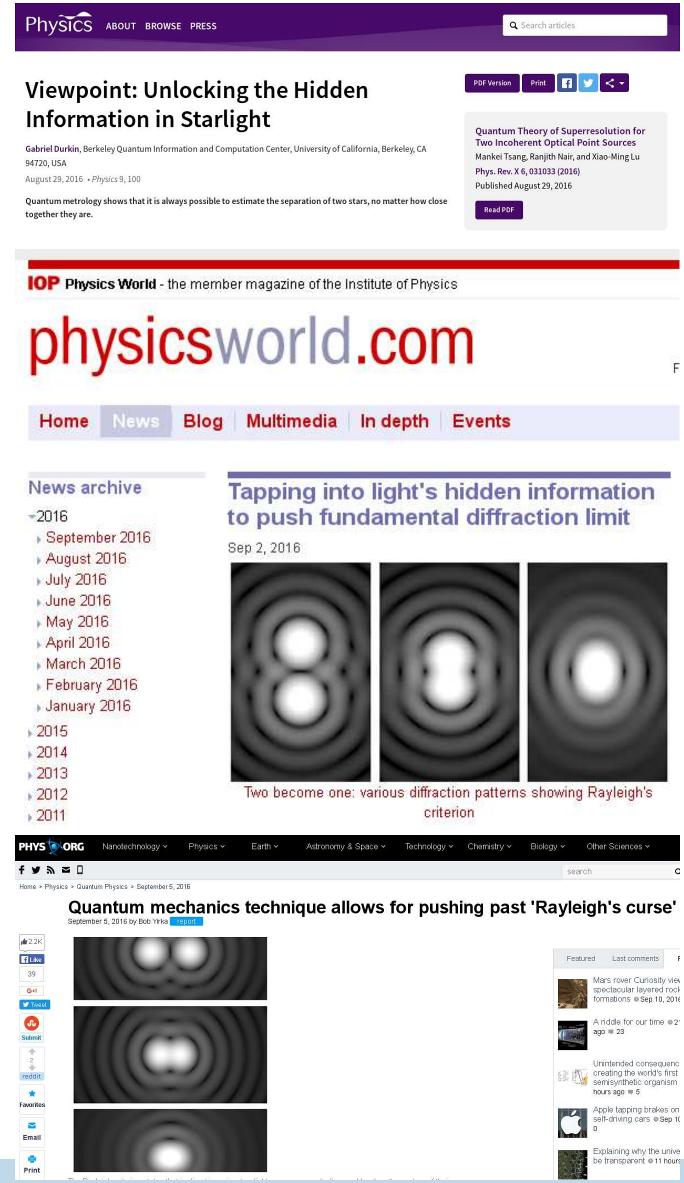
1. Tsang, Nair, and Lu, PRX **6**, 031033 (2016).
2. Nair and Tsang, Opt. Express **24**, 3684 (2016).
3. Tsang, Nair, and Lu, Proc. SPIE **10029**, 1002903 (2016).
4. Nair and Tsang, PRL (Editors' Suggestion) **117**, 190801 (2016).
5. Tsang, arXiv:1605.03799 (2016).
6. Ang, Nair, and Tsang, arXiv:1606.00603 (2016).
7. Tsang, arXiv:1608.03211 (2016).
8. Lu, Nair, Tsang, arXiv:1609.03025 (2016).

Popular Coverage

- Viewpoint in APS Physics
- IoP Physics World and nanotechweb.org:

> **Seth Lloyd of the Massachusetts Institute of Technology in the US is impressed. 'This is awesome work and I am amazed that it hasn't been done before,' he says. 'Perhaps everyone thought it was too good to be true.'**

- APS Physics Central
- Phys.org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Viewpoint: Unlocking the Hidden Information in Starlight

Gabriel Durkin, Berkeley Quantum Information and Computation Center, University of California, Berkeley, CA 94720, USA

August 29, 2016 • Physics 9, 100

Quantum metrology shows that it is always possible to estimate the separation of two stars, no matter how close together they are.

Quantum Theory of Superresolution for Two Incoherent Optical Point Sources

Mankei Tsang, Ranjith Nair, and Xiao-Ming Lu

Phys. Rev. X 6, 031033 (2016)

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Tapping into light's hidden information to push fundamental diffraction limit

Sep 2, 2016

Two become one: various diffraction patterns showing Rayleigh's criterion

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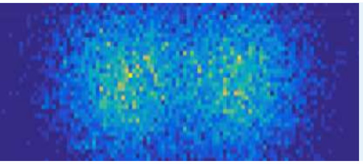
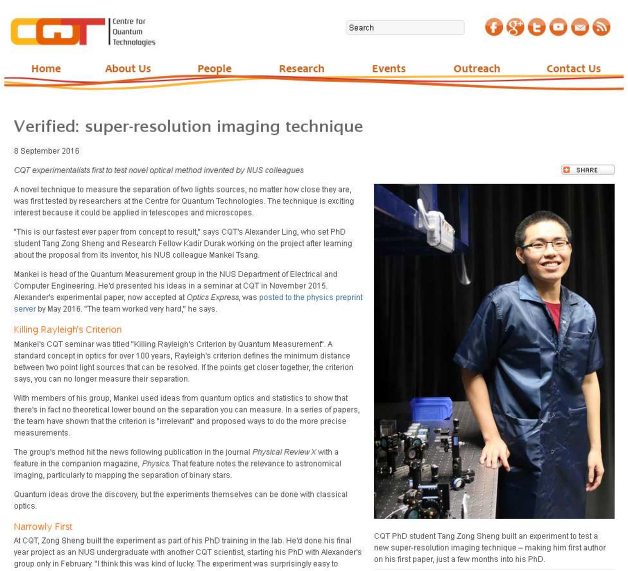
Quantum mechanics technique allows for pushing past 'Rayleigh's curse'

September 5, 2016 by Bob Yirka



Experiments

- Tang, Durak, and Ling, “Fault-tolerant and finite-error localization for point emitters within the diffraction limit,” Optics Express **24**, 22004 (2016).
- Yang, Taschilina, Moiseev, Simon, Lvovsky, “Far-field linear optical superresolution via heterodyne detection in a higher-order local oscillator mode,” Optica **3**, 1148 (2016).
- Tham, Ferretti, Steinberg, “Beating Rayleigh’s Curse by Imaging Using Phase Information,” Phys. Rev. Lett. **118**, 070801 (2017).
- Paúr, Stoklasa, Hradil, Sánchez-Soto, Rehacek, “Achieving the ultimate optical resolution,” Optica **3**, 1144 (2016).



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Verified: super-resolution imaging technique

8 September 2016

CQT experimentalists first to test novel optical method invented by NUS colleagues

A novel technique to measure the separation of two lights sources, no matter how close they are, was first tested by researchers at the Centre for Quantum Technologies. The technique is exciting interest because it could be applied in telescopes and microscopes.

"This is our fastest ever paper from concept to result," says CQT's Alexander Ling, who set PhD student Tang Zong Sheng and Research Fellow Kadir Durak working on the project after learning about the proposal from its inventor, his NUS colleague Mankei Tsang.

Mankei is head of the Quantum Measurement group in the NUS Department of Electrical and Computer Engineering. He'd presented his ideas in a seminar at CQT in November 2015. Alexander's experimental paper, now accepted at *Optics Express*, was posted to the physics preprint server by May 2016. "The team worked very hard," he says.

Killing Rayleigh's Criterion

Mankei's CQT seminar was titled "Killing Rayleigh's Criterion by Quantum Measurement". A standard concept in optics for over 100 years, Rayleigh's criterion defines the minimum distance between two point light sources that can be resolved. If the points get closer together, the criterion says, you can no longer measure their separation.

With members of his group, Mankei used ideas from quantum optics and statistics to show that there's in fact no theoretical lower bound on the separation you can measure. In a series of papers, the team have shown that the criterion is "irrelevant" and proposed ways to do the more precise measurements.

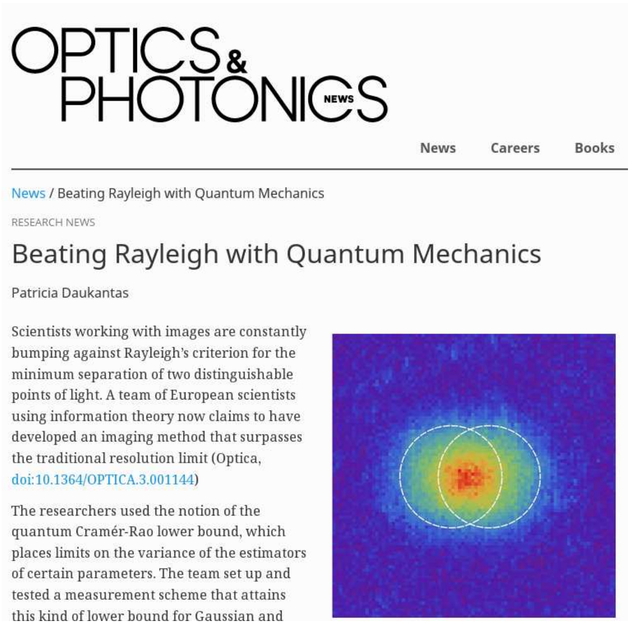
The group's method hit the news following publication in the journal *Physical Review X* with a feature in the companion magazine, *Physics*. That feature notes the relevance to astronomical imaging, particularly to mapping the separation of binary stars.

Quantum ideas drove the discovery, but the experiments themselves can be done with classical optics.

Narrowly First

At CQT, Zong Sheng built the experiment as part of his PhD training in the lab. He'd done his final year project as an NUS undergraduate with another CQT scientist, starting his PhD with Alexander's group only in February. "I think this was kind of lucky. The experiment was surprisingly easy to

CQT PhD student Tang Zong Sheng built an experiment to test a new super-resolution imaging technique – making him first author on his first paper, just a few months into his PhD.



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News / Beating Rayleigh with Quantum Mechanics

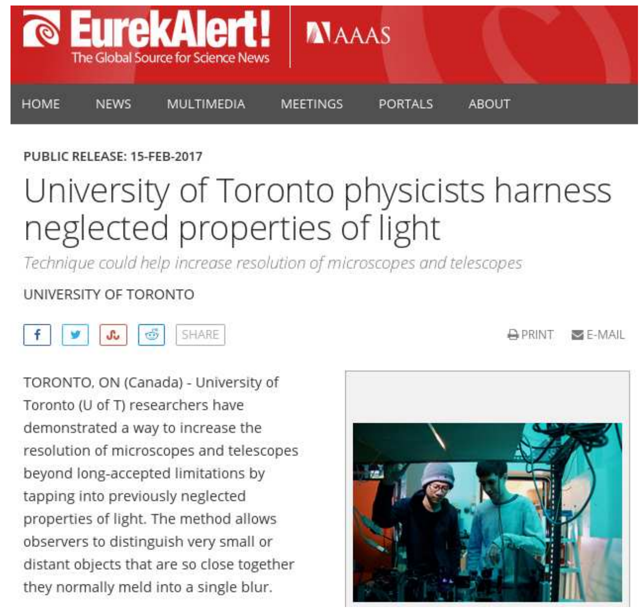
RESEARCH NEWS

Beating Rayleigh with Quantum Mechanics

Patricia Daukantas

Scientists working with images are constantly bumping against Rayleigh's criterion for the minimum separation of two distinguishable points of light. A team of European scientists using information theory now claims to have developed an imaging method that surpasses the traditional resolution limit (Optica, doi:10.1364/OPTICA.3.001144)

The researchers used the notion of the quantum Cramér-Rao lower bound, which places limits on the variance of the estimators of certain parameters. The team set up and tested a measurement scheme that attains this kind of lower bound for Gaussian and



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PUBLIC RELEASE: 15-FEB-2017

University of Toronto physicists harness neglected properties of light

Technique could help increase resolution of microscopes and telescopes

UNIVERSITY OF TORONTO

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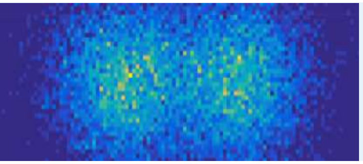
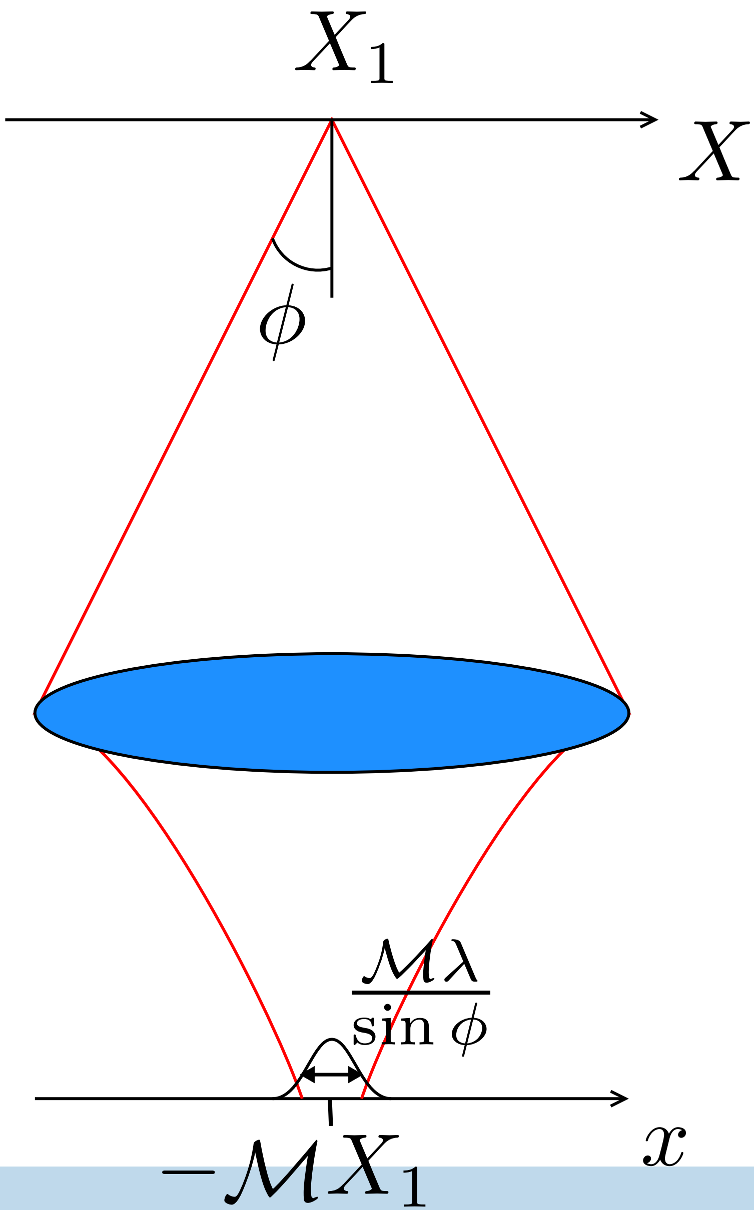
TORONTO, ON (Canada) - University of Toronto (U of T) researchers have demonstrated a way to increase the resolution of microscopes and telescopes beyond long-accepted limitations by tapping into previously neglected properties of light. The method allows observers to distinguish very small or distant objects that are so close together they normally meld into a single blur.



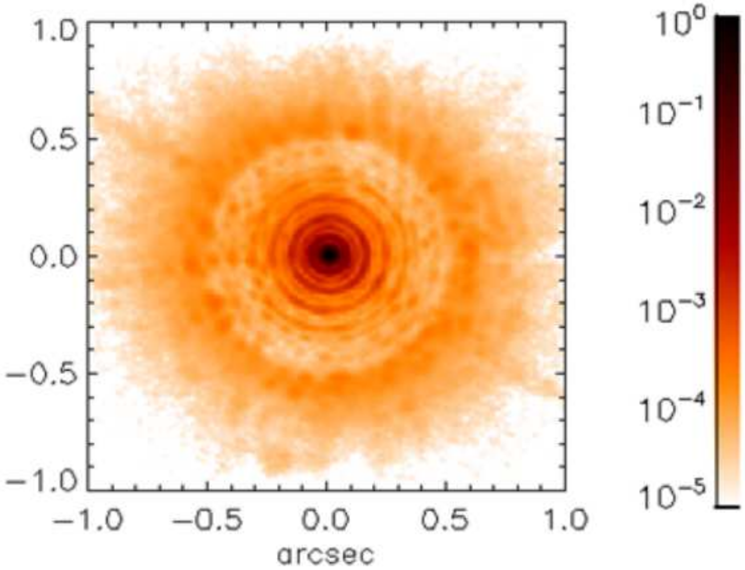
Imaging of One Point Source

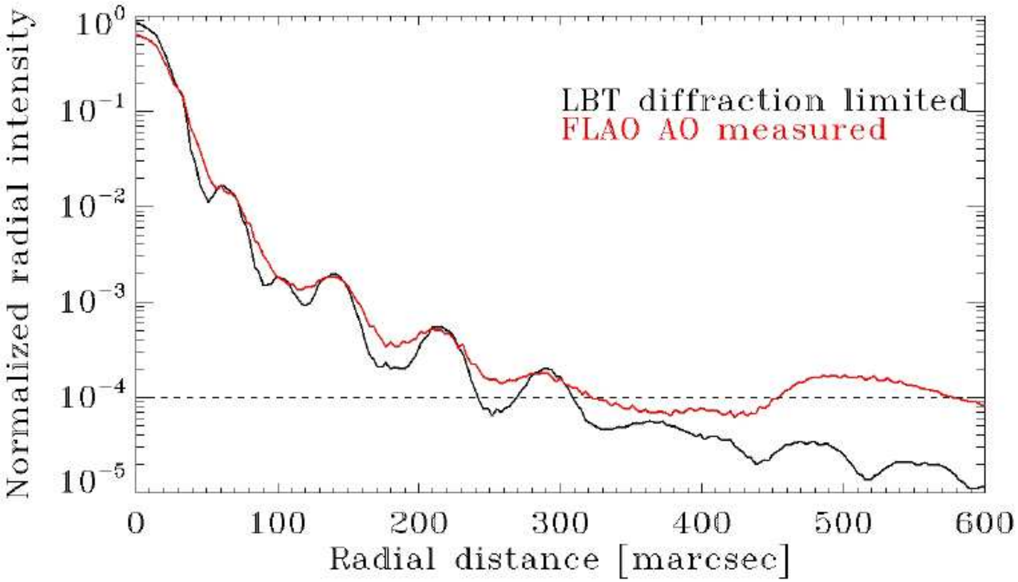


Diffraction limited

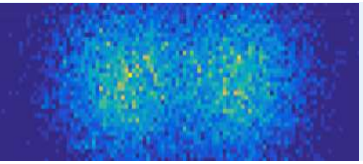
1. Fluorescence microscopy
2. Space telescopes (Webb, \$8.8 billion)
3. Ground-based telescopes:
   (a) Large Binocular Telescope (LBT) (\$120 million)
   (b) Giant Magellan Telescope (GMT) (~\$1 billion)
   (c) Thirty Meter Telescope (TMT) (~\$1.2 billion)
   (d) European Extremely Large Telescope (E-ELT) (~\$1.2 billion)

LBT Point-Spread Function





- Esposito *et al.*, "Large Binocular Telescope Adaptive Optics System: New achievements and perspectives in adaptive optics," Proc. SPIE **8149**, 814902 (2011)
- Strehl ratio $> 80\%$ at infrared



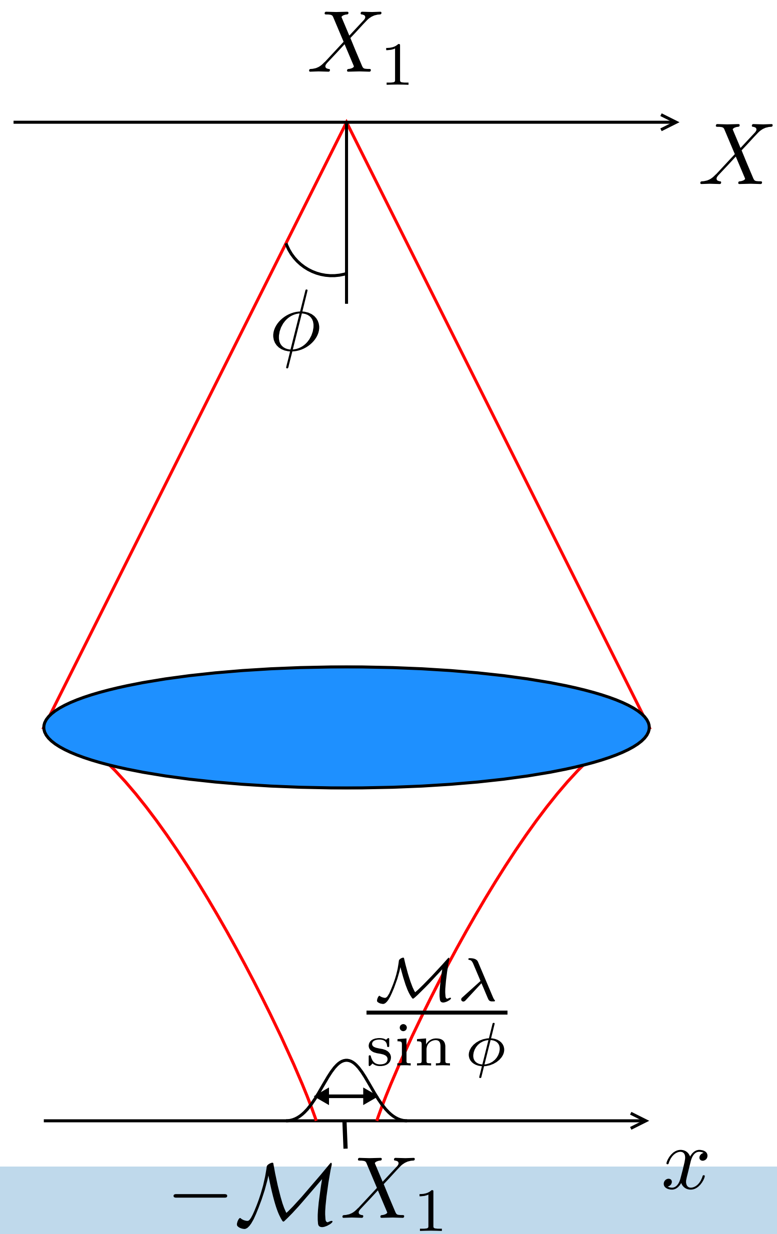
Inferring Position of One Point Source

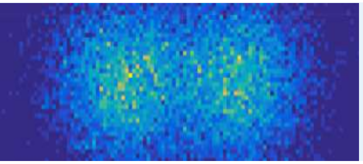
- Classical source
- Given N detected photons, **mean-square error**:

$$\Sigma = \frac{\sigma^2}{N}, \tag{1}$$

$$\sigma \sim \frac{\lambda}{\sin\phi}. \tag{2}$$

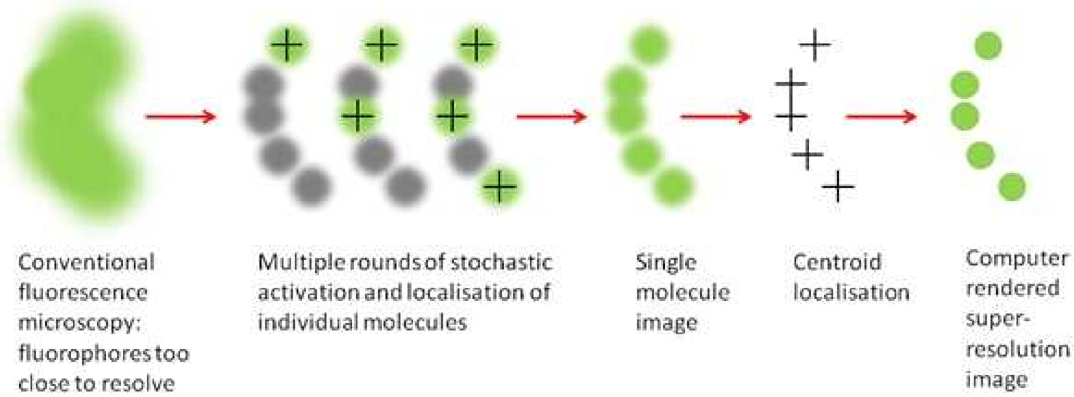
- Optics: Farrell (1966)
- Quantum: Helstrom (1970)
- Astronomy: Lindegren (1978)
- Microscopy: Bobroff (1986)
- Full EM, full quantum: Tsang, Optica **2**, 646 (2015).





Superresolution Microscopy

- PALM, STORM, etc.: **isolate** emitters. Locate **centroids**.



https://cam.facilities.northwestern.edu/588-2/single-molecule-localization-microscopy/

- Special fluorophores
- **slow**
- doesn't work for stars


The Nobel Prize in Chemistry 2014
Eric Betzig, Stefan W. Hell, William E. Moerner

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The Nobel Prize in Chemistry 2014

Photo: A. Mahmoud
Eric Betzig
Prize share: 1/3

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William E. Moerner
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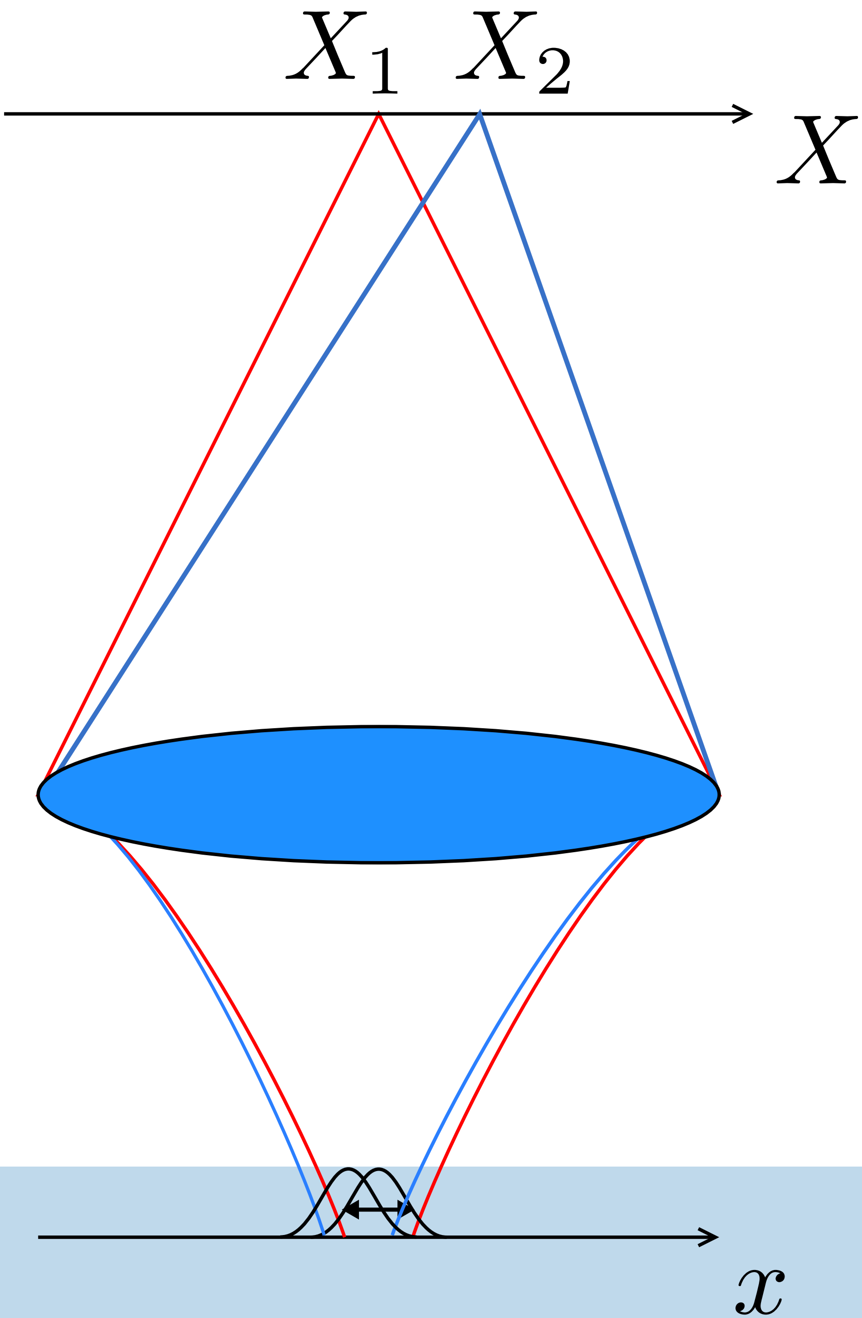
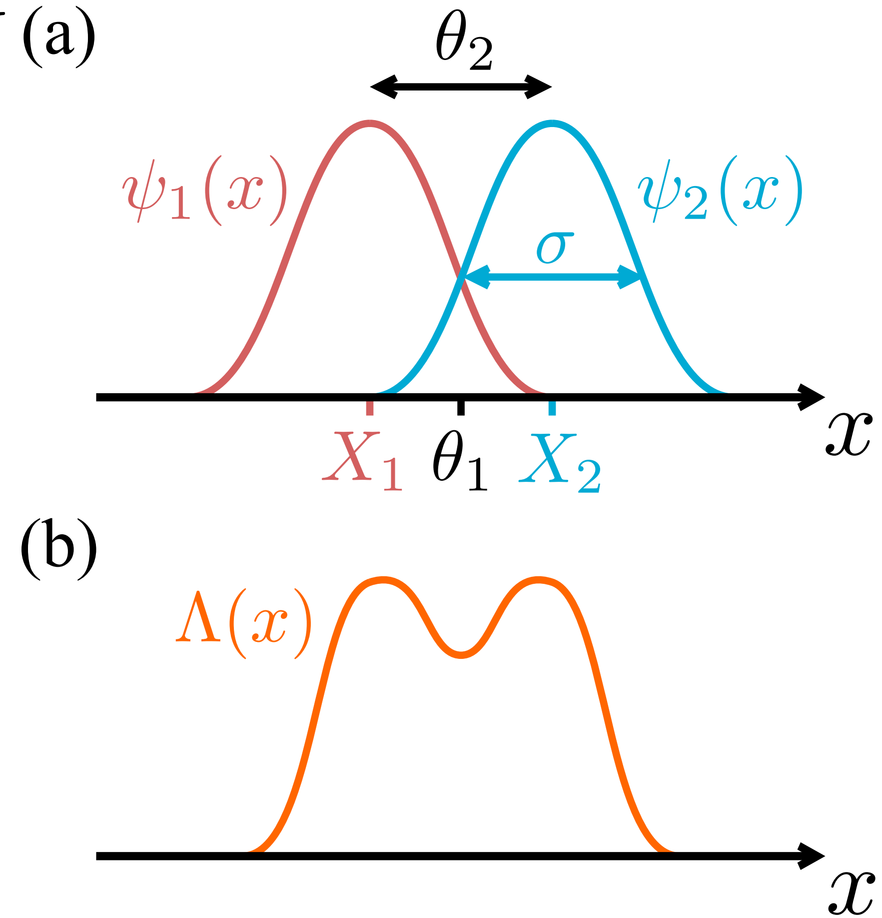
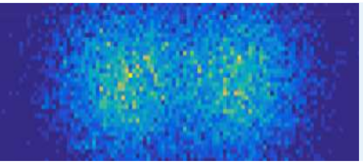
The Nobel Prize in Chemistry 2014 was awarded jointly to Eric Betzig, Stefan W. Hell and William E. Moerner *"for the development of super-resolved fluorescence microscopy"*.





- $\Lambda(x) = \frac{1}{2}\left[|\psi_1(x)|^2 + |\psi_2(x)|^2\right]$
- Rayleigh's criterion (1879): requires $\theta_2 \gtrsim \sigma$ (heuristic)



Centroid and Separation Estimation

- Bettens *et al.*, Ultramicroscopy **77**, 37 (1999); Van Aert *et al.*, J. Struct. Biol. **138**, 21 (2002); Ram, Ward, Ober, PNAS **103**, 4457 (2006).
- Incoherent sources, Poisson statistics
- Cramér-Rao bound (**standard in single-molecule imaging**)
- $X_1 = \theta_1 - \theta_2/2$, $X_2 = \theta_1 + \theta_2/2$.
- **centroid**:

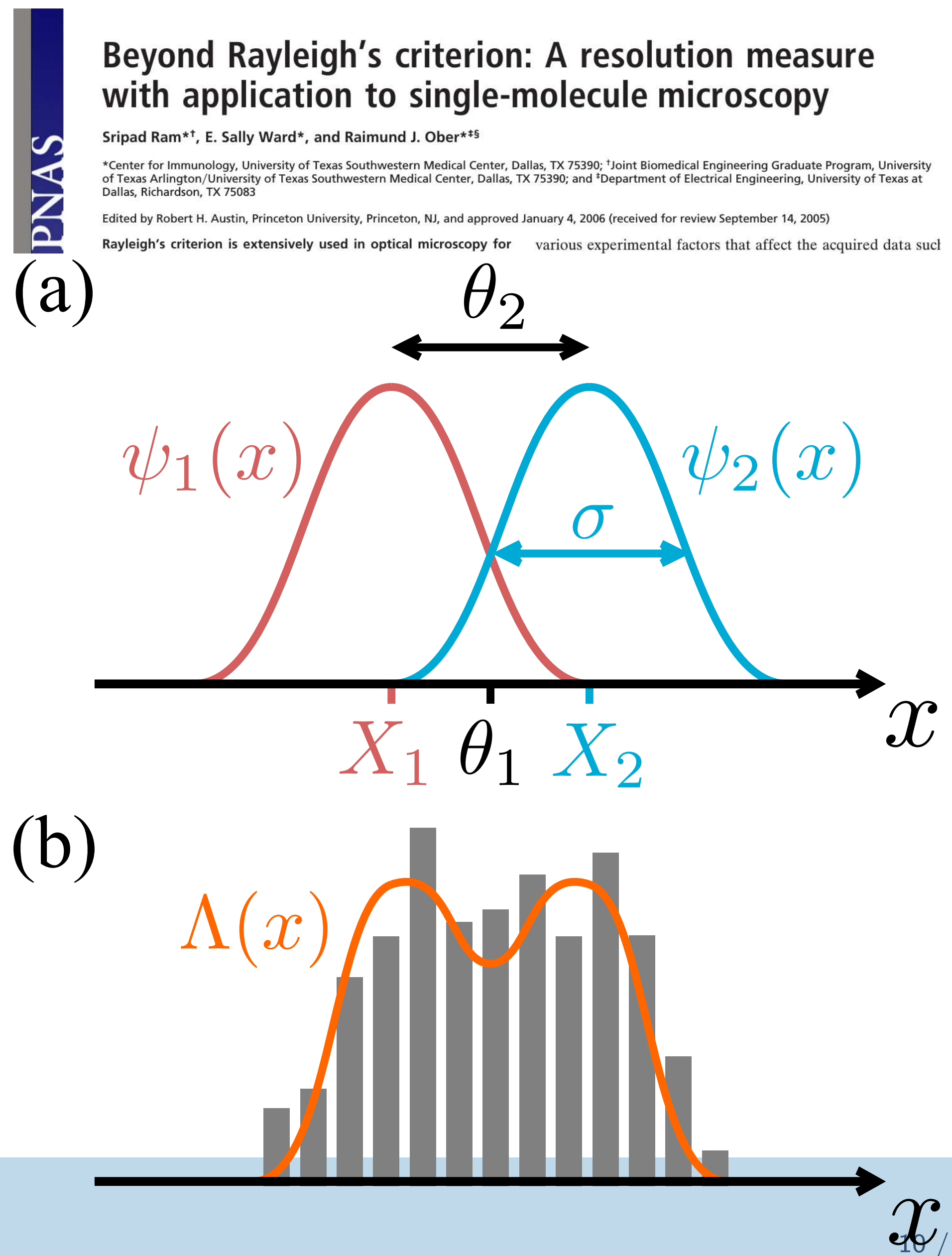
$$\Sigma_{11} \geq \frac{\sigma^2}{N}. \quad (3)$$

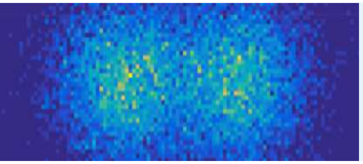
- CRB for **separation estimation**: two regimes
  - $\theta_2 \gg \sigma$:

$$\Sigma_{22} \geq \frac{4\sigma^2}{N}, \quad (4)$$

  - $\theta_2 \ll \sigma$:

$$\boxed{\Sigma_{22} \to \frac{4\sigma^2}{N} \times \infty} \quad (5)$$



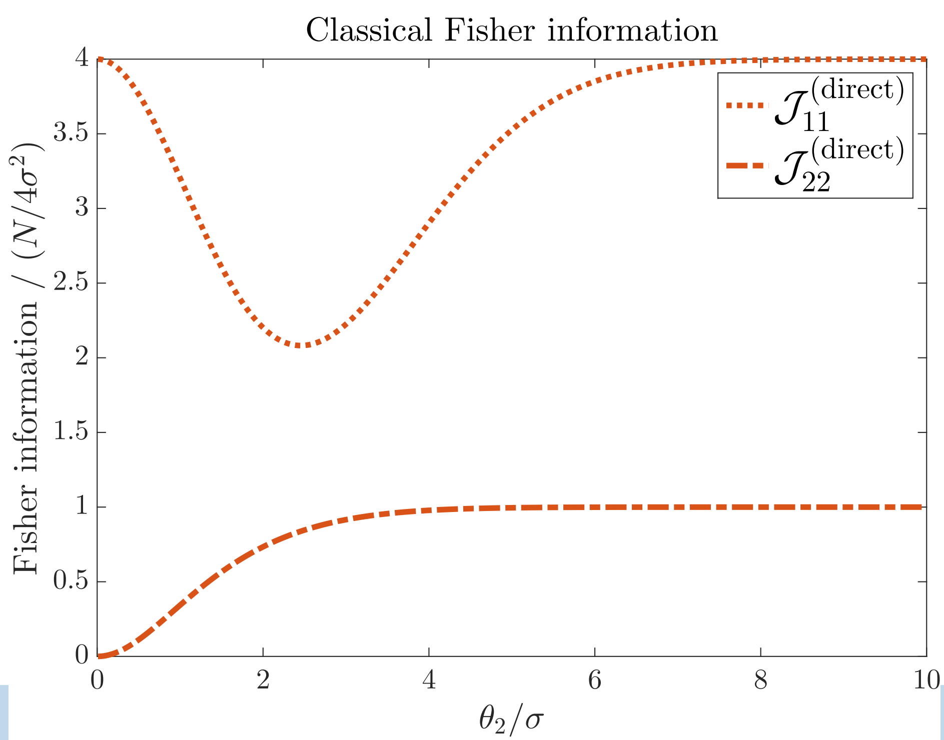


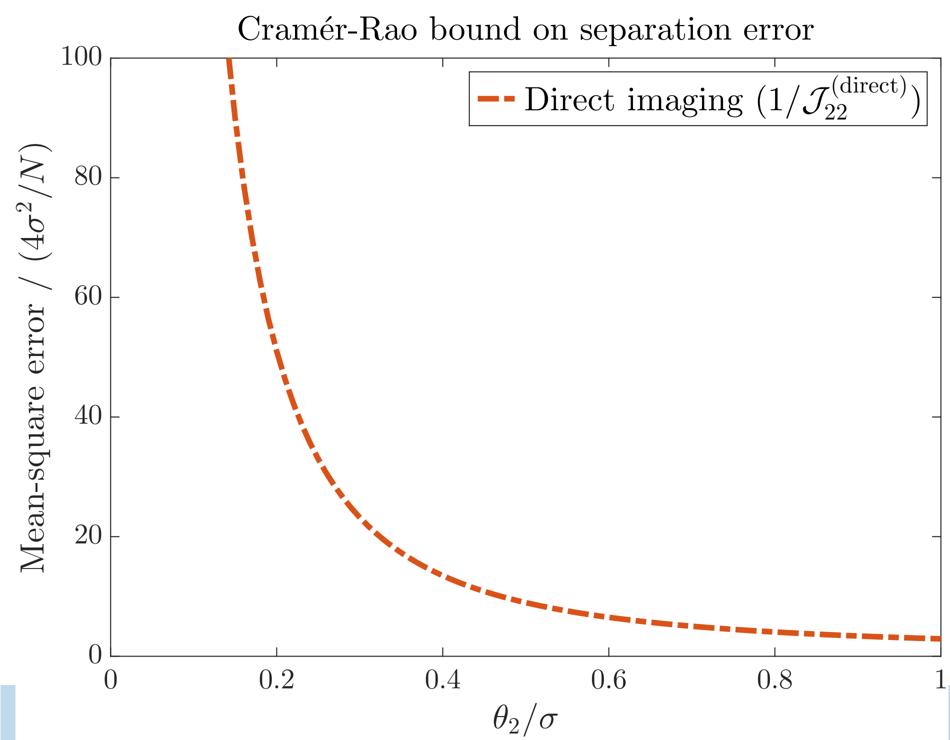
Rayleigh's Curse

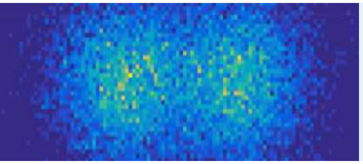
- Cramér-Rao bound:

$$\Sigma_{11} \geq \frac{1}{\mathcal{J}_{11}^{(\text{direct})}} \qquad \Sigma_{22} \geq \frac{1}{\mathcal{J}_{22}^{(\text{direct})}} \tag{6}$$

$\mathcal{J}^{(\text{direct})}$ is Fisher information for direct imaging
- Gaussian PSF, similar behavior for other PSF
- **Rayleigh's curse**
- PALM/STED/STORM: avoid violating Rayleigh







Quantum Information

- Measuring the spatial intensity (direct imaging, CCD) is just one measurement method. **Quantum mechanics allows infinite possibilities.**
- Helstrom (1967): For any measurement

$$\Sigma \geq \mathcal{J}^{-1} \boxed{\geq \mathcal{K}^{-1},} \tag{7}$$

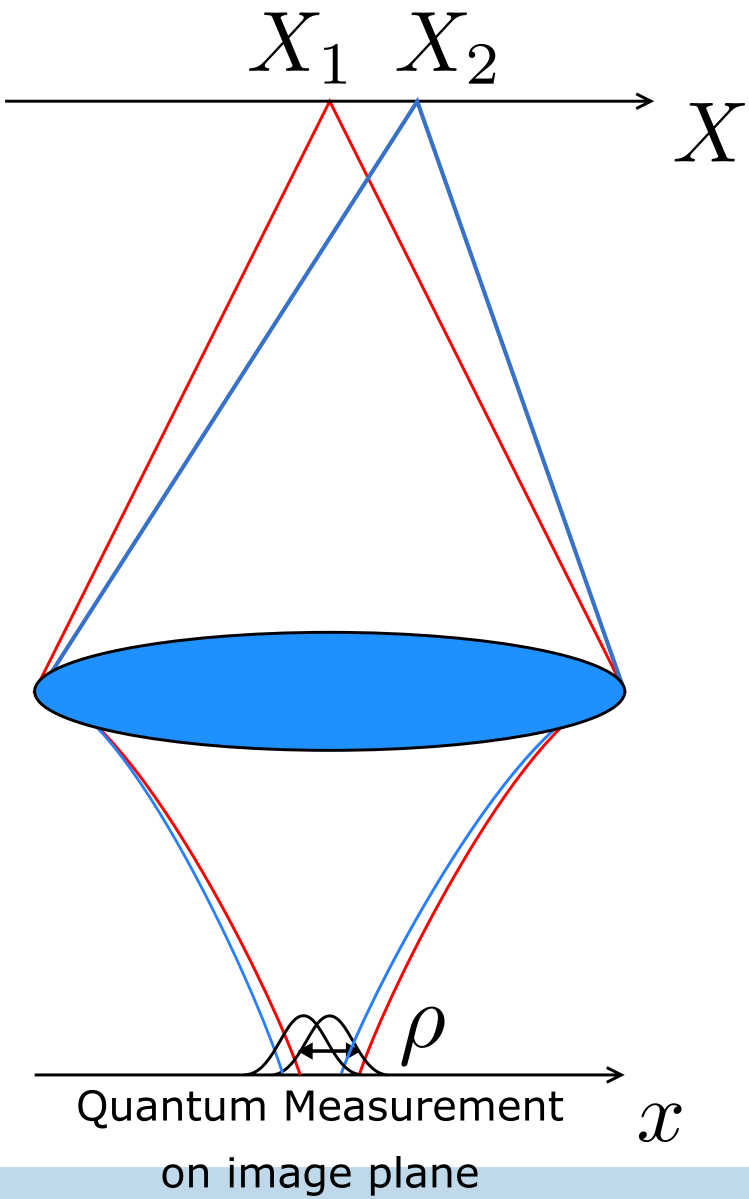
$$\mathcal{K}_{\mu\nu} = M \,\mathrm{Re}\left(\mathrm{tr}\,\mathcal{L}_\mu \mathcal{L}_\nu \rho\right), \tag{8}$$

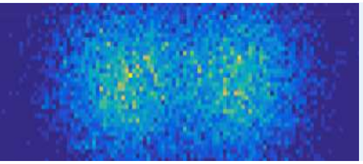
$$\frac{\partial \rho}{\partial \theta_\mu} = \frac{1}{2}\left(\mathcal{L}_\mu \rho + \rho \mathcal{L}_\mu\right). \tag{9}$$

- $\mathcal{K}(\rho)$ is the quantum Fisher information, **the ultimate amount of information in the photons.**
- Coherent sources: Tsang, Optica **2**, 646 (2015).
- Mixed states:

$$\rho = \sum_n D_n \left|e_n\right\rangle \left\langle e_n\right|, \tag{10}$$

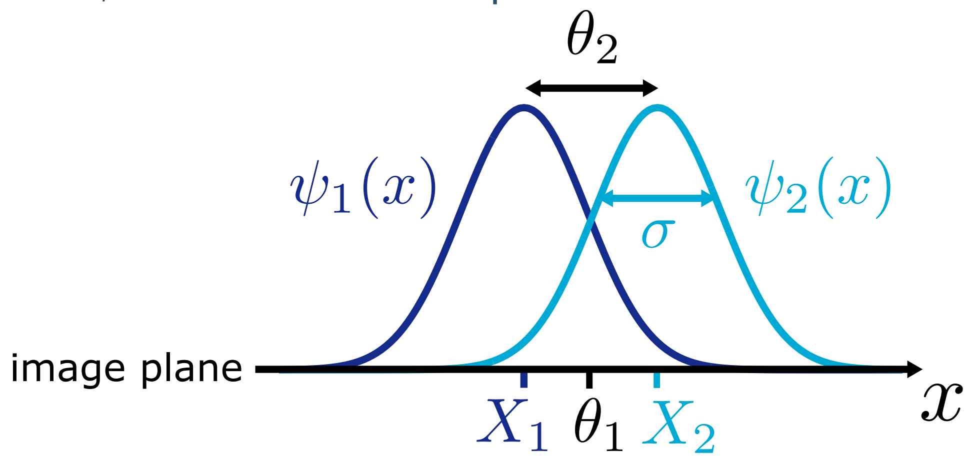
$$\mathcal{L}_\mu = 2 \sum_{n,m;D_n+D_m\neq 0} \frac{\left\langle e_n\right| \frac{\partial \rho}{\partial \theta_\mu} \left|e_m\right\rangle}{D_n + D_m} \left|e_n\right\rangle \left\langle e_m\right|. \tag{11}$$





Quantum Optics

- Mandel and Wolf, *Optical Coherence and Quantum Optics*; Goodman, *Statistical Optics*
- Thermal sources, e.g., stars, fluorescent particles.
- **Average photon number per mode $\epsilon \ll 1$ at optical frequencies** (visible, UV, X-ray, etc.).
- $\epsilon \sim 0.01$ for the sun at visible, $\epsilon \sim 10^{-6}$ for fluorophores.

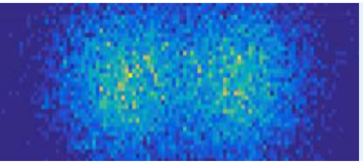


- Quantum state in M temporal modes on **image plane** is $\rho^{\otimes M}$, where

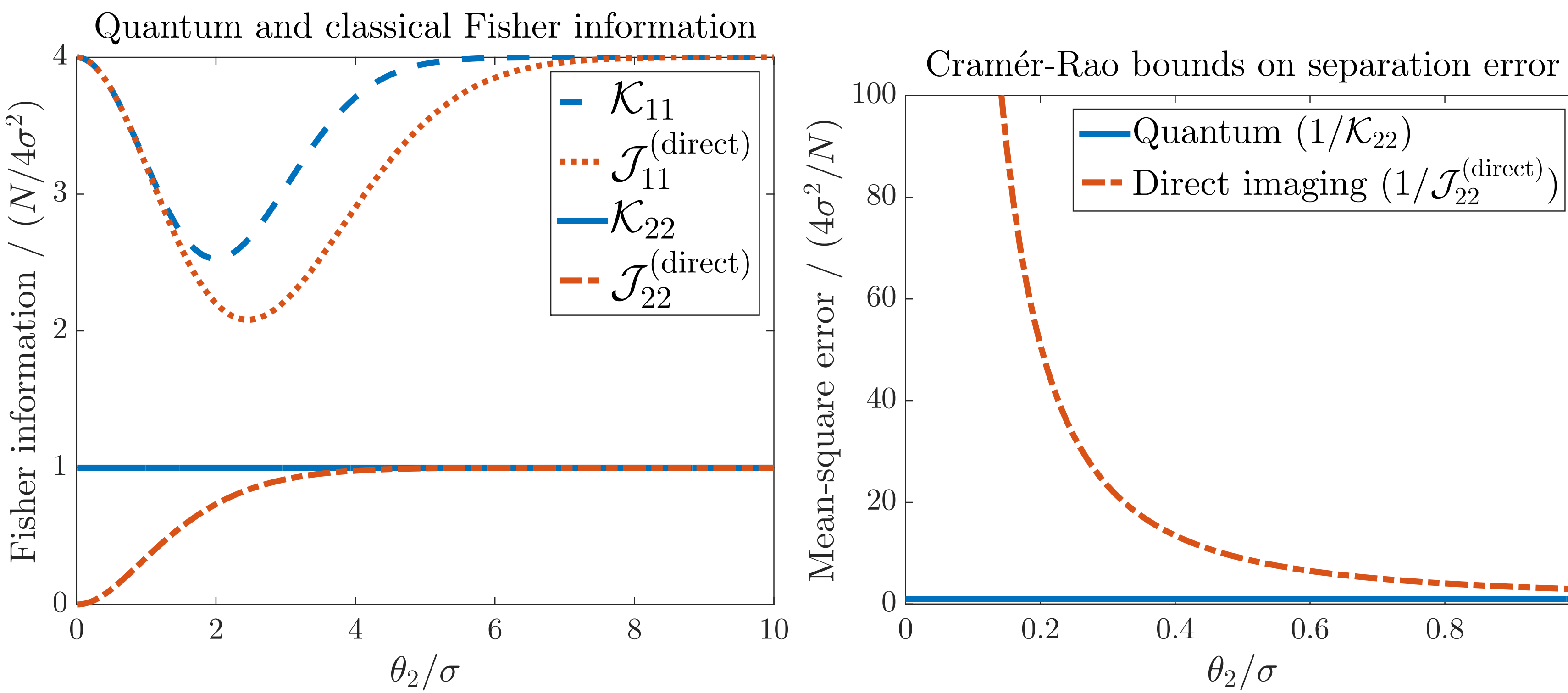
$$\boxed{\rho = (1-\epsilon)\,|\text{vac}\rangle\langle\text{vac}| + \frac{\epsilon}{2}\left(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|\right) + O(\epsilon^2)} \qquad \boxed{\langle\psi_1|\psi_2\rangle \neq 0,} \tag{12}$$

$$|\psi_1\rangle = \int_{-\infty}^{\infty} dx\psi(x - X_1)\,|x\rangle\,, \quad |\psi_2\rangle = \int_{-\infty}^{\infty} dx\psi(x - X_2)\,|x\rangle\,. \tag{13}$$

- derive from zero-mean Gaussian P function, mutual coherence
- Multiphoton coincidence: **rare**, little info as $\epsilon \ll 1$ (**homeopathy**)
- Similar model for stellar interferometry in Gottesman, Jennewein, Croke, PRL **109**, 070503 (2012); Tsang, PRL **107**, 270402 (2011).



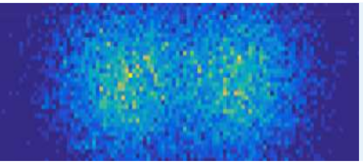
Plenty of Room at the Bottom



- Tsang, Nair, and Lu, Physical Review X **6**, 031033 (2016)

$$\boxed{\Sigma_{22} \geq \frac{1}{\mathcal{K}_{22}} = \frac{1}{N\Delta k^2}.} \tag{14}$$

- **thermal sources with arbitrary** ϵ: Nair and Tsang, PRL (Editors' Suggestion) **117**, 190801 (2016); Lupo and Pirandola, *ibid.* **117**, 190802 (2016).
- Hayashi *ed.*, *Asymptotic Theory of Quantum Statistical Inference*; Fujiwara JPA **39**, 12489 (2006): there exists a POVM such that $\Sigma_{\mu\mu} \to 1/\mathcal{K}_{\mu\mu}$, $N \to \infty$.



Hermite-Gaussian Basis

- project in **Hermite-Gaussian** basis:

$$E_1(q) = |\phi_q\rangle \langle\phi_q| \, , \tag{15}$$

$$|\phi_q\rangle = \int_{-\infty}^{\infty} dx \phi_q(x) \, |x\rangle \, , \tag{16}$$

$$\phi_q(x) = \left(\frac{1}{2\pi\sigma^2}\right)^{1/4} H_q\left(\frac{x}{\sqrt{2}\sigma}\right) \exp\left(-\frac{x^2}{4\sigma^2}\right) . \tag{17}$$

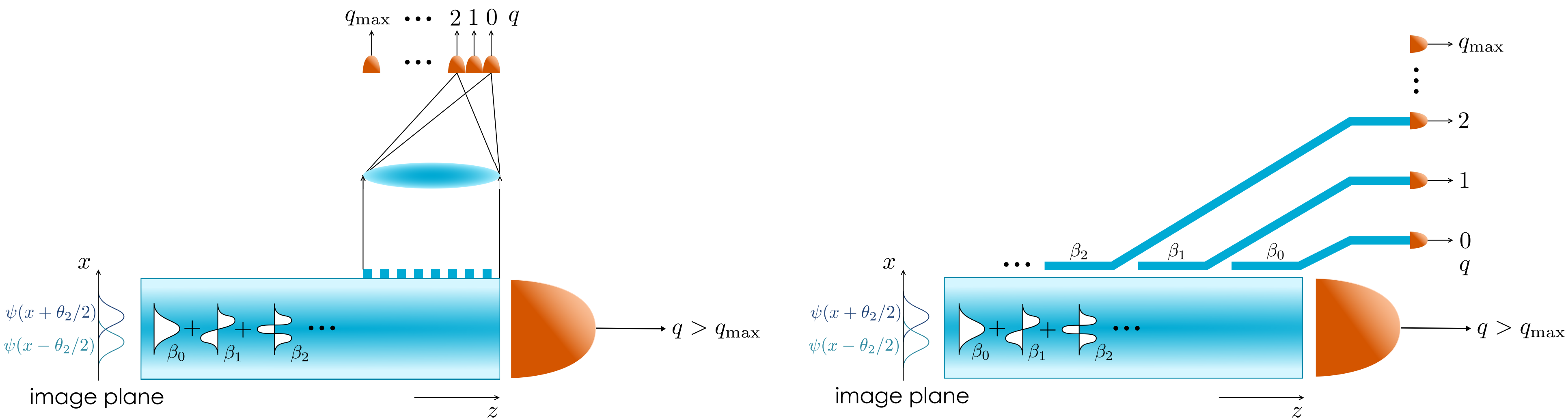
- Assume PSF $\psi(x)$ is Gaussian (common).

$$\boxed{\frac{1}{\mathcal{J}_{22}^{(\mathrm{HG})}} = \frac{1}{\mathcal{K}_{22}} = \frac{4\sigma^2}{N}.} \tag{18}$$

- **Maximum-likelihood estimator** can saturate the classical bound asymptotically for large N.
- arXiv:1605.03799v2: SPADE with Max-Like:

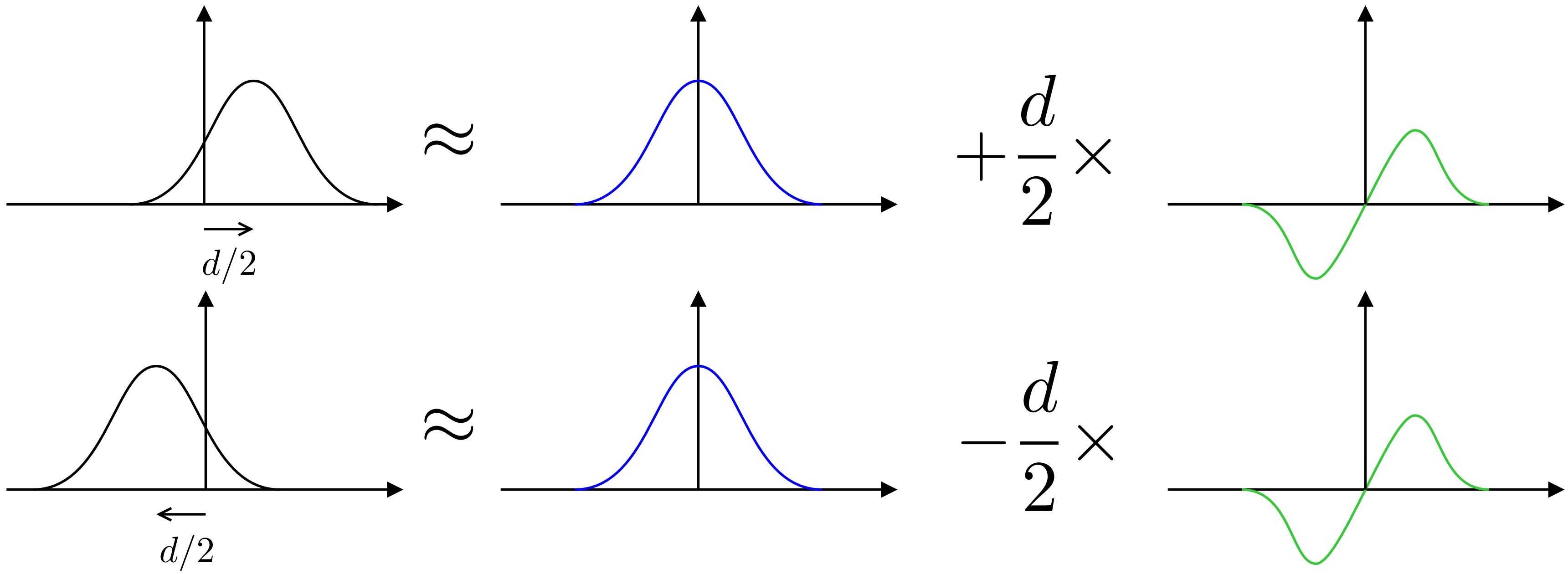
$$\boxed{\Sigma_{22} \leq \frac{16\sigma^2}{N} \text{ for any detected } N.} \tag{19}$$

Spatial-Mode Demultiplexing (SPADE)

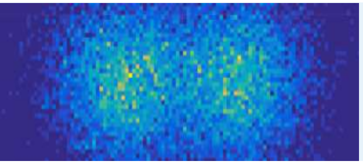


- Many other ways (optical comm.), e.g.,
  - DAB Miller, "Self-configuring universal linear optical components," Photonics Research **1**, 1 (2013).
  - Guifang Li *et al.*, "Space-division multiplexing: the next frontier in optical communication," Adv. Opt. Photon. **6**, 413 (2014).
  - V. A. Soifer, *Computer Design of Diffractive Optics* (CISP/Woodhead, Cambridge, 2013)

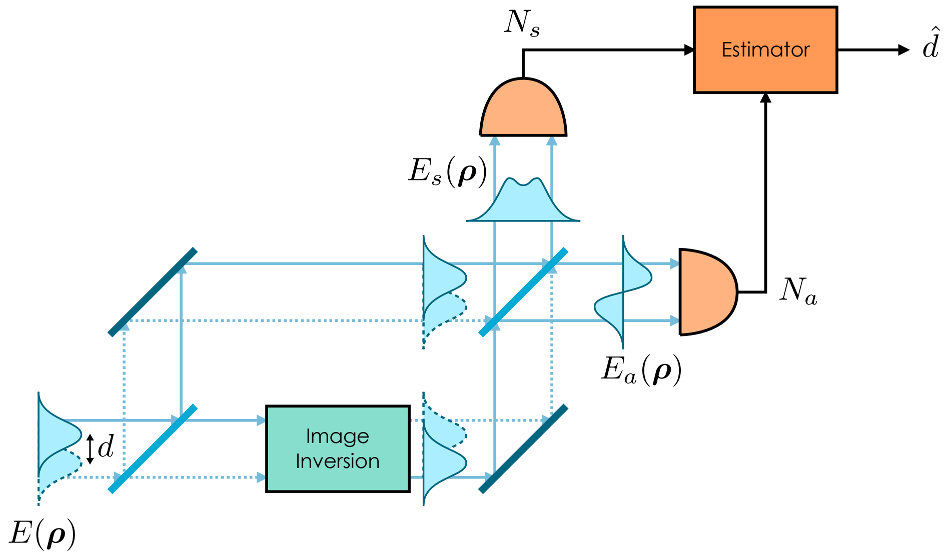
Elementary Explanation



- **Incoherent sources**: energy in **first-order mode** is $\propto (d/2)^2 + (-d/2)^2 = d^2/2$
- **Zeroth-order mode** is just **background noise**, removing it improves SNR.
- Why quantum formalism?
  - Fundamental quantum limit
  - Ensures measurement is physical
  - Discover new possibilities



SLIVER

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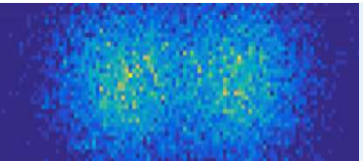
SLIVER to determine separation of two sources closer than Rayleigh limit

02/01/2016

By John Wallace

Senior Editor

- **S**uper**L**ocalization via **I**mage-in**VER**sion interferometry
- **Nair** and Tsang, Opt. Express **24**, 3684 (2016).
- Laser Focus World, Feb 2016 issue.
- Classical theory/experiment of image-inversion interferometer:
  - Wicker, Heintzmann, Opt. Express **15**, 12206 (2007).
  - Wicker, Sindbert, Heintzmann, Opt. Express **17**, 15491 (2009).
  - Weigel, Babovsky, Kiessling, Kowarschik, Opt. Commun. **284**, 2273 (2011).
  - No statistical analysis, predicted <2X resolution enhancement

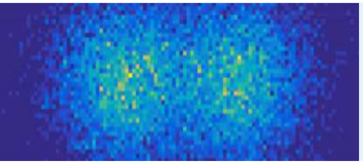


Theoretical Follow-up

		Dimensions	Sources	Theory	Experimental proposals
1.	Tsang, Nair, and Lu, Phys. Rev. X **6**, 031033 (2016)	1D	Weak thermal (optical frequencies and above)	Quantum	SPADE
2.	Nair and Tsang, Optics Express **24**, 3684 (2016)	2D	Thermal (any frequency)	Semiclassical	SLIVER
3.	Tsang, Nair, and Lu, Proc. SPIE **10029**, 1002903 (2016)	N/A	Weak thermal, lasers	Semiclassical	N/A
4.	Nair and Tsang, PRL (Editors' Suggestion) **117**, 190801 (2016)	1D	Thermal	Quantum	SLIVER
5.	Tsang, arXiv:1605.03799	1D	Weak thermal	Quantum, Bayesian, Minimax	SPADE
6.	Ang, Nair, and Tsang, arXiv:1606.00603	2D	Weak thermal	Quantum	SPADE, SLIVER
7.	Tsang, arXiv:1608.03211	2D	Weak thermal, **multiple sources**	Quantum	SPADE
8.	Lu, Nair, Tsang, arXiv:1609.03025	2D	Weak thermal, **one-versus-two**	Quantum, **binary hypothesis testing**	SPADE, SLIVER

Other groups:

- Lupo and Pirandola, PRL (Editors' Suggestion) **117**, 190801 (2016).
- Rehacek *et al.*, Optics Letters **42**, 231 (2017).
- Krovi, Guha, Shapiro, arXiv:1609.00684.
- Kerviche, Guha, Ashok, arXiv:1701,04913.

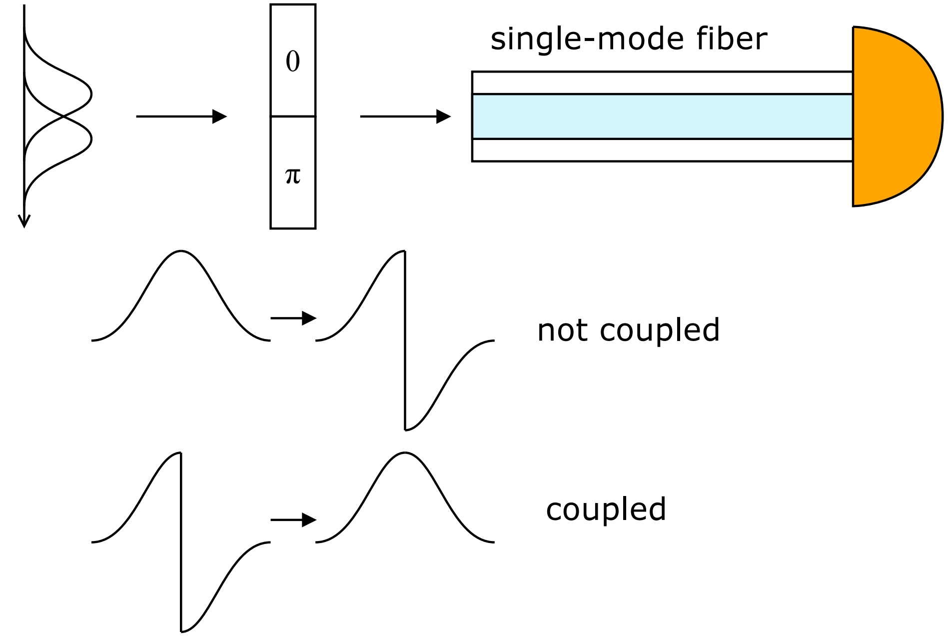


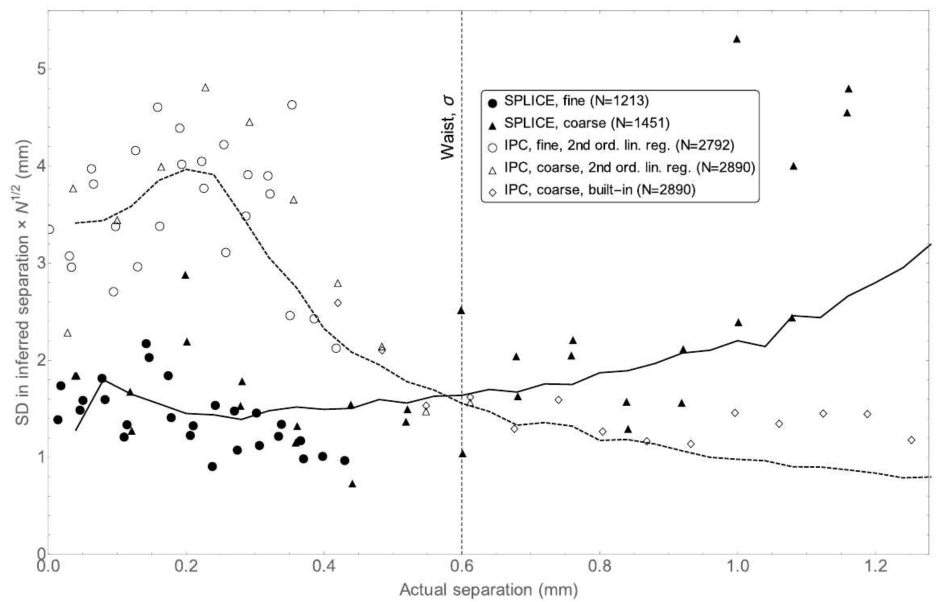
Experiments

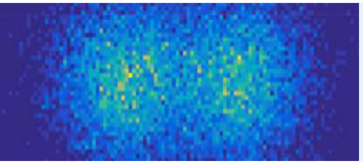
- Tang, Durak, and Ling, "Fault-tolerant and finite-error localization for point emitters within the diffraction limit," Optics Express **24**, 22004 (2016).
  - SLIVER
  - Laser, classical noise
- Yang, Taschilina, Moiseev, Simon, Lvovsky, "Far-field linear optical superresolution via heterodyne detection in a higher-order local oscillator mode," Optica **3**, 1148 (2016).
  - Mode heterodyne
  - Laser
- Tham, Ferretti, Steinberg, "Beating Rayleigh's Curse by Imaging Using Phase Information," Phys. Rev. Lett. **118**, 070801 (2017).
  - variation of SPADE
  - two independent SPDC sources, $\sim 2\times$ **quantum limit**
- Paúr, Stoklasa, Hradil, Sánchez-Soto, Rehacek, "Achieving the ultimate optical resolution," Optica **3**, 1144 (2016).
  - variation of SPADE
  - laser, close to quantum limit (caveat)

- Tham, Ferretti, Steinberg, Phys. Rev. Lett. **118**, 070801 (2017).

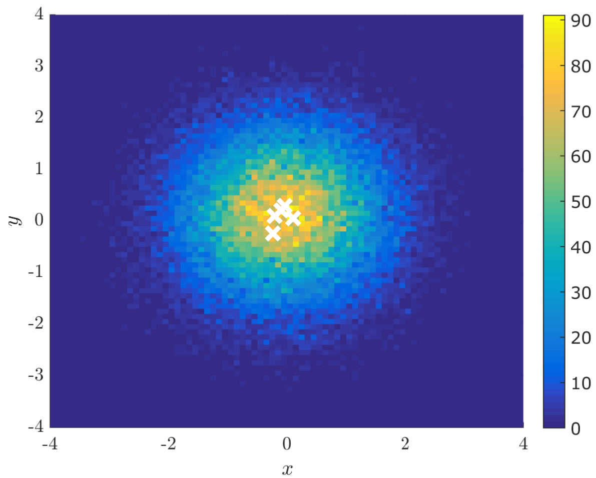
image plane

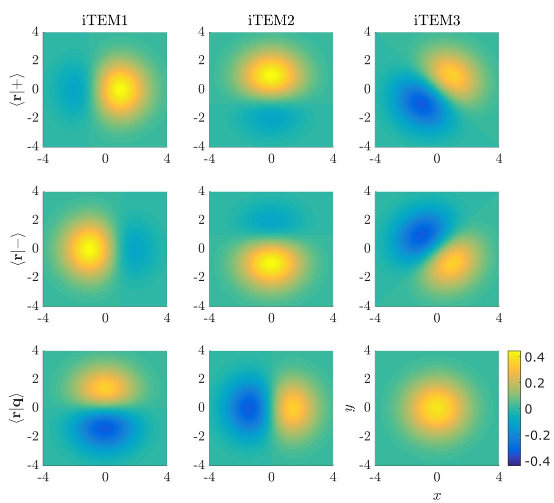


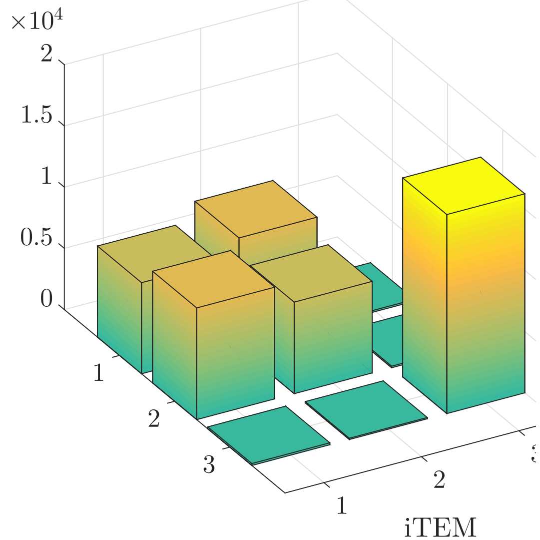


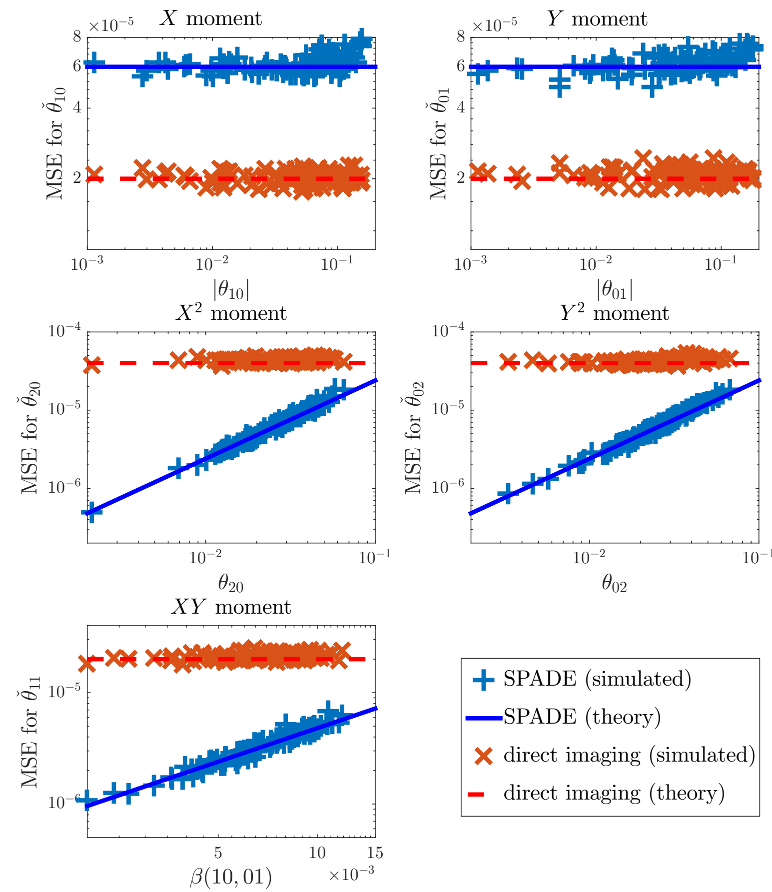


Arbitrary Source Distributions

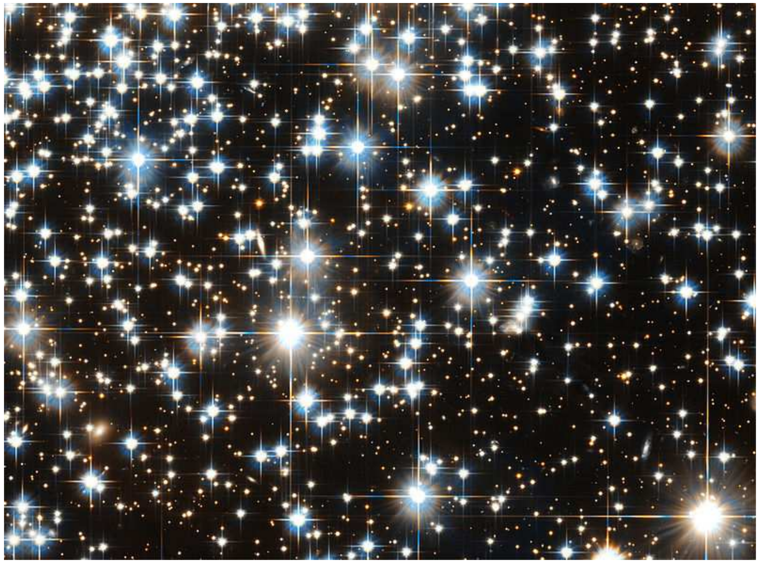


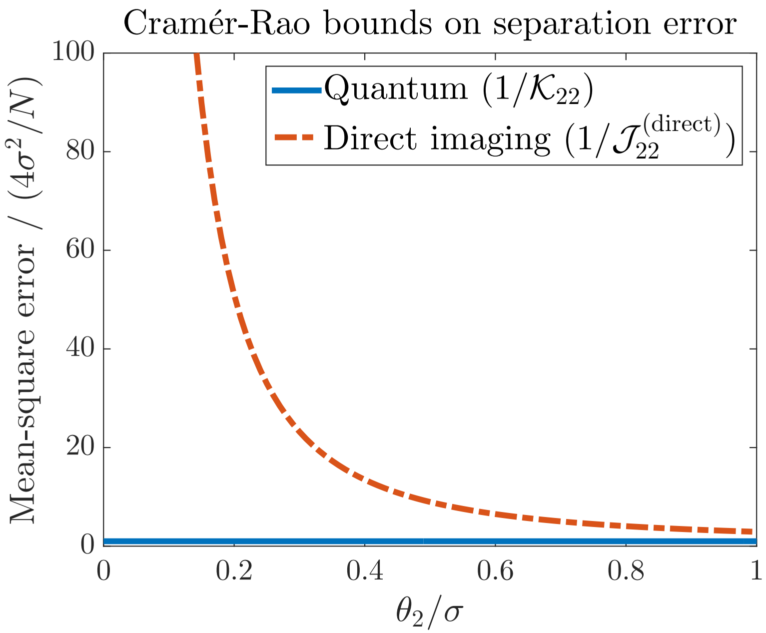


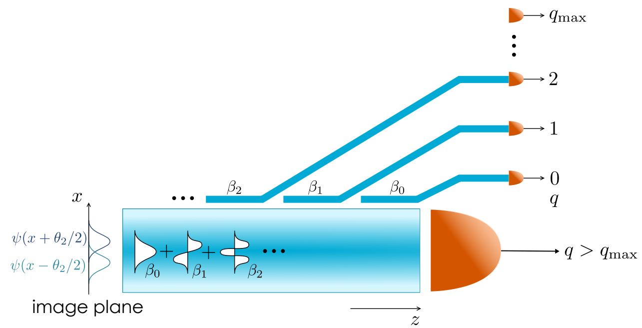


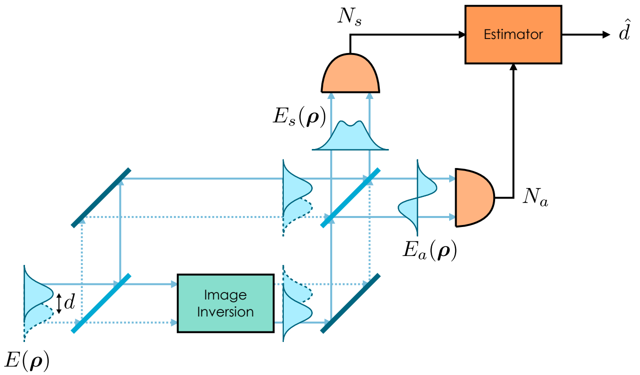


- Yang *et al.*, Optica **3**, 1148 (2016): even moments
- Tsang, arXiv:1608.03211: Generalized SPADE: **Enhanced estimation of 2nd or higher moments**

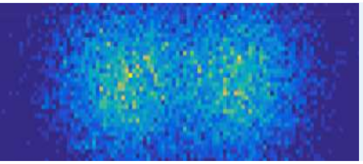








- **FAQ**: `https://sites.google.com/site/mankeitsang/news/rayleigh/faq`
- email: `mankei@nus.edu.sg`



$\epsilon \ll 1$ Approximation

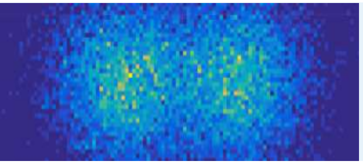
- Chap. 9, Goodman, *Statistical Optics*:

  "If the count degeneracy parameter is much less than 1, it is highly probable that there will be either **zero or one counts** *in each separate coherence interval of the incident classical wave. In such a case the classical intensity fluctuations have a* **negligible "bunching"** *effect on the photo-events, for (with high probability) the light is simply too weak to generate multiple events in a single coherence cell.*

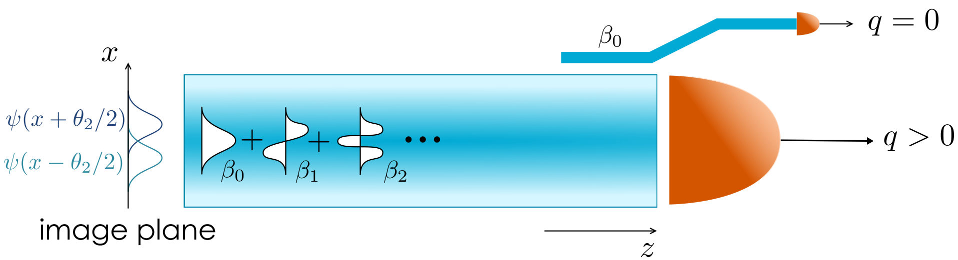
- Zmuidzinas (https://pma.caltech.edu/content/jonas-zmuidzinas), JOSA A **20**, 218 (2003):

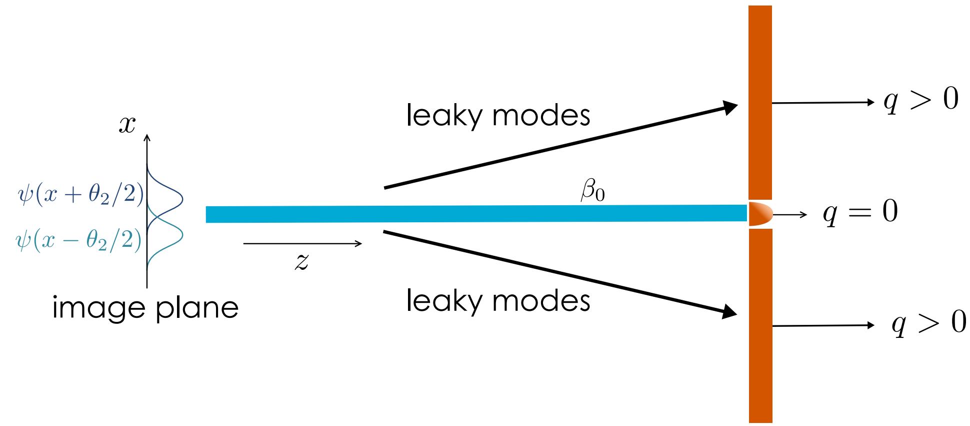
  "It is well established that the photon counts registered by the detectors in an optical instrument follow statistically independent **Poisson** *distributions, so that the fluctuations of the counts in different detectors are uncorrelated. To be more precise, this situation holds for the case of thermal emission (from the source, the atmosphere, the telescope, etc.) in which the mean photon occupation numbers of the modes incident on the detectors are low, $n \ll 1$. In the high occupancy limit, $n \gg 1$, photon bunching becomes important in that it changes the counting statistics and can introduce correlations among the detectors. We will discuss only the first case, $n \ll 1$,* **which applies to most astronomical observations at optical and infrared wavelengths**.*"*

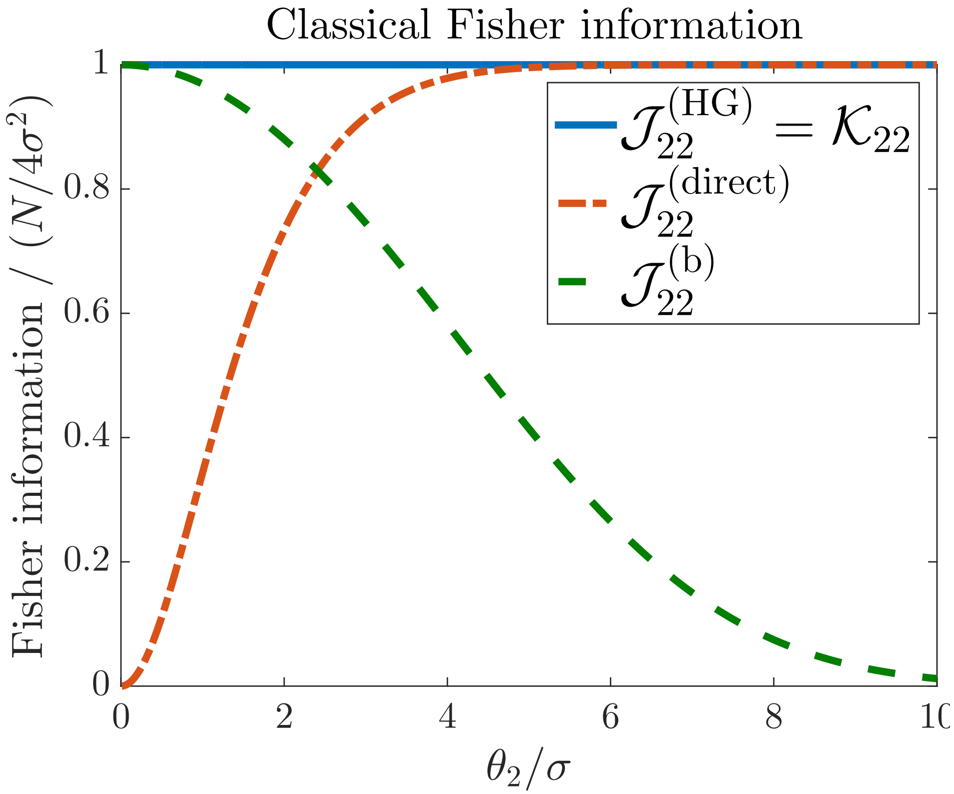
- **Hanbury Brown-Twiss** (**post-selects** on two-photon coincidence, homeopathy): poor SNR, **obsolete for decades** in astronomy.
- See also Labeyrie *et al.*, *An Introduction to Optical Stellar Interferometry*, etc.
- Fluorescent particles: Pawley *ed.*, *Handbook of Biological Confocal Microscopy*, Ram, Ober, Ward (2006), etc., may have **antibunching**, but **Poisson model** is fine and standard because of $\epsilon \ll 1$.

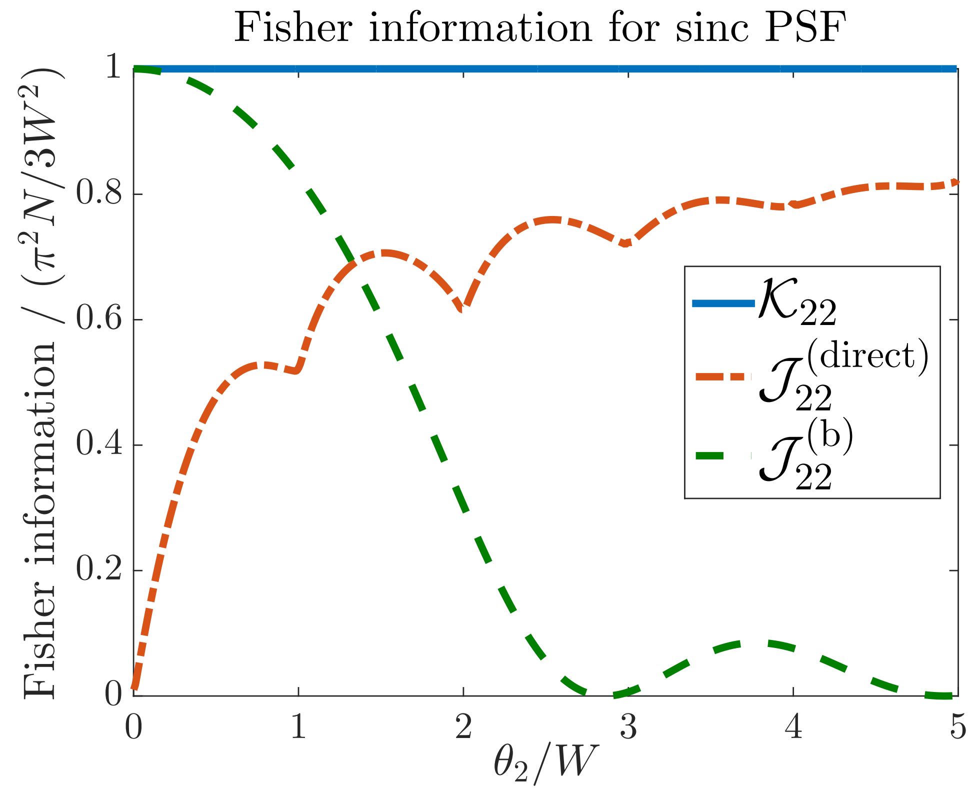


Binary SPADE

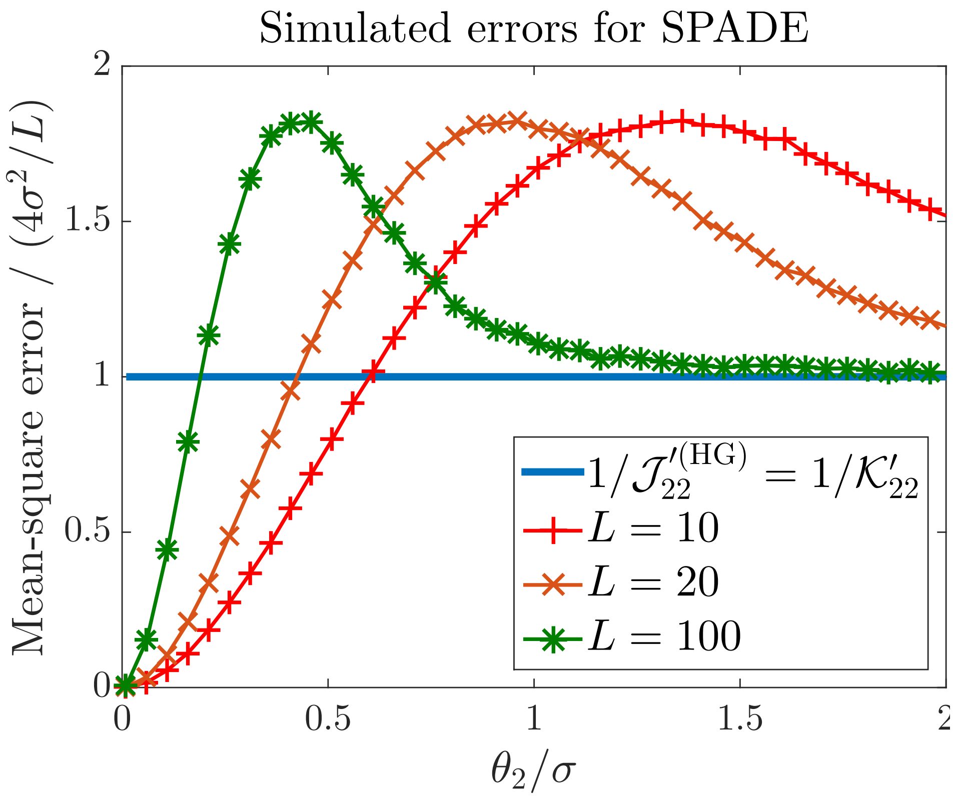


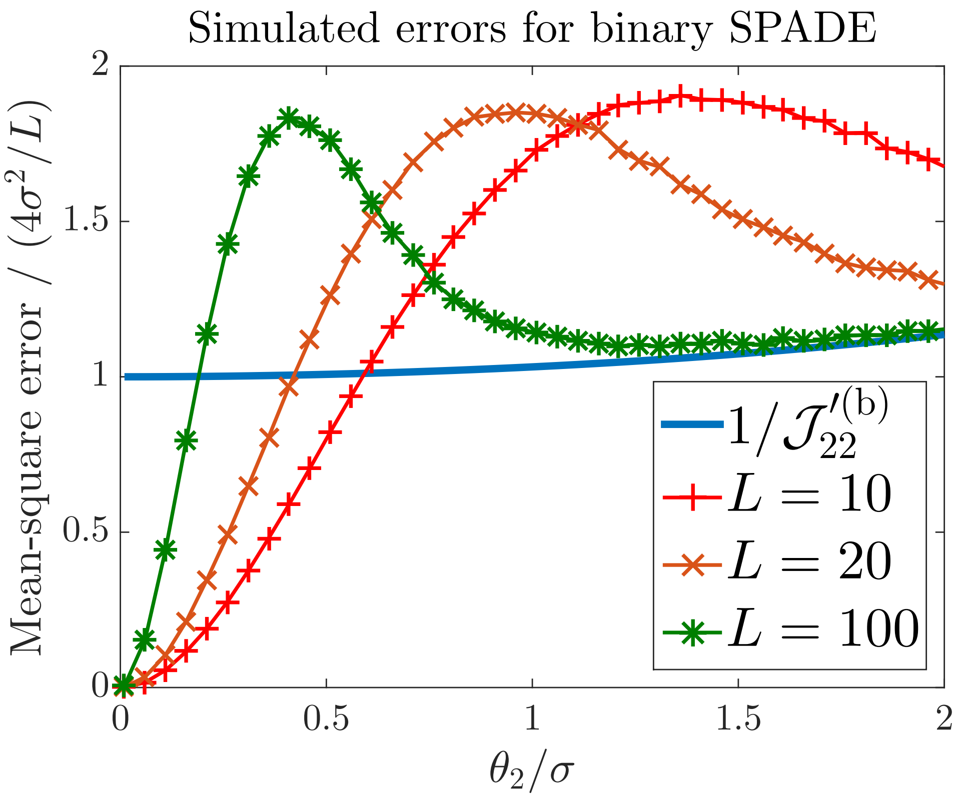




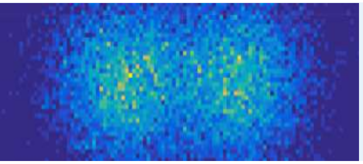


Numerical Performance of Maximum-Likelihood Estimators





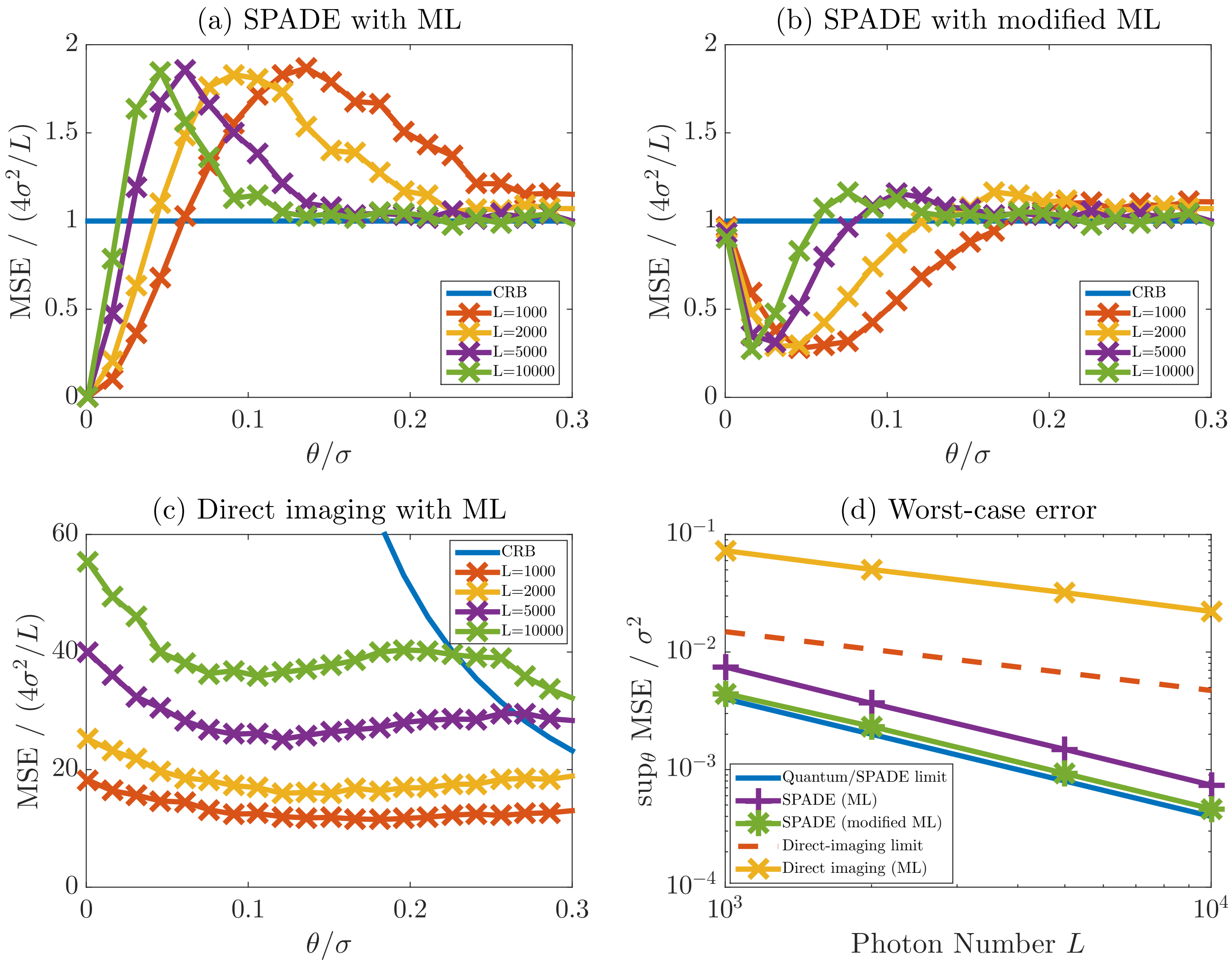
- $L =$ number of detected photons
- **biased** (violate CRB), $< 2\times$CRB.

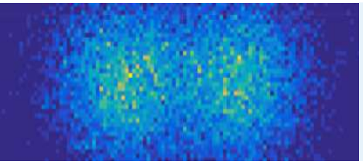


Minimax/Bayesian

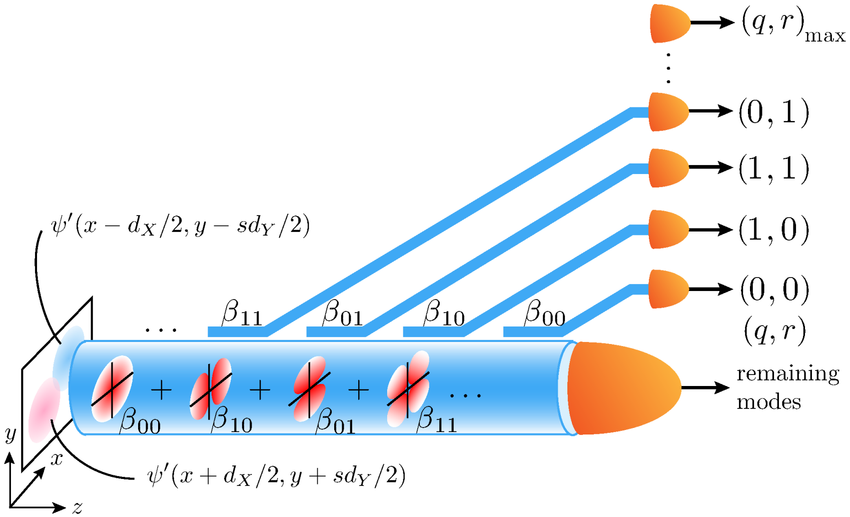
Bayesian CRB for any biased/unbiased estimator (e-print arXiv:1605.03799)

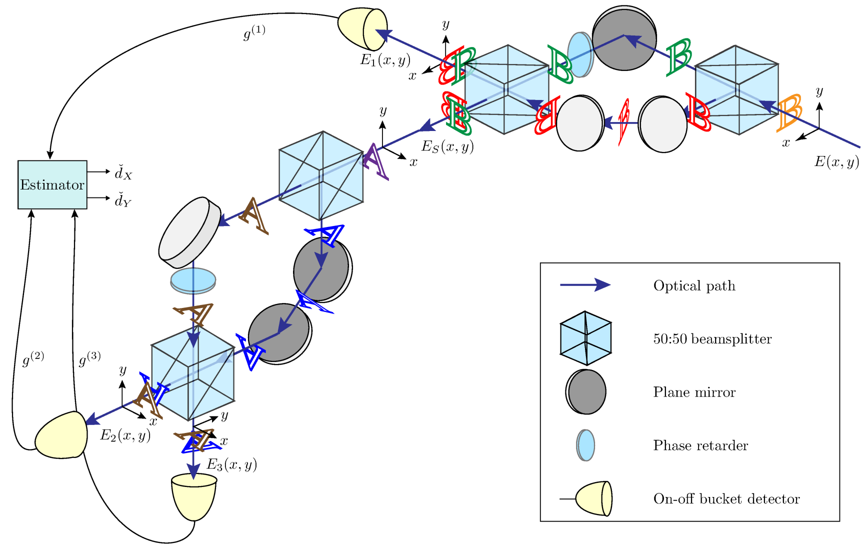
$$\text{Quantum/SPADE:}\ \sup_\theta \Sigma_{22}(\theta) \ge \frac{4\sigma^2}{N}, \qquad \text{Direct imaging:}\ \sup_\theta \Sigma_{22}^{(\text{direct})}(\theta) \ge \frac{\sqrt{2}\sigma^2}{3\sqrt{N}}. \qquad (20)$$



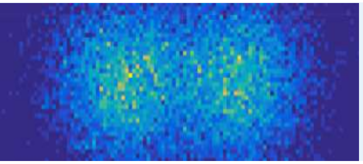


2D SPADE and SLIVER

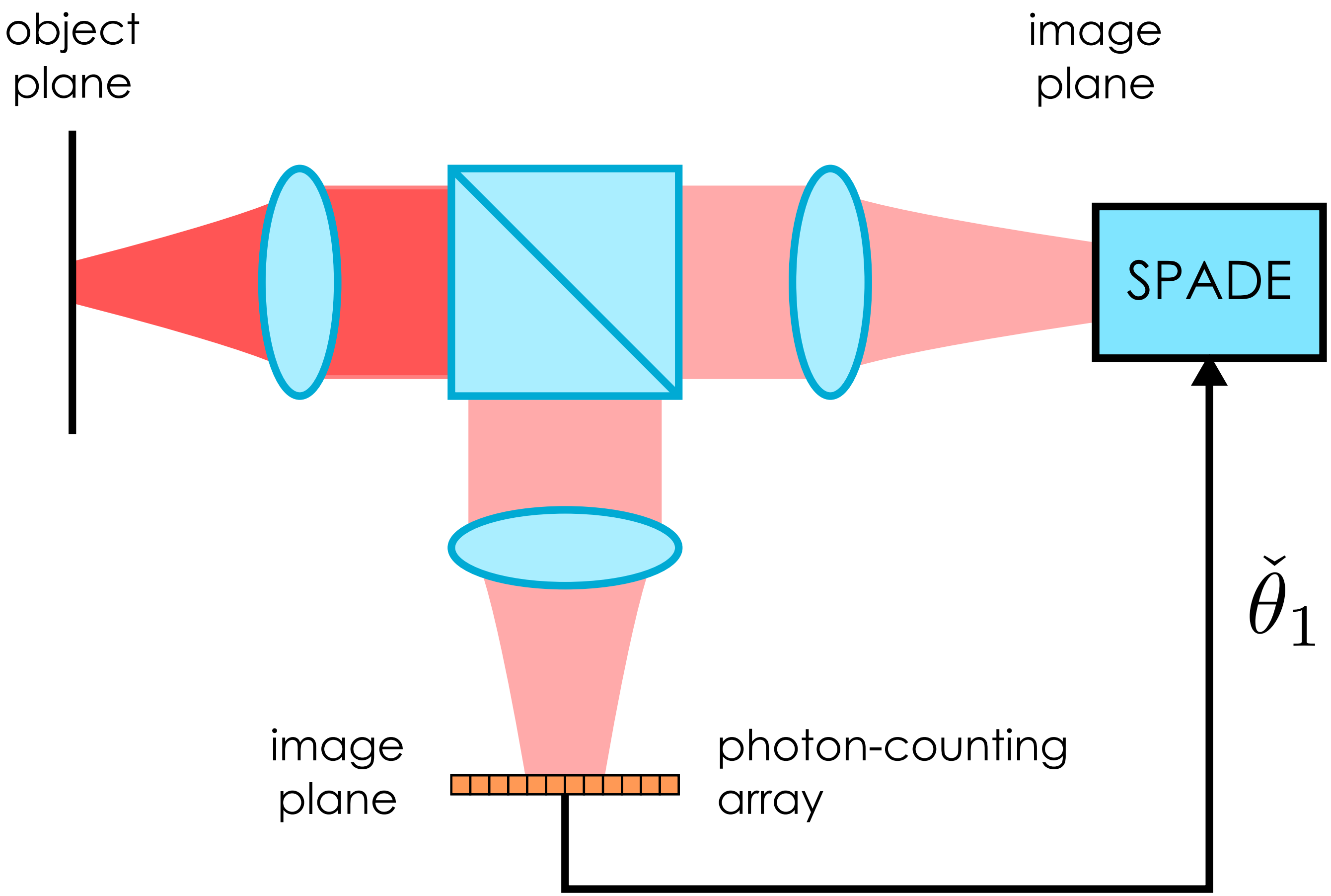


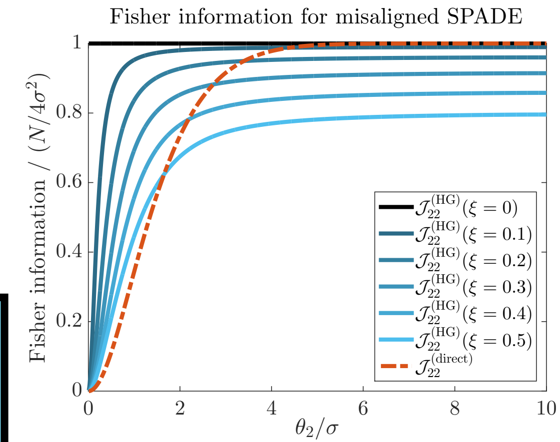


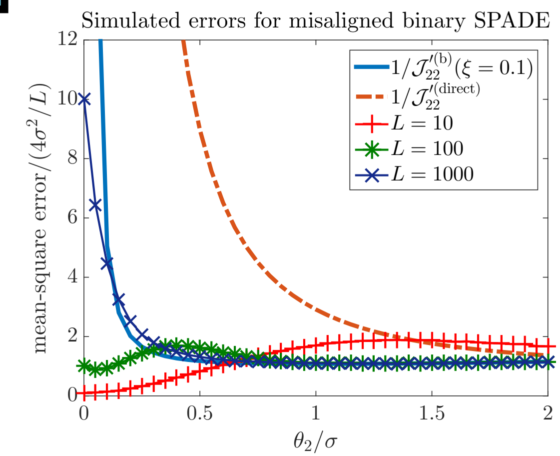
Ang, Nair, Tsang, e-print arXiv:1606.00603

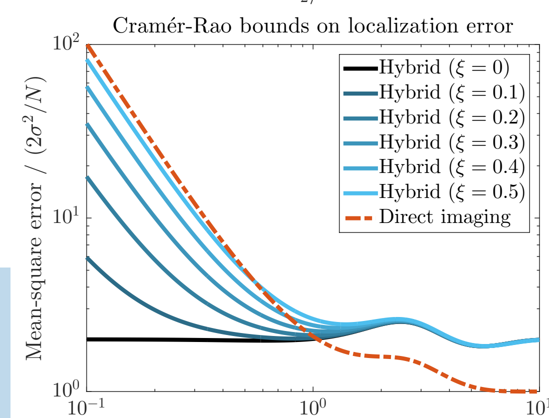


Misalignment

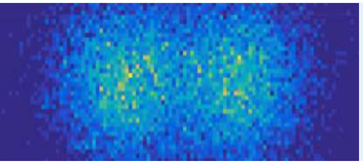




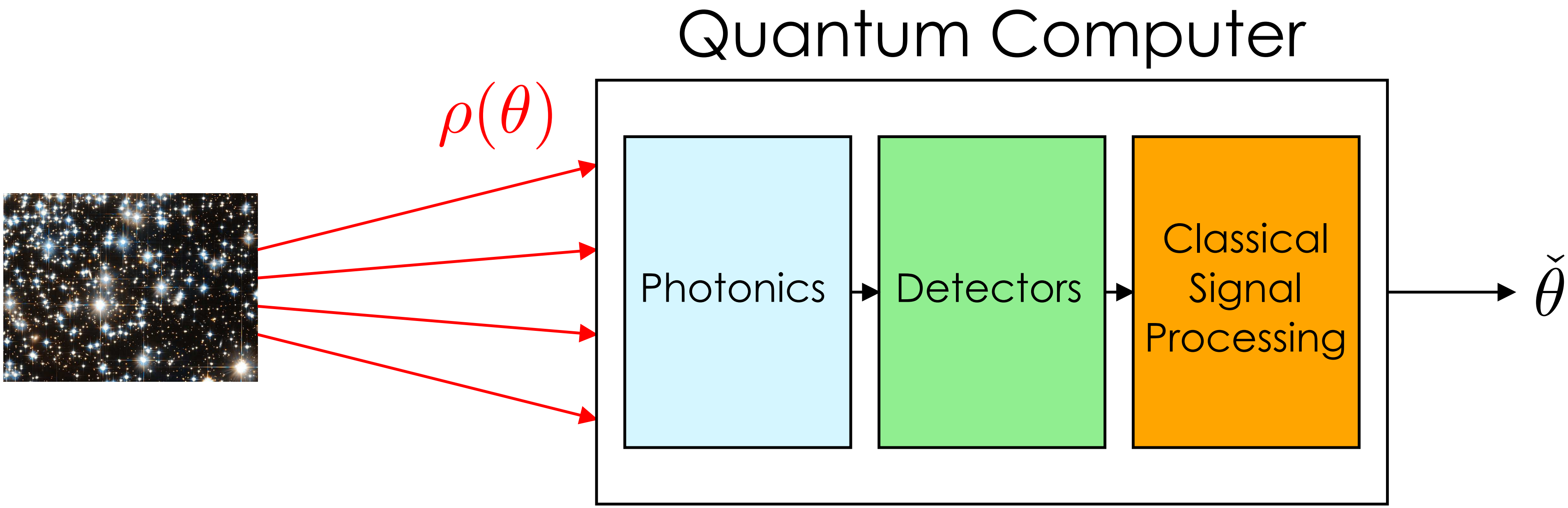




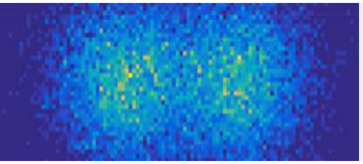
- $\xi \equiv |\check{\theta}_1 - \theta|/\sigma \ll 1$
- Overhead photons $N_1 \sim 1/\xi^2$
- $\xi = 0.1,\ N_1 \sim 100.$
- CRB for $X_s = \theta_1 \pm \theta_2/2$



Quantum Computational Imaging



- Design quantum computer to
  - Maximize information extraction
  - Reduce classical computational complexity



Quantum Technology 1.5