# Cavity Quantum Electro-optics

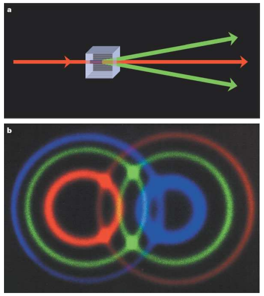
Mankei Tsang

`mankei@unm.edu`

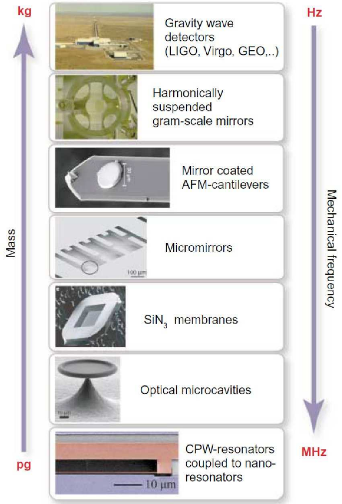
Center for Quantum Information and Control, UNM

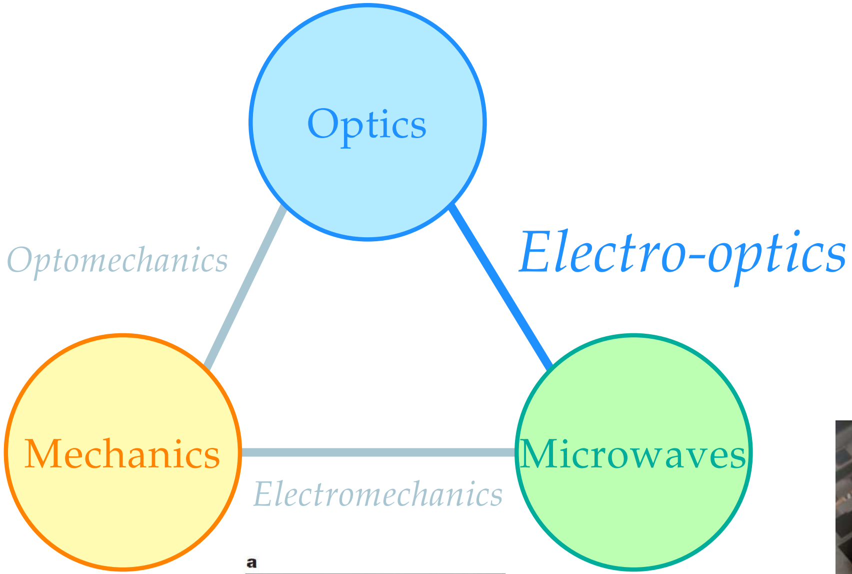
M. Tsang, Physical Review A **81**, 063837 (2010).

# What’s Next



Kippenberg and Vahala,
Science 321, 1172 (2008)





Optics

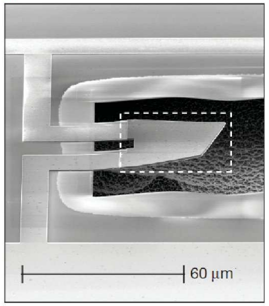
*Optomechanics*

*Electro-optics*

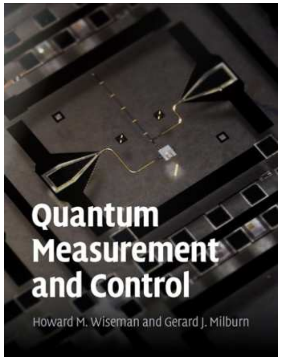
Mechanics

Microwaves

*Electromechanics*



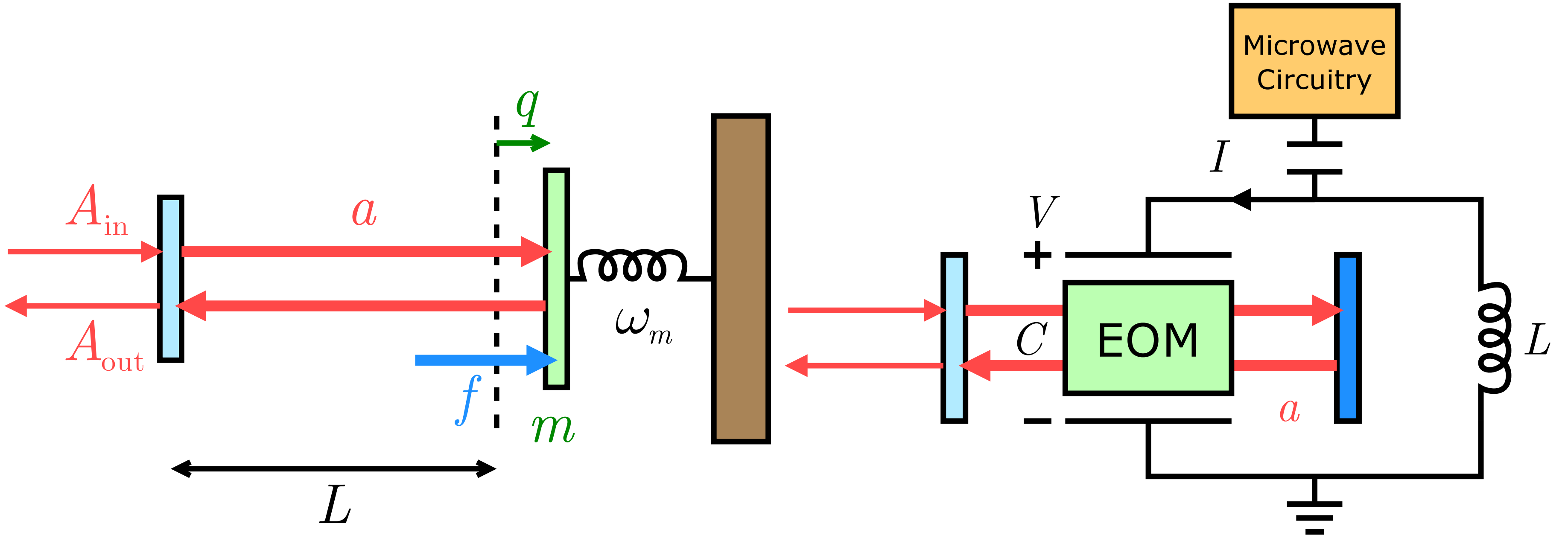
O'Connell et al., Nature 464, 697 (2010)



Quantum
Measurement
and Control

Howard M. Wiseman and Gerard J. Milburn

# Optomechanics and Electro-optics

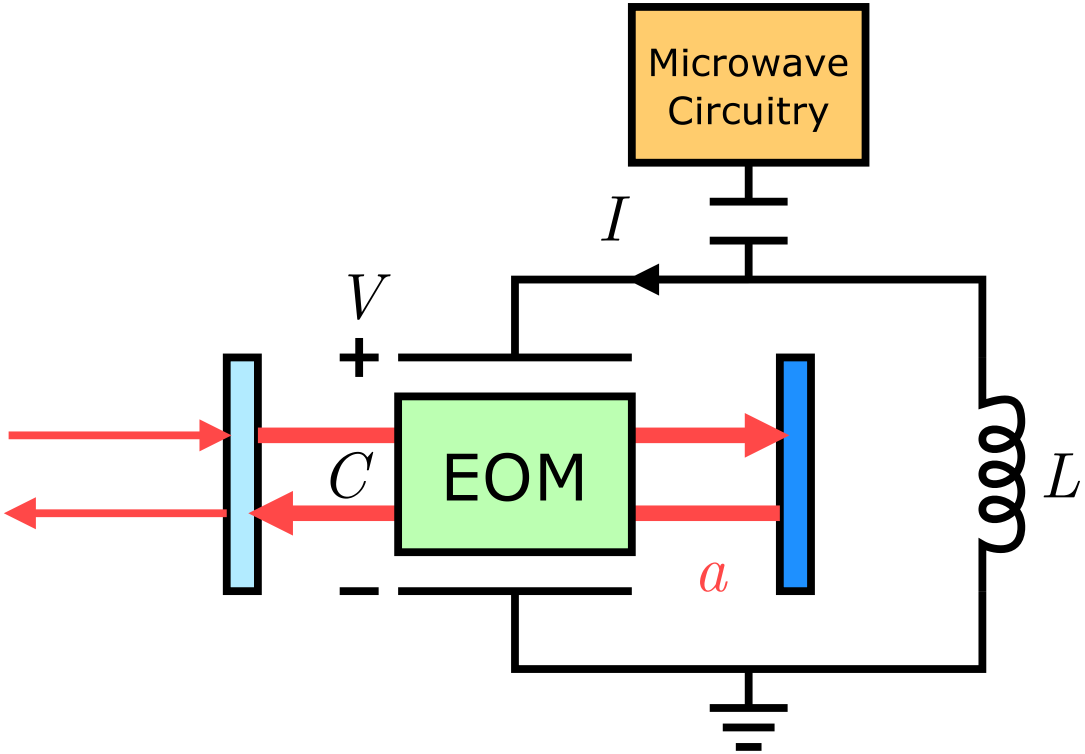


- Interaction Hamiltonian:

$$H = -\frac{\phi}{\tau} a^\dagger a \tag{1}$$

- $\phi$: optical phase shift per round trip
- $\tau$: optical round-trip time

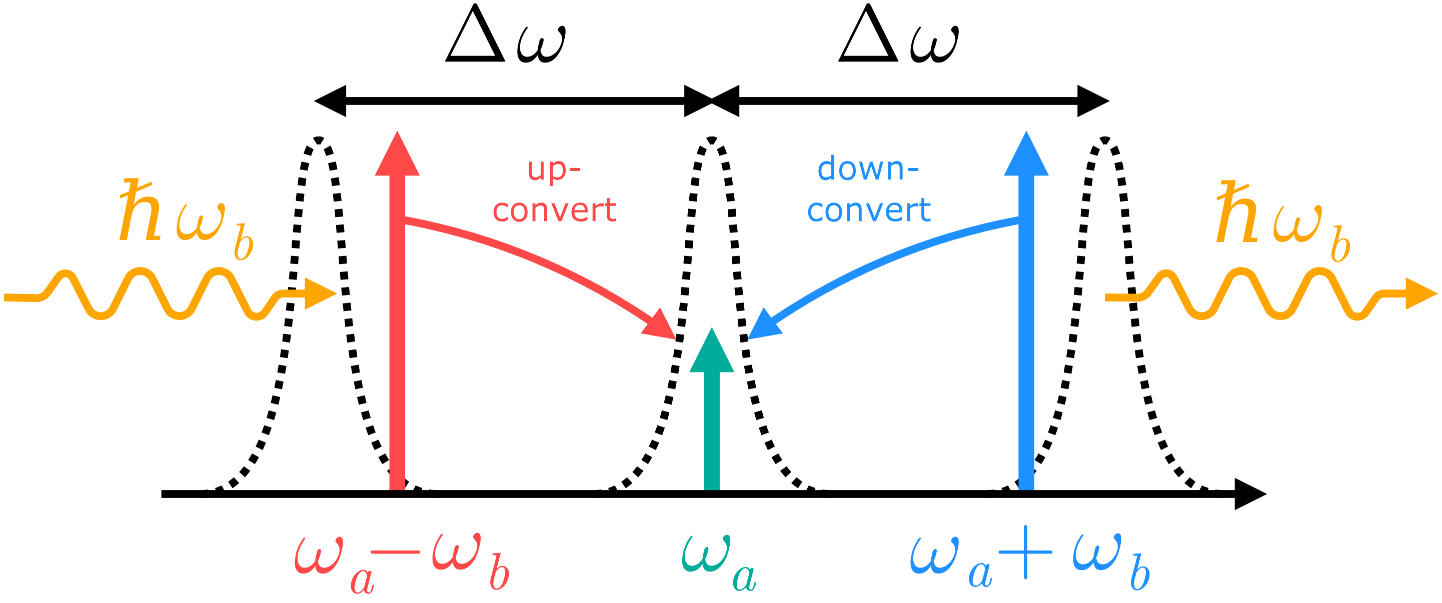
# Quantizing Microwave Resonator



$$\phi = \frac{\omega_a n^3 r l}{c d} V, \qquad V = \left(\frac{\hbar\omega_b}{2C}\right)^{1/2}\left(b + b^\dagger\right), \tag{2}$$

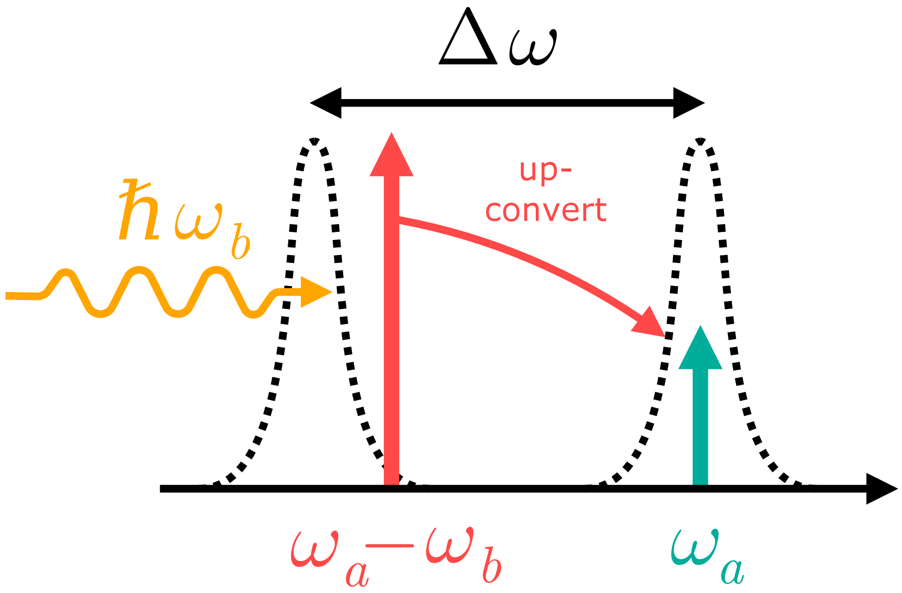
$$H = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b - \hbar g\left(b + b^\dagger\right) a^\dagger a, \qquad g \equiv \frac{\omega_a n^3 r l}{c\tau d}\left(\frac{\hbar\omega_b}{2C}\right)^{1/2}. \tag{3}$$

# Multiple Optical Modes



$$H = \hbar\omega_a a^\dagger a + \hbar(\omega_a - \Delta\omega)a_1^\dagger a_1 + \hbar(\omega_a + \Delta\omega)a_2^\dagger a_2$$
$$+ \hbar\omega_b b^\dagger b - \hbar g(b + b^\dagger)(a + a_1 + a_2)^\dagger(a + a_1 + a_2). \tag{4}$$

# Cooling

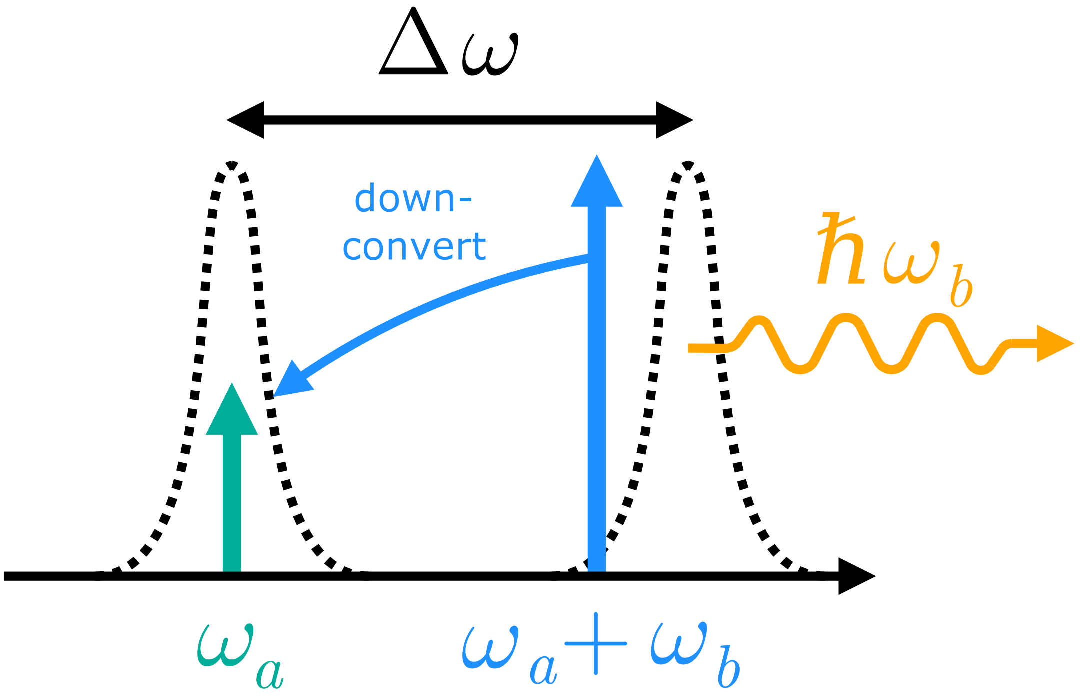


$$H_C \equiv -\hbar g\left(\alpha_- \tilde{a}^\dagger \tilde{b} + \alpha_-^\dagger \tilde{a} \tilde{b}^\dagger\right), \qquad \langle \tilde{b}^\dagger \tilde{b} \rangle_{t\to\infty} = \frac{N(\omega_b) + G N(\omega_a)}{1+G}, \tag{5}$$

$$G \equiv \frac{G_0}{1 + (\gamma_b/\gamma_a)(1+G_0)}, \qquad G_0 \equiv \frac{4g^2|\alpha_-|^2}{\gamma_a \gamma_b}. \tag{6}$$

- Potentially allows observation of QED phenomena at higher background temperatures or lower microwave/radio frequencies.

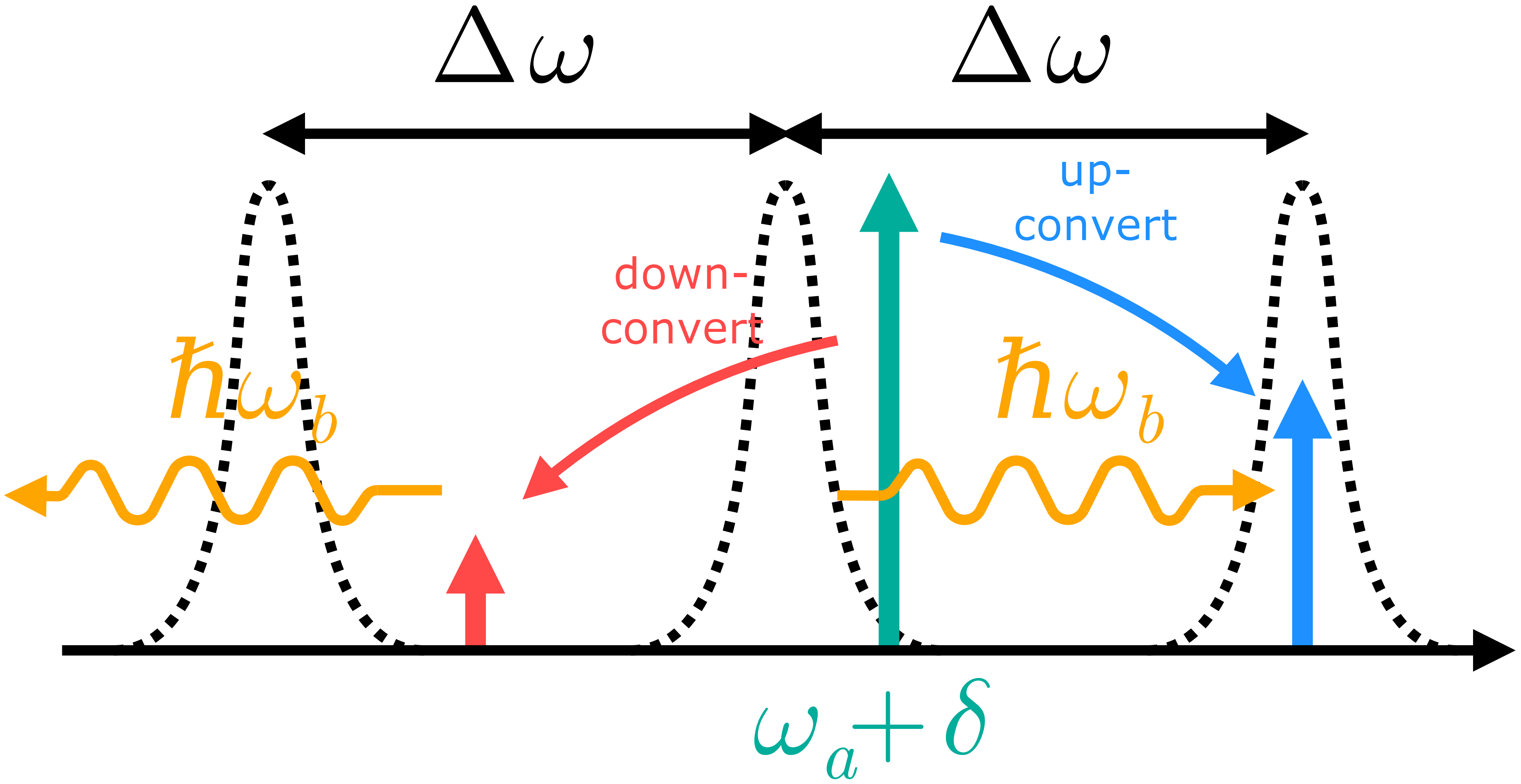
# Parametric Amplification



$$H_A \equiv -\hbar g\left(\alpha_+\tilde{a}^\dagger\tilde{b}^\dagger + \alpha_+^\dagger\tilde{a}\tilde{b}\right), \qquad \text{Threshold}: 4g^2|\alpha_+|^2/(\gamma_a\gamma_b) \geq 1. \tag{7}$$

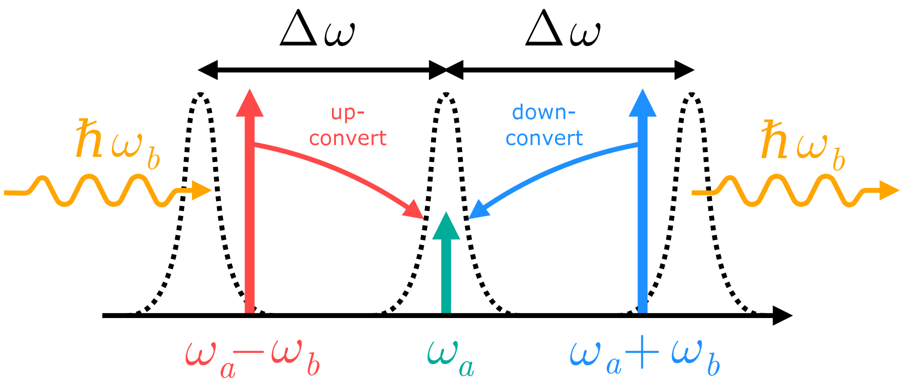
- can entangle optical and microwave modes and create entangled optical and microwave photons.

# Effect of Parasitic Down Conversion on Cooling



$$\langle\tilde{b}^\dagger\tilde{b}\rangle_{t\to\infty} \approx \frac{N(\omega_b)+\Gamma\mu}{1+\Gamma}, \qquad \Gamma \equiv \frac{\Gamma_0}{1+\mu}, \qquad \Gamma_0 \equiv \frac{4g^2|\alpha_0|^2}{\gamma_a\gamma_b}, \qquad \mu \equiv \frac{\gamma_a^2}{16\delta^2}. \tag{8}$$

# Back-Action-Evading Measurements



- Defining $\theta \equiv \frac{\theta_+ + \theta_-}{2}$, $\nu \equiv \frac{\theta_+ - \theta_-}{2}$,

$$X_a \equiv \exp(-i\theta)\tilde{a} + \exp(i\theta)\tilde{a}^\dagger, \qquad Y_a \equiv -i\left[\exp(-i\theta)\tilde{a} - \exp(i\theta)\tilde{a}^\dagger\right], \tag{9}$$

$$X_b \equiv \exp(-i\nu)\tilde{b} + \exp(i\nu)\tilde{b}^\dagger, \qquad Y_b \equiv -i\left[\exp(-i\nu)\tilde{b} - \exp(i\nu)\tilde{b}^\dagger\right], \tag{10}$$

$$\xi \equiv \exp(-i\theta)A + \exp(i\theta)A^\dagger, \qquad \eta \equiv -i\left[\exp(-i\theta)A - \exp(i\theta)A^\dagger\right], \tag{11}$$

$$\frac{dX_a}{dt} = -\frac{\gamma_a}{2}X_a + \sqrt{\gamma_a}\xi, \qquad \frac{dY_b}{dt} = 2g|\alpha|X_a, \tag{12}$$

$$\frac{dX_b}{dt} = 0, \qquad \frac{dY_a}{dt} = 2g|\alpha|X_b - \frac{\gamma_a}{2}Y_a + \sqrt{\gamma_a}\eta. \tag{13}$$

- Efficient continuous QND, BAE measurements of a microwave quadrature

# Experimental Feasibility

- For the EOM made by Ilchenko *et al.* [Ilchenko *et al.*, JOSAB **20**, 333 (2003)],

$$\omega_b \approx 2\pi \times 9 \text{ GHz}, \quad g \approx 2\pi \times 20 \text{ Hz}, \quad \gamma_a \approx 2\pi \times 40 \text{ MHz}, \quad \gamma_b \approx 2\pi \times 90 \text{ MHz}. \tag{14}$$

- assume pump power $P$ of $2$ mW, $\lambda_0 \equiv 2\pi c/\omega_a = 1550$ nm, and $\gamma_a^2/(16\delta^2) = 0.5$,

$$|\alpha_-|^2 = \frac{\gamma_a P}{\hbar\omega_a(\delta^2 + \gamma_a^2/4)} \approx 1.7 \times 10^8, \qquad G \approx 2 \times 10^{-5}. \tag{15}$$

- One can improve the $g$ coefficient by reducing the size and capacitance of the microwave resonator. $n^3 r \sim 300$ pm/V in lithium niobate, make $d \sim 10$ $\mu$m instead of the 150 $\mu$m, and assume $l/(c\tau) \sim 0.5$ and $C \sim 1$ pF, $g$ can be as high as $2\pi \times 5$ kHz, which would make $G \sim 0.3$.
- One can reduce $\gamma_b$ if the microwave resonator is unloaded or made of a better conductor. The quality factor of superconducting microwave resonators can be as high as $2 \times 10^6$ to $3 \times 10^8$ and $\gamma_b$ can then be reduced and $G$ be enhanced by a factor of $10^4$ to $10^6$.

# Conclusion

- Quantum electro-optics has the same dynamics as optomechanics.
- Cooling
- Parametric Amplification
- BAE Measurements of Microwave Quadratures
- Experimental implementation challenging but not impossible.
- M. Tsang, Physical Review A **81**, 063837 (2010).
- Discussions with Tobias Kippenberg, Elanor Huntington, Matthew Woolley, Carlton Caves, and Jeffrey Shapiro are gratefully acknowledged.
- `http://mankei.tsang.googlepages.com/`
- `mankei@unm.edu`