Evading quantum mechanics

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Quantum mechanics is potentially advantageous for certain information-processing tasks, but its probabilistic nature and requirement of measurement back action often limit the precision of conventional classical information-processing devices, such as sensors and atomic clocks. Here we show that by engineering the dynamics of coupled quantum systems, it is possible to construct a subsystem that evades the laws of quantum mechanics, at all times of interest, and obeys any classical dynamics, linear or nonlinear, that we choose.

To do so, let us revisit the concept of quantum nondemolition (QND) [1]. A QND observable is represented by a Heisenberg-picture operator $O(t)$ that commutes with itself at times t and t' when the observable is measured:

$$[O(t), O(t')] = 0 \,. \tag{1}$$

The most well-known QND observables are ones that remain *static* in the absence of classical signals, viz.,

$$O(t) = O(t') \,. \tag{2}$$

Nowadays it is often assumed that Eqs. (1) and (2) are interchangeable as the QND condition [2, 3].

To show that there exists a much wider class of QND observables, we generalize the concept of a QND observable to that of a *quantum-mechanics-free subsystem* (QMFS), which is a set of observables $\mathcal{O} = \{O_1, O_2, \ldots, O_N\}$ that obey, in the Heisenberg picture,

$$[O_j(t), O_k(t')] = 0 \text{ for all } j \text{ and } k, \tag{3}$$

at all times t and t' when the observables are measured. The operators can then be mapped to a classical stochastic processes by virtue of the spectral theorem [4] and become immune to the laws of quantum mechanics, including the Heisenberg principle and measurement invasiveness.

To construct a QMFS, consider two sets of canonical positions and momenta, $\{Q, P\} = \{Q_1, Q_2, \ldots, Q_M, P_1, P_2, \ldots, P_M\}$ and $\{\Phi, \Pi\} = \{\Phi_1, \Phi_2, \ldots, \Phi_M, \Pi_1, \Pi_2, \ldots, \Pi_M\}$, which obey the usual canonical commutation relations. Suppose the Hamiltonian has the form $H = \frac{1}{2}\sum_{j=1}^{M}(P_j f_j + f_j P_j + \Phi_j g_j + g_j \Phi_j) + h$, where $f_j = f_j(Q, \Pi, t)$, $g_j = g_j(Q, \Pi, t)$, and $h = h(Q, \Pi, t)$ are arbitrary, Hermitian-valued functions. The equations of motion for $Q_j(t)$ and $\Pi_j(t)$ become

$$\dot{Q}_j = f_j\big(Q(t), \Pi(t), t\big) \,, \quad \dot{\Pi}_j = -g_j\big(Q(t), \Pi(t), t\big) \,. \tag{4}$$

The Q and Π variables are dynamically coupled to each other, but not to the incompatible set $\{\Phi, P\}$, and thus obey Eq. (3) and form a QMFS, as depicted in Fig. 1. A prime example arises when one measures the collective position of a pair of quantum harmonic oscillators $\{q, p\}$ and $\{q', p'\}$, one with positive mass and one with negative mass, with $Q = q + q'$ and $\Pi = (p - p')/2$. This QMFS, behaving as a classical harmonic oscillator, has been experimentally demonstrated with atomic spin ensembles [5] and also proposed to remove back-action noise in optomechanics [6]. See Ref. [7] for a more in-depth discussion.

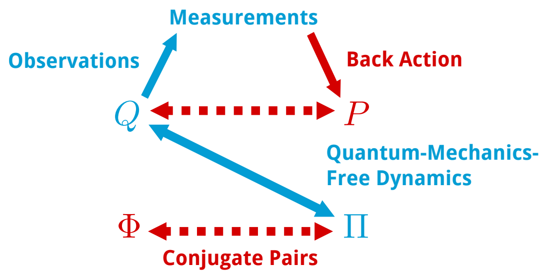

Figure 1: A quantum-mechanics-free subsystem.

It is possible to construct discrete-variable QMFSs as well. Consider a three-qubit quantum Toffoli gate [8], which transforms the Heisenberg-picture Pauli Z operators according to $Z_1' = Z_1$, $Z_2' = Z_2$, $Z_3' = (I - (I - Z_1)(I - Z_2)/2)Z_3$, where I is the identity operator. The input and output Z operators all commute, so the Z operators can be mapped to classical bits that undergo classical information processing, and one can use a circuit of Toffoli gates as a universal classical computer to implement arbitrary classical discrete-variable dynamics in discrete time.

References

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