Cavity Quantum Electro-Optic Transduction

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Abstract. The input-output relation between the optical and microwave fields coupled via a cavity electro-optic modulator with red-sideband optical pumping is derived. It is shown that the photon-flux conversion efficiency can in principle reach 100% under a critical coupling condition.

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INTRODUCTION

Future classical and quantum information processing applications will require coherent conversion of signals between the microwave and optical frequencies. A promising way to achieve the transduction is via the electro-optic effect. Ilchenko *et al.* demonstrated a photonic microwave receiver using a whispering-gallery-mode electro-optic modulator with resonant optical pumping [1], while the author has recently proposed an alternative method of electro-optic transduction using red-sideband optical pumping [2]. The side-band pumping method has the advantage of converting the microwave to an optical wave with a single carrier frequency, rather than one with multiple sidebands as demonstrated in Ref. [1], and can work equally well as a microwave transmitter that converts an optical signal to a microwave. While Ref. [2] focuses on the cooling of thermal microwave fields that can be achieved via red sideband pumping, this paper focuses on the input-output relation between the traveling optical and microwave fields and the resulting conversion efficiency.

ANALYSIS

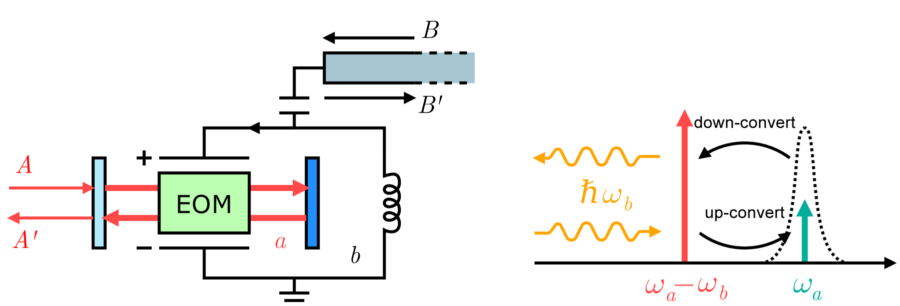

FIGURE 1. Schematic of electro-optic transduction via a cavity electro-optic modulator (EOM).

The setup is illustrated in Fig. 1. Let a and b be the intra-cavity optical and microwave annihilation operators at frequencies ω_a and ω_b, respectively. If the optical cavity is pumped at the red-detuned frequency $\omega_a - \omega_b$ with amplitude α_- (taken to be real here without loss of generality), we obtain the following equations of motion [2]:

$$\frac{da}{dt} = ig\alpha_- b - \frac{\gamma_a}{2} a + \sqrt{\gamma_a} A, \qquad A' = \sqrt{\gamma_a} a - A, \tag{1}$$

$$\frac{db}{dt} = ig\alpha_- a - \frac{\gamma_b}{2} b + \sqrt{\gamma_b} B, \qquad B' = \sqrt{\gamma_b} b - B, \tag{2}$$

where g is the electro-optic coupling constant defined in Ref. [2], $\gamma_{a,b}$ are the cavity decay rates, A and B are the input optical and microwave fields and A' and B' are the output fields. Our goal is to express A' and B' in terms of A and B only. Let us first diagonalize the system matrix given by

$$F \equiv \begin{pmatrix} -\gamma_a/2 & ig\alpha_- \\ ig\alpha_- & -\gamma_b/2 \end{pmatrix}, \tag{3}$$

which has eigenvalues given by

$$\lambda_\pm = -\frac{1}{2}\left(\frac{\gamma_a}{2} + \frac{\gamma_b}{2}\right) \pm ig\alpha_- \cos\theta, \qquad \cos\theta \equiv \sqrt{1 - \left[\frac{1}{2g\alpha_-}\left(\frac{\gamma_a}{2} - \frac{\gamma_b}{2}\right)\right]^2}. \tag{4}$$

The eigenvector matrix is

$$V = \begin{pmatrix} 1 & 1 \\ e^{-i\theta} & -e^{i\theta} \end{pmatrix}, \tag{5}$$

so we can re-express $c \equiv (a,b)^T$ in terms of eigenmode operators $u \equiv (u_+, u_-)^T$ as

$$c \equiv \begin{pmatrix} a \\ b \end{pmatrix} = Vu = \begin{pmatrix} 1 & 1 \\ e^{-i\theta} & -e^{i\theta} \end{pmatrix} \begin{pmatrix} u_+ \\ u_- \end{pmatrix}. \tag{6}$$

The transformed equations of motion become

$$\frac{du}{dt} = V^{-1} F V u + V^{-1} \Gamma C, \qquad \Gamma \equiv \begin{pmatrix} \sqrt{\gamma_a} & 0 \\ 0 & \sqrt{\gamma_b} \end{pmatrix}, \qquad C \equiv \begin{pmatrix} A \\ B \end{pmatrix}, \tag{7}$$

where $V^{-1} F V$ is diagonal with components $\lambda_\pm$. Neglecting the initial conditions for u, which decay exponentially, u in the frequency domain becomes

$$u(\omega) = H(\omega) V^{-1} \Gamma C(\omega), \qquad H(\omega) \equiv \begin{pmatrix} 1/(-i\omega - \lambda_+) & 0 \\ 0 & 1/(-i\omega - \lambda_-) \end{pmatrix}. \tag{8}$$

The output fields can hence be expressed in terms of the input fields by

$$C'(\omega) \equiv \begin{pmatrix} A'(\omega) \\ B'(\omega) \end{pmatrix} = \Gamma c(\omega) - C(\omega) = S(\omega) C(\omega), \quad S(\omega) \equiv \Gamma V H(\omega) V^{-1} \Gamma - I, \tag{9}$$

I being the identity matrix and S being the scattering matrix. Explicitly,

$$\begin{pmatrix} A'(\omega) \\ B'(\omega) \end{pmatrix} = \begin{pmatrix} S_{AA}(\omega) & S_{AB}(\omega) \\ S_{BA}(\omega) & S_{BB}(\omega) \end{pmatrix} \begin{pmatrix} A(\omega) \\ B(\omega) \end{pmatrix}. \tag{10}$$

For signals with bandwidths much smaller than $|\lambda_\pm|$, we can consider the scattering matrix S at $\omega = 0$ only:

$$S(0) = \begin{pmatrix} \cos\phi & i\sin\phi \\ i\sin\phi & \cos\phi \end{pmatrix}, \quad \cos\phi \equiv \frac{1-G_0}{1+G_0}, \quad \sin\phi \equiv \frac{2\sqrt{G_0}}{1+G_0}, \quad G_0 \equiv \frac{4g^2\alpha_-^2}{\gamma_a\gamma_b}. \tag{11}$$

Note that this G_0 is exactly the same as the G_0 defined in Ref. [2] that quantifies the cooling efficiency. The conversion efficiency in terms of the photon flux is thus

$$|S_{AB}|^2 = |S_{BA}|^2 = \frac{4G_0}{(1+G_0)^2}, \tag{12}$$

which is the same for both up-conversion and down-conversion. The efficiency reaches 100% for a finite G_0:

$$G_0 = 1, \qquad\qquad |S_{AB}|^2 = |S_{BA}|^2 = 1, \tag{13}$$

which can be understood as a *critical coupling* phenomenon. Furthermore, since S is unitary, the transduction is in principle a *noiseless* operation.

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