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Polynomial complexity optimal multiuser detection for wider class of problems

M. Motani

Optimal multiuser detection for code division multiple access has complexity which grows exponentially with the number of users. Recently proposed algorithms enable polynomial complexity optimal detection for certain special spreading sequences. It is shown that these algorithms are applicable to a wider class of spreading sequences than initially proposed.

Introduction and motivation: The symbol synchronous code division multiple access (CDMA) system model of [1] is considered, assuming K users, binary modulation and a spreading factor of N. In each symbol period, the maximum likelihood (ML) detector chooses the combination of users' bits $\mathbf{b} = [b_1, \ldots, b_K]^\top$ which maximises the conditional probability distribution function of the received signal given $\mathbf{b}$. The matched filter outputs, which form sufficient statistics for making decisions about $\mathbf{b}$ based on the received signal, are given by [1]

$$\mathbf{y} = \mathbf{RAb} + \sigma\mathbf{w} \tag{1}$$

where (i) $\mathbf{R}$ is the normalised spreading sequence correlation matrix, i.e. $\mathbf{R} = [\rho_{ij}]$: $\rho_{ii} = 1$ and $\rho_{ij} \leq 1$ for $i \neq j$, (ii) $\mathbf{A}$ is the diagonal matrix of received amplitudes, i.e. $\mathbf{A} = \mathrm{diag}(a_1, \ldots, a_K)$ and (iii) $\mathbf{w}$ is a jointly Gaussian zero-mean real vector with covariance matrix $\mathbf{R}$ and σ^2 is the noise variance. The ML estimate for $\mathbf{b}$ is given by [1]

$$\hat{\mathbf{b}}_{ML} = \arg\max_{\mathbf{b}\in\{\pm1\}^K} p_{\mathbf{y}|\mathbf{b}}(\mathbf{y}|\mathbf{b}) = \arg\max_{\mathbf{b}\in\{\pm1\}^K}\{2\mathbf{y}^\top\mathbf{Ab} - \mathbf{b}^\top\mathbf{ARAb}\} \tag{2}$$

In general, solving (2) requires 2^K function evaluations (for all possible binary vectors of length K) and so the computational complexity grows exponentially, i.e. $O(2^K)$, with the number of users [1].

There exist sets of spreading sequences for which optimum multiuser detection (MUD) can be performed with polynomial complexity (PC).

- If the sequences are orthogonal, i.e. the cross-correlation between distinct pairs of sequences is zero, then optimum detection reduces to solving K single user problems and can be done with linear complexity, i.e. $O(K)$.
- If the sequences are such that the cross-correlation between distinct pairs of sequences is non-positive, then optimal detection requires cubic complexity, i.e. $O(K^3)$ [2, 3].
- If the sequences are such that the cross-correlation between distinct pairs of sequences is constant and we have perfect power control, then optimal detection requires log-linear complexity, i.e. $O(K \log K)$ [4].

In [5], it was shown that the approach in [4] is not robust to large fluctuations in the users' powers and a modification was presented which improved performance significantly with only a mild increase in complexity. This Letter shows that the PC algorithms in [2–4] are applicable to a wider set of spreading sequences than initially proposed.

Applicability of PC algorithms: In a K user synchronous CDMA system with cross-correlation matrix $\tilde{\mathbf{R}}$ and amplitude matrix $\mathbf{A}$, the maximum-likelihood estimate is given by

$$\hat{\mathbf{b}}_{ML} = \arg\max_{\mathbf{b}\in\{\pm1\}^K}\{2y^\top\mathbf{Ab} - \mathbf{b}^\top\mathbf{A}\tilde{\mathbf{R}}\mathbf{Ab}\} \tag{3}$$

where $\tilde{\mathbf{R}}$ does not necessarily satisfy the assumptions made in [2–4], and so these algorithms cannot be directly applied. The following result establishes conditions on $\tilde{\mathbf{R}}$ which facilitates the applicability of these algorithms.

Proposition 1: Suppose that $\tilde{\mathbf{R}} = \mathbf{Q}^\top\mathbf{RQ}$ where $\mathbf{Q} = \mathrm{diag}(q_1, q_2, \ldots, q_K)$ and $q_i \in \{\pm1\}$ for all i. If the off-diagonal elements of $\mathbf{R}$ are either (i) strictly non-positive or (ii) constant, then (3) can be solved with polynomial complexity.

Proof: Using the assumptions given, (3) can be rewritten as

$$\hat{\mathbf{b}}_{ML} = \arg\max_{\mathbf{b}\in\{\pm1\}^K}\{2\mathbf{y}^\top\mathbf{Q}^\top\mathbf{Ab} - \mathbf{b}^\top\mathbf{AQ}^\top\mathbf{RQAb}\} \tag{4}$$

$$= \arg\max_{\mathbf{b}\in\{\pm1\}^K}\{2\mathbf{y}^\top\mathbf{Q}^\top\mathbf{AQb} - \mathbf{b}^\top\mathbf{Q}^\top\mathbf{ARAQb}\} \tag{5}$$

where (4) and (5) follow by noting that $\mathbf{Q}^\top\mathbf{Q} = I$ and $\mathbf{QA} = \mathbf{AQ}$, respectively.

Letting $\mathbf{c} = \mathbf{Qb}$ and $\mathbf{z} = \mathbf{Qy}$, (5) can be written as

$$\hat{\mathbf{c}}_{ML} = \arg\max_{\mathbf{c}\in\{\mathbf{Qb}:\mathbf{b}\in\{\pm1\}^K\}}\{2\mathbf{z}^\top\mathbf{Ac} - \mathbf{c}^\top\mathbf{ARAc}\}$$

$$= \arg\max_{\mathbf{c}\in\{\pm1\}^K}\{2\mathbf{z}^\top\mathbf{Ac} - \mathbf{c}^\top\mathbf{ARAc}\} \tag{6}$$

where the last equality follows from noting that $\{\mathbf{Qb}\text{: } \mathbf{b} \in \{\pm1\}^K\} = \{\pm1\}^K$. The ML solution for $\mathbf{b}$ then follows from

$$\hat{\mathbf{b}}_{ML} = \mathbf{Q}^\top\hat{\mathbf{c}}_{ML} \tag{7}$$

If $\mathbf{R}$ satisfies the non-positive cross-correlation property, [2, 3] can be used to solve (6) and equivalently (3) with polynomial complexity. If $\mathbf{R}$ satisfies the constant cross-correlation property and we have perfect power control, [4] can be used to solve (6) and equivalently (3) with polynomial complexity. □

Comment: In Proposition 1, it was assumed that $\mathbf{Q} = \mathrm{diag}(q_1, q_2, \ldots, q_K)$ and $q_i \in \{\pm1\}$ for all i. However, if there is perfect power control, i.e. $\mathbf{A} = I$, then the only necessary conditions are $\mathbf{Q}^\top\mathbf{Q} = I$ and $\{\mathbf{Qb}\text{: } \mathbf{b} \in \{\pm1\}^K\} = \{\pm1\}^K$. If $\mathbf{A} \neq I$, then the assumptions made in Result 1 are necessary because the additional condition that $\mathbf{QA} = \mathbf{AQ}$ is required. If a square matrix $\mathbf{Q}$ commutes with the square diagonal matrix $\mathbf{A}$, then either $\mathbf{A}$ is the identity matrix or $\mathbf{Q}$ is a diagonal matrix. This is easily seen by equating the (i, m)th entries of $\mathbf{QA}$ and $\mathbf{AQ}$:

$$\sum_j \mathbf{Q}_{ij}\mathbf{A}_{jm} = \sum_j \mathbf{A}_{ij}\mathbf{Q}_{jm}$$

Since $\mathbf{A}$ is diagonal, $\mathbf{Q}_{im}\mathbf{A}_{mm} = \mathbf{A}_{ii}\mathbf{Q}_{im}$. The condition $i = m$ yields no constraints. The condition $i \neq m$ implies that either $\mathbf{A}_{mm} = \mathbf{A}_{ii}$ or $\mathbf{Q}_{im} = 0$. So if $\mathbf{A} \neq I$ and $\mathbf{QA} = \mathbf{AQ}$, then $\mathbf{Q} = \mathrm{diag}(q_1, q_2, \ldots, q_K)$. Finally since $\mathbf{Q}^\top\mathbf{Q} = I$, it is clear that $q_i \in \{\pm1\}$ for all i.

Example: Consider the seven-user synchronous CDMA system using the following set of non-orthogonal spreading sequences:

$$\tilde{\mathbf{S}} = \begin{bmatrix} -1 & -1 & -1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 & 1 & 1 & 1 \\ -1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 & -1 \end{bmatrix}$$

The cross-correlation matrix, $\tilde{\mathbf{R}}=\tilde{\mathbf{S}}^T\tilde{\mathbf{S}}$, is not of the form required by [2–4]. However, noting that $\tilde{\mathbf{R}}=\mathbf{QRQ}$, where $\mathbf{Q}$ is the diagonal matrix defined by $\mathbf{Q}=\text{diag}\{-1, 1, 1, -1, -1, -1, 1\}$, and $\mathbf{R}$ is of the form required by [4], the original problem can be solved with polynomial (log-linear) complexity.

Spreading sequences: The existence of the spreading sequences for which PC algorithms exist is now considered. [3] showed that for any N there exist at most $(3/2)N$ sequences with non-positive cross-correlations. For the constant cross-correlation case, there exist constructions based on maximal-length sequences, Gold sequences, and cyclic difference sets [6]. The following constructive lower bound is also proved.

Proposition 2: For any N, there exist at least N sequences of length N with the constant cross-correlation property.

Proof: It is proved that N is a lower bound by explicit construction. Given N, consider the set of unit basis vectors in an N-dimensional linear vector space, $S=\{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_N\}$ where $\mathbf{e}_k=\{0, 0, \ldots, 0, 1, 0, \ldots, 0\}$ with the 1 in the kth place. Since any two sequences in this set differ in exactly two places, the cross-correlation between any two distinct sequences is $\rho=N-4$. □

Conclusion: In this Letter, it has been shown that the polynomial complexity algorithms in [2–4] are applicable to a wider class of multiuser detection problems. By explicit construction, a lower bound on the size of sequence sets has been presented for which certain optimal polynomial complexity algorithms exist. The low-complexity detection techniques in this Letter are applicable to other maximum-likelihood sequence detection problems, e.g. single-user duobinary signalling.

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3.48 ps ECL ring oscillator using over-300 GHz f_T/f_{max} InP DHBTs

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An ECL gate delay of 3.48 ps/stage in a 19-stage conventional ring oscillator fabricated in an over-300 GHz f_T/f_{max} InP DHBT technology has been demonstrated. This is the first report (to the authors' knowledge) of sub-4 ps ECL gate delay. Large collector current density contributes to the very short gate delay.

Introduction: The improvement of operation speed of transistors is an essential requirement for over 100 Gbit/s-class logic applications. To evaluate the high-speed characteristics of transistors, f_T and f_{max} are commonly used as figures of merit. However, these values are not directly relevant to the performance of digital circuits. High current density capability is also indispensable for ultra-high-speed logic circuits using bipolar transistors. Ring oscillators have been widely used for evaluating device technologies for logic applications. SCFL ring oscillators with 4.6 ps gate delay have been demonstrated using InP HEMTs [1], and ECL ring oscillators in SiGe HBT technology have achieved gate delay of 4.2 ps [2]. Moreover, very short gate delay of 2 ps has been reported for a CML ring oscillator fabricated in a 170 GHz f_T, 150 GHz f_{max} InP HBT technology [3]. In the CML oscillator, emitter feedback and zero-peaking techniques [3] made it possible to attain such a small gate delay using production-level HBTs.

This Letter describes 19-stage ECL ring oscillators with ultra-high-speed InP DHBTs with 150 and 200 nm-thick collectors. The operation speed of an ECL gate is less sensitive to fanout than that of a CML gate due to the ECL gate's large current driving capability. Therefore, ECL gates are basically suitable for estimating the operation speed of actual ultra-high-speed logic ICs, which have a fanout greater than one. We used a conventional circuit configuration so as to clarify the potential of the transistor technology.

Table 1: Base layer parameters

	Thickness (nm)	GaAs mole fraction (collector → emitter)	Sheet resistance (Ω/sq.)
C150-HBT	30	0.47 → 0.56	573
C200-HBT	35	0.47 → 0.60	449

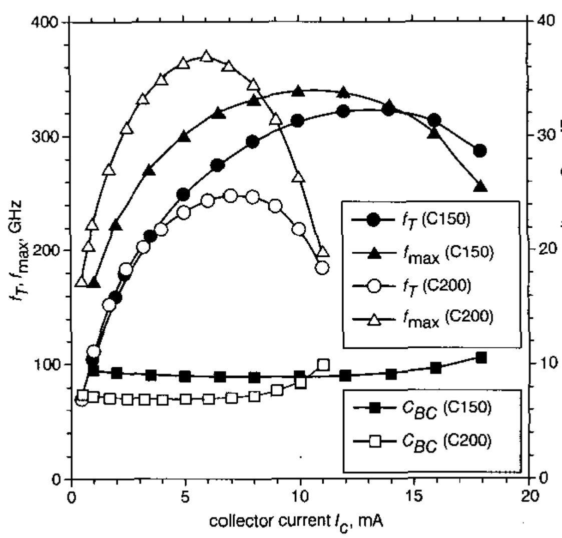

Fig. 1 *f_T, f_{max}, and total collector capacitance against collector current at V_{CE} of 1.3 V*

InP DHBT and IC fabrication: The HBT layers were grown on 3-inch InP substrates by low-pressure MOVPE. Collector thicknesses were 150 (C150-HBT) or 200 nm (C200-HBT). The C150-HBT was designed for high-current-density operation to achieve high transconductance. A step-graded InGaAs/InGaAsP/InP collector [4, 5] was utilised to obtain a high breakdown voltage for such thin collectors. The base layers were compositionally graded InGaAs [6] heavily carbon-doped to 6×10^{19} cm^{-3}. The thickness, GaAs composition, and sheet resistance of the base layers are shown in Table 1. The InP emitter layer was doped to 3 and 6×10^{17} cm^{-3} for the C200- and C150-HBTs, respectively. The C150-HBT was doped to such a high density to compensate for the space charge of electrons in the high-current-density operation. For the device layout, we used the base-pad isolation structure in order to eliminate the extrinsic collector capacitance at the base pad area [7]. The emitter size is 0.8×3 μm^2. The