

The Capacity of the Single Source Multiple Relay Single Destination Mesh Network

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Abstract

In this paper, we derive the information theoretic capacity of a special class of mesh networks. A mesh network is a heterogeneous wireless network in which the transmission among power limited nodes is assisted by powerful relays, which use the same wireless medium. We investigate the mesh network when there is one source, one destination, and multiple relays, which we call the single source multiple relay single destination (SSMRSD) mesh network. We derive the asymptotic capacity of the SSMRSD mesh network when the relays' powers get large. Our approach is as follows. We first look at an upper bound on the information theoretic capacity of these networks in a Gaussian setting. We then show that this bound is achievable asymptotically using the compress-forward strategy for the multiple relay channel. While this result is derived for the asymptotic case when the relays' powers grow to infinity, we extend our analysis to the case with finite but large relay's power (using the single relay network as an example). We show that even when the relay's power is only a few times larger than the source's power, the compress-forward rate gets close to the capacity. The results indicate the value of cooperation in wireless networks. The capacity characterization quantifies how the relays can cooperate, using the compress-forward strategy, to either conserve node energy or to increase transmission rate.

Key words: Mesh network, multiple relay channel, capacity

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1 Introduction

Wireless networks have been finding more applications and capturing much research attention in recent years. The advantage of mobile clients makes ad-hoc wireless networking an attractive solution for home and enterprise users. However, with an almost unlimited number of ways of interacting and cooperating, analysis of these multi-terminal networks is difficult. To date, the capacity of even the simple three-node channel [1] is not known, except for special cases, for example, the multiple access channel [2][3], the degraded relay channel [4], the degraded broadcast channel [5]. However, this did not hinder research in channel with more nodes.

Recently, *mesh networks* have been capturing interests from both the research arena and the industry. Mesh networks ([6] and the references therein) are peer-to-peer multihop wireless networks with powerful relays. One practical setup of the mesh network is depicted in Fig. 1. It consists of mesh routers (stationary and line powered) and mesh clients (mobile and battery powered). This type of setup is adopted by 802.11s [7], the IEEE standard (waiting for approval) for wireless mesh networks. The standard defines a network of wireless access points (mesh routers) communicating wirelessly with each other and serving mesh clients in their proximity. In other words, mesh routers act as relays. The access points forward data packets via multi-hopping. They do not need to connect to the wired backbone as in IEEE 802.11a/b/g. Although there is no restriction in the definition of the mesh network that prevents a mesh client to forward data for other mesh clients, this will raise the issues of security and fairness when implemented.

In this paper, we study the single source multiple relay single destination (SSMRSD) mesh network. We model the SSMRSD mesh network by the multiple relay channel [8][9][10][11][12]. The multiple relay channel captures the scenario where the transmission from the source to the destination is aided by several relay nodes, which themselves have no data to send. As such, the SSMRSD mesh network is a multiple relay channel where the relays can transmit at large power. One can treat the SSMRSD mesh network as an excerpt of the general mesh network where we consider just one of the source-destination pairs. We note that the capacity of the general multiple relay channel has not been found, except for special cases, e.g., the degraded multiple relay channel. The capacity of the SSMRSD mesh network, which is not a degraded multiple relay channel [8, Theorem 2.3], has not been found.

We investigate rates achievable by the compress-forward coding strategy on the SSMRSD mesh network. The coding technique was first introduced by Cover and El Gamal [4] for the single relay channel and later extended to the multiple relay channel by Kramer *et al.* [9], where it is called the compress-

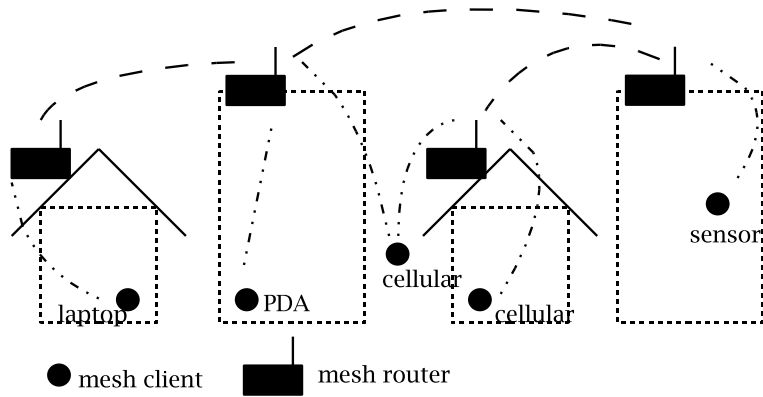


Fig. 1. A mesh network.

and-forward strategy. In the compress-forward strategy, the source transmits to both the relay and the destination. The relay does not decode the data, but simply quantizes, compresses, and sends them to the destination. We categorize this coding strategy as a cooperative coding strategy, in which the nodes participate in some rich form of cooperation. To explain this rich form of cooperation, we contrast it with a non-cooperative coding strategy, the *multihop strategy*. In the multihop strategy, when one router transmits, only the next hop router listens. Though other routers over-hear the transmission, they regard it as noise. On the other hand, in the cooperative strategy, instead of ignoring the useful transmission, nodes that over-hear the transmissions of other nodes (though the information might not be intended for them) help to forward, fully or partially, the data to the destination.

We show that the compress-forward strategy approaches the capacity of the SSMRSD mesh network as the relays' powers grow. Our approach is as follows. First, we study an upper bound on the capacity of the SSMRSD mesh network, which is derived from the max-flow min-cut theorem. We study achievable rates of the compress-forward coding strategy on the multiple relay channel. We show that when the power constraints of the relays are relaxed (which is the case of the SSMRSD mesh network), the compress-forward technique approaches the capacity upper bound asymptotically. Using the single relay channel as an example, we investigate the performance of the compress-forward strategy as the relay's power grows.

Our contributions in this paper are as follows:

- (1) We derive an achievable rate expression of the compress-forward strategy on the Gaussian multiple relay channel.
- (2) We derive the capacity (asymptotic result) of the SSMRSD Gaussian mesh network as the relays' powers grow to infinity.
- (3) We show, using numerical analyses, how the compress-forward rate approaches the capacity as the relay's power grows. More specifically, we show that rates achievable by the compress-forward strategy is close to

the capacity even when the relay's power is only a few times higher than the source's power.

- (4) We study and compare coding strategies on the SSMRSD mesh network. The comparison of different coding strategies motivates code designers to design practical coding schemes based on certain types of cooperation, to drive the transmission rate toward the theoretical limits. Knowing the network capacity as a function of transmission powers and node positions, and understanding how nodes should cooperate help network designers in network deployment.

Analysis on the capacity of multiple source multiple destination networks is hard. Gupta and Kumar [13] found an upper bound and an lower bound on the capacity of such channels. However, the analyses cannot be readily applied to mesh networks. This is because in [13], every node has data to send to a random destination, but in mesh networks, relays (mesh routers) do not generate data.

Most research on the capacity or transmission rate of mesh networks, e.g., Jun *et al.* [14][15], Kodialam and Nandagopal [16], Roy *et al.* [17], are based on non-cooperative multihop coding strategies. Data are forwarded hop-by-hop from the mesh clients to the gateway through relays. Transmissions in proximity use orthogonal channels, e.g., using time division multiple access (TDMA) or frequency division multiple access (FDMA), to avoid interference. In this paper, we give examples to show that the rate achievable by the TDMA multihop strategy is lower than that by the compress-forward strategy. The FDMA multihop strategy can be analyzed similarly.

The rest of the paper is organized as follows. Section 2 introduces the channel model. In Section 3, we establish a useful theorem that we will need in later sections. In Section 4, we investigate an upper bound on the capacity of the SSMRSD mesh network. This is followed an investigation of the rates achievable by the compress-forward strategy, in Section 5. By allowing the power constraint of the relays to increase, we show, in Section 6, that the rate achievable by the compress-forward strategy approaches the capacity asymptotically. In Section 7, we investigate the behavior of the compress-forward rate when the relay's power is finite. In addition, we compare the performance of the compress-forward and the decode-forward strategies for different relay's positions. We conclude this paper in Section 8.



Fig. 2. The multiple relay channel.

2 Channel Model

Fig. 2 depicts the multiple relay channel. The multiple relay channel can be completely described by the channel distribution

$$p^*(y_2, y_3, \dots, y_T | x_1, x_2, \dots, x_{T-1}) \quad (1)$$

on $\mathcal{Y}_2 \times \mathcal{Y}_3 \times \dots \times \mathcal{Y}_T$, for each $(x_1, x_2, \dots, x_{T-1}) \in \mathcal{X}_1 \times \mathcal{X}_2 \times \dots \times \mathcal{X}_{T-1}$. In this paper, we only consider memoryless channels. Node 1 (on the extreme left) is the source node and node T (on the extreme right) is the destination node. Nodes 2 to $T - 1$ are purely relay nodes. Message W is generated at node 1 and is to be transferred to the sink at node T . We follow the definitions of capacity, achievable rate (R_W) used by Kramer *et al.* [9, Section III.A].

In a Gaussian multiple relay channel, node j , $j = 2, \dots, T$, receives

$$Y_j = \sum_{\substack{i=1, \dots, T-1 \\ i \neq j}} \sqrt{\lambda_{ij}} X_i + Z_j, \quad (2)$$

where X_i , input to the channel from node i , is a random variable with power constraint $E[X_i^2] \leq P_i$. P_i is the average transmission power constraint on node i . Y_j is the received signal at node j . Z_j , the receiver noise at node t , is an independent zero mean Gaussian random variable with variance N_j . λ_{ij} is the channel gain from node i to node j . λ_{ij} depends on the antenna gains, the carrier frequency of the transmission, and the distance between the transmitter and the receiver.

We study achievable rates of the T -node Gaussian multiple relay channel. More specifically, we investigate the performance of the compress-forward strategy (i.e, achievable rates) when the relays' powers grow large. This models the SSMRSD mesh network, in which the relays are line powered and can transmit at high power. We define $\mathcal{R} \triangleq \{2, 3, \dots, T - 1\}$ as the set of all relay nodes. We use the notation $X_{\{1, \dots, m\}} \triangleq (X_1, \dots, X_m)$.

3 A Cut-Set Bound is Attained by Independent Gaussian Inputs

In this section, we establish a useful theorem which we will need in the sequel. We consider a T -node multiple relay channel where nodes $1, \dots, T - 1$ send X_1, \dots, X_{T-1} into the channel respectively. The channel inputs are subject to

power constraints $E[X_i] \leq P_i$ for $i = 1, \dots, T - 1$. Without loss of generality, nodes $2, \dots, T$ receive the following signals from the channel.

$$Y_j = \sum_{i \in \{1\} \cup \mathcal{R} \setminus \{j\}} X_i + Z_j \quad (3)$$

where $Z_j \sim \mathcal{N}(0, N_j)$, $j = 2, 3, \dots, T$ are independent Gaussian noise. Without loss of generality, we set the channel gains to be 1.

We consider the cut-set bound on the rate at which we can transmit information from the source to the relays and the destination, assuming that the relays and the destination can cooperate. The following theorem establishes that the optimal input distribution to maximize this bound is such that the the source and the relays send independent Gaussian inputs.

Theorem 1 *Consider a T -node Gaussian multiple relay channel. A sufficient condition on the input distribution that achieves*

$$\max_{p(x_1, x_2, \dots, x_{T-1})} I(X_1; Y_{\mathcal{R}}, Y_T | X_{\mathcal{R}}) \quad (4)$$

is that the inputs are Gaussian and X_1 is independent of $X_{\mathcal{R}}$. It follows that independent Gaussian inputs X_1, \dots, X_{T-1} also achieve (4).

Proof of Theorem 1: First, we consider the case $T = 3$, which means there is one relay. We want to show that

$$\max_{p(x_1, x_2)} I(X_1; Y_2, Y_3 | X_2) \quad (5)$$

is achieved when X_1 and X_2 are independent Gaussian inputs.

From [9, Proposition 2], we know the optimal input distribution is Gaussian. We let

$$X_1 = \alpha X_2 + W, \quad (6)$$

where W and X_2 are independent Gaussian random variables, such that $E[W^2] = P_W$ and $P_1 = \alpha^2 P_2 + P_W$.

Now,

$$H(Y_2, Y_3 | X_1, X_2) = \frac{1}{2} \log(2\pi e)^2 N_2 N_3, \quad (7)$$

and

$$H(Y_2, Y_3 | X_2) = \frac{1}{2} \log(2\pi e)^2 \begin{vmatrix} P_W + N_2 & P_W \\ P_W & P_W + N_3 \end{vmatrix} \quad (8a)$$

$$= \frac{1}{2} \log(2\pi e)^2 (P_W N_2 + P_W N_3 + N_2 N_3). \quad (8b)$$

Hence,

$$I(X_1; Y_2, Y_3 | X_2) = H(Y_2, Y_3 | X_2) - H(Y_2, Y_3 | X_1, X_2) \quad (9a)$$

$$= \frac{1}{2} \log \left[1 + \frac{P_1 - \alpha^2 P_2}{N_2} + \frac{P_1 - \alpha^2 P_2}{N_3} \right]. \quad (9b)$$

Setting $\alpha = 0$ maximizes the mutual information. This completes the proof for $T = 3$.

Now, we extend this result to $T = 4$ or the two-relay channel. The generalization from the two-relay channel to the multiple-relay channel is straight forward. We need to show that a sufficient condition on the input distribution function to achieve

$$\max_{p(x_1, x_2, x_3)} I(X_1; Y_2, Y_3, Y_4 | X_2, X_3) \quad (10)$$

is that X_1 and (X_2, X_3) are independent Gaussian inputs.

From [9, Proposition 2], (10) is achieved by Gaussian inputs X_1 , X_2 , and X_3 . Now, we combine the relays' inputs to form $X_{\mathcal{R}} = X_2 + X_3$. The destination receives $Y_4 = X_1 + X_{\mathcal{R}} + N_4$. From the single relay case $T = 3$, we know that choosing X_1 to be independent of $X_{\mathcal{R}}$ is optimal. Certainly, choosing independent X_1 , X_2 , and X_3 maximizes the mutual information term. This proves the case of $T = 4$.

Now, we demonstrate that (10) can indeed be achieved with any correlation between X_2 and X_3 , as long as X_1 is independent of (X_2, X_3) . We let $X_2 = \beta X_3 + W$, where X_1 , X_3 and W are independent Gaussian inputs. Here, $E[W^2] = P_W$ and $P_2 = \beta^2 P_3 + P_W$.

Now,

$$H(Y_2, Y_3, Y_4 | X_1, X_2, X_3) = \frac{1}{2} \log(2\pi e)^3 N_2 N_3 N_4. \quad (11)$$

Also,

$$H(Y_2, Y_3, Y_4 | X_2, X_3) = \frac{1}{2} \log(2\pi e)^3 \begin{vmatrix} P_1 + N_2 & P_1 & P_1 \\ P_1 & P_1 + N_3 & P_1 \\ P_1 & P_1 & P_1 + N_4 \end{vmatrix} \quad (12a)$$

$$= \frac{1}{2} \log(2\pi e)^3 [P_1(N_2 N_3 + N_2 N_4 + N_3 N_4) + N_2 N_3 N_4]. \quad (12b)$$

Hence,

$$I(X_1; Y_2, Y_3, Y_4 | X_2, X_3) = \frac{1}{2} \log \left[1 + P_1 \left(\frac{1}{N_2} + \frac{1}{N_3} + \frac{1}{N_4} \right) \right]. \quad (13)$$

We note that this is independent of β . This means that (10) can be achieved with any correlation between X_2 and X_3 .

We can easily generalize this result to any $T > 4$ and hence obtain Theorem 1. \square

4 A Capacity Upper Bound of the Multi-Terminal Network

4.1 In the Multi-Terminal Network

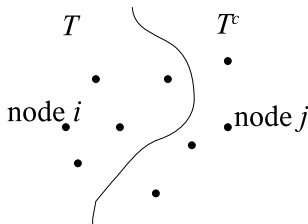


Fig. 3. A cut in the multi-terminal network.

Consider a T -node multi-terminal network where node i transmits X_i and node j receives Y_j . The channel is characterized by the channel transition probability $p(y_1, \dots, y_T | x_1, \dots, x_T)$. If the rate from node i to node j , R_{ij} , is achievable, then the following must be satisfied [18, Theorem 14.10.1]

$$\sum_{i \in \mathcal{T}, j \in \mathcal{T}^c} R_{ij} \leq \max_{p(x_1, \dots, x_T)} I(X_{\mathcal{T}}; Y_{\mathcal{T}^c} | X_{\mathcal{T}^c}), \quad (14)$$

for some joint probability function $p(x_1, \dots, x_T)$ for all $\mathcal{T} \subset \{1, \dots, T\}$ where $i \in \mathcal{T}$ and $j \notin \mathcal{T}$. \mathcal{T}^c is the complement of \mathcal{T} in $\{1, \dots, T\}$.

We can interpret this theorem as follows. The achievable rate from node i to node j must be smaller than the rate of all possible cuts separating nodes i and j . Fig. 3 depicts a possible cut. We define *cut rate* for the cut separating \mathcal{T} and \mathcal{T}^c as the right side of (14). It is the maximum achievable rate from nodes in \mathcal{T} to nodes in \mathcal{T}^c when all nodes on the same side of the cut are allowed to cooperate.

4.2 In the SSMRSD Mesh Network

Consider a T -node Gaussian SSMRSD mesh network where

- Node 1 is the source node with power constraint $E[X_1^2] \leq P_1$. The source only transmits and does not receive.

- Node T is the destination node, which only receives and does not transmit.
- Nodes 2 to $T - 1$ are relays with large power constraints, $E[X_i^2] \leq P_i$ and $P_i \gg P_1, \forall i = 2, \dots, T - 1$. The relays can transmit and receive at the same time.

We note that any cut rate with $1 \in \mathcal{T}$ and $T \in \mathcal{T}^c$ is an upper bound of the rate from the source to the destination. Since the relays can transmit with large power, if we include any relay node in set \mathcal{T} , the cut rate (defined as (14)) is large. Hence the minimum cut rate occurs when the cut separates $\mathcal{T} = \{1\}$ and $\mathcal{T}^c = \{2, \dots, T\}$. So the upper bound of the capacity of the SSMRSD reduces to

$$C_{\text{SSMRSDMesh}} \leq \max_{p(x_1, \dots, x_{T-1})} I(X_1; Y_{\mathcal{R}}, Y_T | X_{\mathcal{R}}), \quad (15)$$

for some joint probability function $p(x_1, \dots, x_{T-1})$. From Theorem 1, independent Gaussian inputs maximize this upper bound in the Gaussian channel.

5 Achievable Rates

5.1 In the Discrete Memoryless Multiple Relay Channel

In this section, we investigate achievable rates of the multiple relay channel using the compress-forward strategy. Setting $U_t = X_t, \forall t \in \mathcal{R}$ in the compress-forward strategy for the multiple relay channel introduced by Kramer *et al.* [9, Theorem 3], we can achieve rates up to

$$R = I(X_1; \tilde{Y}_{\mathcal{R}}, Y_T | X_{\mathcal{R}}), \quad (16)$$

where

$$I(\tilde{Y}_{\mathcal{S}}; Y_{\mathcal{S}} | X_{\mathcal{R}}, \tilde{Y}_{\mathcal{S}^c}, Y_T) \leq \sum_{m=1}^M I(X_{\mathcal{B}_m}; Y_{r(m)} | X_{\mathcal{B}_m^c}), \quad (17)$$

with the joint probability distribution function

$$p(x_1) \left[\prod_{t \in \mathcal{R}} p(x_t) p(\tilde{y}_t | x_{\mathcal{R}}, y_t) \right] p^*(y_{\mathcal{R}}, y_T | x_1, x_{\mathcal{R}}). \quad (18)$$

for all $\mathcal{S} \subseteq \mathcal{R}$, all partitions $\{\mathcal{B}_m\}_{m=1}^M$ of \mathcal{S} , and all $r(m) \in \{2, \dots, T\} \setminus \mathcal{B}_m$. We define \mathcal{S}^c as the complement of \mathcal{S} in \mathcal{R} and \mathcal{B}_m^c as the complement of \mathcal{B}_m in \mathcal{R} . U is the part which is to be decoded by all relays. Setting $U_t = X_t$ means each relay decodes all other relays' codewords. We note that in the compress-forward strategy, all channel inputs X_1, \dots, X_{T-1} are independent.

5.2 In the Gaussian Multiple Relay Channel

We consider the Gaussian multiple relay channel. The inputs to the channel from node i , X_i , is constrained by $E[X_i] \leq P_i$. We let the input distribution to the channel be independent Gaussian. Using the compress-forward strategy with $U_j = X_j$, the received signal of node $r(m)$ can be written as

$$Y_{r(m)} = \sqrt{\lambda_{1r(m)}}X_1 + \sum_{\substack{i \in \mathcal{R} \\ i \neq r(m)}} \sqrt{\lambda_{ir(m)}}X_i + Z_{r(m)} \quad (19a)$$

$$\begin{aligned} &= \sqrt{\lambda_{1r(m)}}X_1 + \sum_{\substack{i \in \mathcal{B}_m \\ i \neq r(m)}} \sqrt{\lambda_{ir(m)}}X_i + \sum_{\substack{i \in \mathcal{B}_m^c \\ i \neq r(m)}} \sqrt{\lambda_{ir(m)}}X_i \\ &\quad + Z_{r(m)}. \end{aligned} \quad (19b)$$

The term inside the summation on the RHS of (17) can be evaluated as

$$I(X_{\mathcal{B}_m}; Y_{r(m)} | X_{\mathcal{B}_m^c}) = \frac{1}{2} \log \left[1 + \frac{\sum_{\substack{i \in \mathcal{B}_m \\ i \neq r(m)}} \lambda_{ir(m)} P_i}{\lambda_{1r(m)} P_1 + N_{r(m)}} \right]. \quad (20)$$

We note that all X_i are independent, as seen from (18).

Using the compress-forward strategy, the node j 's quantized received signal is

$$\tilde{Y}_j = Y_j + W_j = \sum_{\substack{i=1, \dots, T-1 \\ i \neq j}} \sqrt{\lambda_{ij}} X_i + Z_j + W_j, \quad (21)$$

where $W_j \sim \mathcal{N}(0, Q_j)$ are independent quantization noise.

The LHS of (17) is

$$I(\tilde{Y}_S; Y_S | X_{\mathcal{R}}, \tilde{Y}_{S^c}, Y_T) = H(\tilde{Y}_S | X_{\mathcal{R}}, \tilde{Y}_{S^c}, Y_T) - H(\tilde{Y}_S | Y_S, X_{\mathcal{R}}, \tilde{Y}_{S^c}, Y_T) \quad (22a)$$

$$= H(\tilde{Y}_S | X_{\mathcal{R}}, \tilde{Y}_{S^c}, Y_T) - H(W_S) \quad (22b)$$

$$\leq H(\tilde{Y}_S | X_{\mathcal{R}}) - H(W_S). \quad (22c)$$

The first term in (22c) is

$$H(\tilde{Y}_S | X_{\mathcal{R}}) = \frac{1}{2} \log \left[2\pi e^D \Lambda(D) \right], \quad (23)$$

where $\Lambda(D)$ is defined as follows,

$$\Lambda(D) = \begin{vmatrix} \lambda_{1s(1)}P_1 + N_{s(1)} + Q_{s(1)} & \cdots & \sqrt{\lambda_{1s(1)}\lambda_{1s(D)}}P_1 \\ \vdots & \ddots & \vdots \\ \sqrt{\lambda_{1s(1)}\lambda_{1s(D)}}P_1 & \cdots & \lambda_{1s(D)}P_1 + N_{s(D)} + Q_{s(D)} \end{vmatrix} \quad (24)$$

$s(i)$ are ordered elements in \mathcal{S} and $D = |\mathcal{S}|$.

The second term in (22c) is

$$H(W_{\mathcal{S}}) = \frac{1}{2} \log \left[2\pi e^D Q_{s(1)} \cdots Q_{s(D)} \right]. \quad (25)$$

Now a sufficient condition for (17) is

$$H(\tilde{Y}_{\mathcal{S}}|X_{\mathcal{R}}) - H(W_{\mathcal{S}}) \leq \sum_{m=1}^M I(X_{\mathcal{B}_m}; Y_{r(m)}|X_{\mathcal{B}_m^c}), \quad (26)$$

or in the Gaussian channel,

$$Q_{s(1)} \cdots Q_{s(D)} \geq \frac{\Lambda(D)}{\prod_{m=1}^M \left[1 + \frac{\sum_{i \in \mathcal{B}_m} \lambda_{ir(m)} P_i}{\lambda_{1r(m)} P_1 + N_{r(m)}} \right]}. \quad (27)$$

The achievable rate is given by

$$I(X_1; \tilde{Y}_{\mathcal{R}}, Y_T | X_{\mathcal{R}}) = H(\tilde{Y}_{\mathcal{R}}, Y_T | X_{\mathcal{R}}) - H(\tilde{Y}_{\mathcal{R}}, Y_T | X_1, X_{\mathcal{R}}) \quad (28a)$$

$$= \frac{1}{2} \log \left[2\pi e^{T-1} \Psi(T-1) \right] \\ - \frac{1}{2} \log \left[2\pi e^{T-1} (N_2 + Q_2) \cdots (N_{T-1} + Q_{T-1}) N_T \right] \quad (28b)$$

$$= \frac{1}{2} \log \left[1 + \frac{\lambda_{12} P_1}{N_2 + Q_2} + \cdots + \frac{\lambda_{1T-1} P_1}{N_{T-1} + Q_{T-1}} + \frac{\lambda_{1T} P_1}{N_T} \right], \quad (28c)$$

where

$$\Psi(T-1) = \begin{vmatrix} \lambda_{12} P_1 + N_2 + Q_2 & \cdots & \sqrt{\lambda_{12} \lambda_{1T}} P_1 \\ \vdots & \ddots & \vdots \\ \sqrt{\lambda_{12} \lambda_{1T}} P_1 & \cdots & \lambda_{1T} P_1 + N_T \end{vmatrix}. \quad (29)$$

Hence we have the following theorem for the T -node Gaussian multiple relay channel.

Theorem 2 Consider a memoryless T -node Gaussian multiple relay channel. Using independent Gaussian input $X_i, i = 1, \dots, T-1$, with power constraints $E[X_i^2] \leq P_i$, the compress-forward strategy achieves rates up to

$$R = \frac{1}{2} \log \left[1 + \frac{\lambda_{12}P_1}{N_2 + Q_2} + \dots + \frac{\lambda_{1T-1}P_1}{N_{T-1} + Q_{T-1}} + \frac{\lambda_{1T}P_1}{N_T} \right], \quad (30)$$

where

$$\tilde{Y}_j = Y_j + W_j \quad (31)$$

and $W_j \sim \mathcal{N}(0, Q_j)$ are independent quantization noise. The rate equation is subject to the constraints

$$Q_{s(1)} \cdots Q_{s(D)} \geq \frac{\Lambda(D)}{\prod_{m=1}^M \left[1 + \frac{\sum_{\substack{i \in \mathcal{B}_m \\ i \neq r(m)}} \lambda_{ir(m)} P_i}{\lambda_{1r(m)} P_1 + N_{r(m)}} \right]}, \quad (32)$$

for all $\mathcal{S} \subseteq \mathcal{R}$, $\{s(1) \dots s(D)\} = \mathcal{S}$, all partitions $\{\mathcal{B}_m\}_{m=1}^M$ of \mathcal{S} , and all $r(m) \in \{2, \dots, T\} \setminus \mathcal{B}_m$. \mathcal{R} is the set of all relays.

We note that the achievability of (30) makes use of the Markov lemma [19, Lemma 4.1], which requires strong typicality. Though strong typicality does not extend to continuous random variables, one can generalize the Markov lemma for Gaussian inputs and thus show that (30) is achievable [9].

6 The Capacity of the Gaussian SSMRSD Mesh Network

Recalling that the Gaussian SSMRSD mesh network is a multiple relay channel with powerful relays, we investigate how (32) can be satisfied for some small Q_i in the mesh network. To see the effect of these powerful nodes, we will look at the scenario when the relays' power constraints grow without bound, i.e., $P_i \rightarrow \infty, \forall i \in \mathcal{R}$. While this may not be practical, it allows to characterize the capacity and to study how the rates scale with the relays' powers. We assume that all the channel gains λ_{ij} , the transmit power P_1 , and the receiver noise N_i are finite. Under these conditions:

- (1) $\Lambda(D)$ is finite when P_1, N_i, λ_{1i} are finite and Q_i are approaching zero, $\forall i \in \mathcal{R}$.
- (2) $(\lambda_{1j}P_1 + N_j), \forall j \in \mathcal{R} \cup \{T\}$ are finite.
- (3) The RHS of (32) approaches zero as $\lambda_{ij}P_i \rightarrow \infty, \forall i \in \mathcal{R}, \forall j \in \mathcal{R} \cup \{T\}$.

Hence, we can set

$$Q_i \rightarrow 0, \forall i \in \mathcal{R}, \quad (33)$$

while (32) can still be satisfied for all $\mathcal{S} \subseteq \mathcal{R}$, all partitions $\{\mathcal{B}_m\}_{m=1}^M$ of \mathcal{S} , and all $r(m) \in \{2, \dots, T\} \setminus \mathcal{B}_m$. When $Q_i \rightarrow 0$, the quantized received signals approach the received signals, that is

$$\tilde{Y}_i = Y_i + W_i \rightarrow Y_i, \quad (34)$$

for all $\forall i \in \mathcal{R}$. The achievable rate in (30) becomes

$$R \rightarrow \max_{\substack{\text{independent Gaussian inputs} \\ E[X_1^2] \leq P_1}} I(X_1; Y_{\mathcal{R}}, Y_T | X_{\mathcal{R}}). \quad (35)$$

We see that (35) has the same form as the capacity upper bound (15) of the SSMRSD mesh network. The upper bound (15) is maximized over all possible input distributions but the achievable rate (35) is achievable with independent Gaussian inputs. However, Theorem 1 states that the cut-set upper bound is maximized by using independent Gaussian inputs. Hence, the compress-forward strategy approaches a cut-set upper bound of the SSMRSD mesh network asymptotically. This is summarized in the following theorem.

Theorem 3 *The achievable rate of the compress-forward strategy approaches the capacity of the Gaussian SSMRSD mesh network asymptotically as the relays' powers grow. The capacity is given by*

$$C_{SSMRSDMesh} = \max_{\substack{\text{independent Gaussian inputs} \\ E[X_1^2] \leq P_1}} I(X_1; Y_{\mathcal{R}}, Y_T | X_{\mathcal{R}}). \quad (36)$$

Similar to Theorem 2, the achievability of (36) requires generalization of the Markov lemma to Gaussian input distributions [9].

7 The Effect of Relay Position Power

7.1 Preliminary

In the previous section, we have shown that when the relays' powers get large, the achievable rate of the compress-forward strategy approaches the capacity of the multiple relay channel. The result was derived for the extreme condition when the relays' powers grow to infinity. While this assumption is not valid in any practical scenario, we investigate the case of finite relays' powers in this section. We study, using the single relay channel as an example, how close the achievable rate of the compress-forward strategy gets near to the capacity when we increase the relay's powers. In addition, we investigate how the relay's position affects the result.

7.2 Comparing Different Coding Strategies and The Cut-Set Bound

We recall that the cut-set bound for the single relay channel is

$$R_{CS} = \max_{p(x_1, x_2)} \min\{I(X_1; Y_2, Y_3|X_2), I(X_1, X_2; Y_3)\} \quad (37a)$$

$$= \max_{0 \leq \alpha_{CS} \leq 1} \min \left\{ L \left(\left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2} \right) (1 - \alpha_{CS}) \right), \right. \\ \left. L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{CS} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3} \right) \right\}, \quad (37b)$$

where $L(x) = \frac{1}{2} \log(1 + x)$. In the numerical analyses in this section, we use this cut-set bound as a basis of comparison. We compare achievable rates of different coding strategies normalized to this bound.

The maximum achievable rate of the decode-forward strategy is [4]

$$R_{DF} = \max_{p(x_1, x_2)} \min\{I(X_1; Y_2|X_2), I(X_1, X_2; Y_3)\} \quad (38a)$$

$$= \max_{0 \leq \alpha_{DF} \leq 1} \min \left\{ L \left(\frac{P_1 \lambda_{12}}{N_2} (1 - \alpha_{DF}) \right), \right. \\ \left. L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{DF} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3} \right) \right\}. \quad (38b)$$

In the decode-forward strategy, the source transmits to both the relay and the destination. The relay fully decodes the data sent by the source, and helps the source to transmit to the destination.

The maximum achievable rate of the compress-forward strategy is

$$R_{CF} = I(X_1; \tilde{Y}_2, Y_3|X_2), \quad \text{where } I(X_2; Y_3) \geq I(Y_2; \tilde{Y}_2|X_2, Y_3) \quad (39a)$$

$$= L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_2 \lambda_{12}}{N_2 + Q} \right), \quad (39b)$$

$$\text{where } Q = \frac{(\lambda_{13} N_2 + \lambda_{12} N_3) P_1 + N_2 N_3}{P_2 \lambda_{23}}. \quad (39c)$$

The maximum achievable rate of the TDMA multihop strategy is

$$R_{MH} = \max_{0 \leq \alpha_{MH} \leq 1} \min \left\{ (1 - \alpha_{MH}) L \left(\frac{P_1 \lambda_{12}}{(1 - \alpha_{MH}) N_2} \right), \alpha_{MH} L \left(\frac{P_2 \lambda_{23}}{\alpha_{MH} N_3} \right) \right\}. \quad (40)$$

In this strategy, during $(1 - \alpha_{MH})$ fraction of the time, the source transmits to the relay; during α_{MH} fraction of the time, the relay transmits to the destination. This is the strategy proposed in [14][15].

7.3 Channel Setup

In order to compare different strategies at different relay's positions and relay's powers, we need to select a model for the channel gain λ_{ij} . Using Friis free space path loss model, the channel gain is given by

$$\lambda_{ij} = \frac{P_j}{P_i} = \frac{G}{(4\pi f d_{ij})^2}, \quad (41)$$

where P_j is the received power, P_i is the transmit power, G is the antenna gain, f is the carrier frequency, d_{ij} is the distance between the transmitter and the receiver. In non-free space, different models are used to model the signal propagation attenuation in different environments. However, in most models, the channel gain is inversely proportional to $d_{ij}^{-\eta}$, where η ranges from 2 to 8. Capturing the main characteristic of how the channel gain varies with distance, one can simplify these path loss models to the following simplified path loss (SPL) model.

$$\lambda_{ij} = \kappa d_{ij}^{-\eta}, \quad (42)$$

where η is the path loss exponent, and $\eta \geq 2$ with equality for free space transmission. κ is a positive constant as far as the analyses in this section are concerned. The SPL is a widely accepted model and commonly used in the information theoretic literature [9][13][20][21].

However, the Friis model or the SPL are only valid for distances in the far field. When one attempts to use this model for small distance, one ends up with a large channel gain, i.e, $d_{ij} \rightarrow 0$, $\lambda_{ij} \rightarrow \infty$. Near field signal propagation models [22] suffer the same problem when we consider asymptotic cases when the distance approaches zero. We will attempt to tweak the SPL to rectify this problem. We note the following characteristics of signal propagation.

- (1) The channel gain is bounded $\forall d_{ij} \geq 0$.
- (2) The channel gain decreases monotonically as d_{ij} increases.
- (3) The channel gain $\lambda_{ij} \rightarrow \kappa d_{ij}^{-\eta}$ as $d_{ij} \rightarrow \infty$.

We modified the SPL model as follows, which we term the modified path loss (MPL) model.

$$\lambda_{ij} = \kappa(1 + d_{ij})^{-\eta}. \quad (43)$$

Though the MPL model might not be accurate for general environments, it captures the above mentioned properties of signal propagation. It fixes the undesirable behavior of the channel gain when $d_{ij} \rightarrow 0$, and allows us to compare different coding strategies with different relay's positions, especially in this extreme condition.

Remark 1 For the single relay channel [9] and the multiple relay channel [23], it has been noted that when the relay(s) moves toward the destination, the

compress-forward strategy approaches the capacity. In all these scenarios the relays' powers are finite. At a first look, that the relay is close to the destination seems equivalent to the case when the relay's power gets large. In this section, we will also show that the two are not equivalent. We note that the compress-forward strategy achieves the capacity when the relay approaches the destination because the channel gain from the relay to the destination approaches infinity using the SPL model for signal propagation. As discussed earlier, the SPL model is not valid when distance approaches zero. We show that the compress-forward strategy does not approach the capacity as the relay-destination distance goes to zero using another model, e.g., the MPL model, in which the channel gain is bounded. However, Theorem 3 holds regardless of the choice of path loss model.

7.4 Scenarios for Comparison

Now, we investigate how the two factors, i.e., the relay's power and the relay's position, affect the rates of the coding strategies. We consider the single relay channel for illustration. We assume that the source, the relay, and the destination are located in a straight line with the relay placed between the source and the destination. We fix the source-destination distance $d_{13} = 1$. Now, let us consider eight cases, using different path loss models, for different relay's powers P_2 , the relay-source distances d_{12} , the relay-destination distances d_{23} .

- (1) SPL: $P_2 \rightarrow \infty, d_{12} \rightarrow 0, \lambda_{12} = \kappa d_{12}^{-\eta} \rightarrow \infty,$
 - (a) $\frac{P_2}{\lambda_{12}} \rightarrow \infty,$
 - (b) $\frac{P_2}{\lambda_{12}} = K_3,$
 - (c) $\frac{P_2}{\lambda_{12}} \rightarrow 0,$
- (2) SPL: $P_2 \rightarrow \infty, d_{12} = K_2,$
- (3) SPL: $P_2 \rightarrow \infty, d_{23} \rightarrow 0,$
- (4) SPL: $P_2 = K_1, d_{12} \rightarrow 0,$
- (5) SPL: $P_2 = K_1, d_{12} = K_2,$
- (6) SPL: $P_2 = K_1, d_{23} \rightarrow 0,$
- (7) MPL: $P_2 \rightarrow \infty$
- (8) MPL: $P_2 = K_1$

for some $K_1 \approx P_1, K_2 \approx 0.5, K_3 \approx 1$.

7.5 On the SPL Model

Table 1 summarizes the results for cases (1a)–(6), i.e., those using the SPL model. The proof for these cases can be found in Appendix A. The cases when

Table 1

The effect of the relay's distance and power on achievable rates and the capacity \mathcal{C} , in the SPL model.

	$d_{12} \rightarrow 0$			$d_{12} \approx 0.5$	$d_{23} \rightarrow 0$
	$\frac{P_2}{d_{12}^{-\eta}} \rightarrow \infty$	$\frac{P_2}{d_{12}^{-\eta}} = K_3$	$\frac{P_2}{d_{12}^{-\eta}} \rightarrow 0$		
$P_2 \rightarrow \infty$	case (1a) $R_{DF} \rightarrow \mathcal{C}(d_{12} \downarrow)$ $R_{DF} \approx \mathcal{C}(P_2 \uparrow)$ $R_{CF} \rightarrow \mathcal{C}$	case (1b) $R_{DF} \rightarrow \mathcal{C}(d_{12} \downarrow)$ $R_{DF} \rightarrow \mathcal{C}(P_2 \uparrow)$ $R_{CF} ?$	case (1c) $R_{DF} \rightarrow \mathcal{C}(d_{12} \downarrow)$ $R_{DF} \rightarrow \mathcal{C}(P_2 \uparrow)$ $R_{CF} \rightarrow \mathcal{C}(d_{12} \downarrow)$ $R_{CF} \rightarrow \mathcal{C}(P_2 \uparrow)$	case (2) $R_{DF} < \mathcal{C}$ $R_{CF} \rightarrow \mathcal{C}$	case (3) $R_{DF} < \mathcal{C}$ $R_{CF} \rightarrow \mathcal{C}$
$P_2 < \infty$	case (4) $R_{DF} \rightarrow \mathcal{C}$ $R_{CF} < \mathcal{C}$			case (5) $R_{DF} < R_{CS}$ $R_{CF} < R_{CS}$	case (6) $R_{DF} < \mathcal{C}$ $R_{CF} \rightarrow \mathcal{C}$

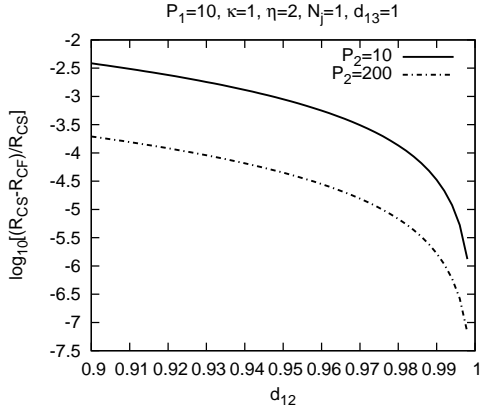


Fig. 4. The performance of the compress-forward strategy in the SPL model when the relay is close to the destination.

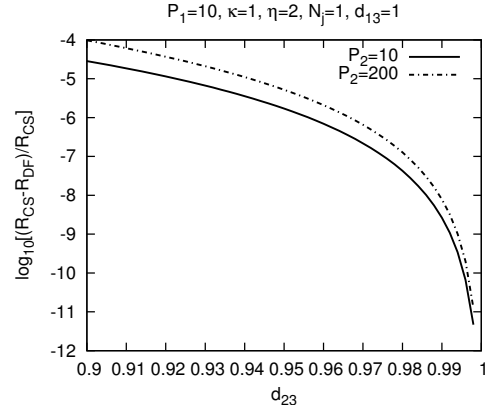


Fig. 5. The performance of the decode-forward strategy in the SPL model when the relay is close to the source.

the relay's power is not large has been considered [9] but we included them here for completeness sake. The decode-forward strategy achieves the capacity when the relay-source distance goes to zero and the compress-forward strategy approaches the capacity when the relay-destination goes to zero.

However, when the relay's power is large, the situation gets complicated. The compress-forward strategy still achieves the capacity asymptotically when the relay is near the destination. It also approaches the capacity for relay not near both the source and the destination. When the relay moves nearer to the source, with high relay's power, different strategies approach the capacity depending on the $\frac{P_2}{\lambda_{12}}$ ratio. Furthermore, different strategies approach the capacity, and possibly at a different convergence rate, when we vary P_2 or d_{12} .

Figs. 4 and 5 show the performance of the compress-forward strategy when

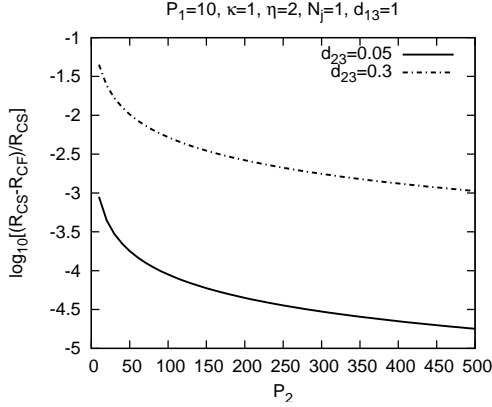


Fig. 6. The performance of the compress-forward strategy in the SPL model when the relay's power is high and the relay is close to the destination.

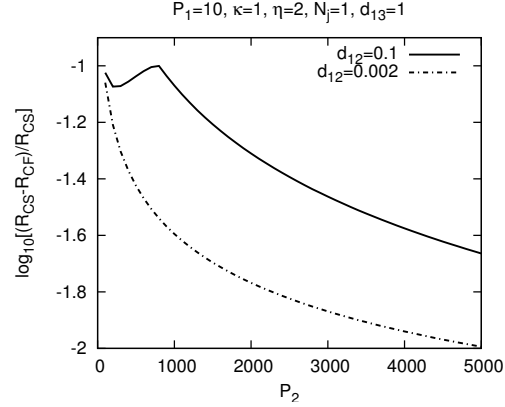


Fig. 7. The performance of the compress-forward strategy in the SPL model when the relay's power is high and the relay is close to the source.

the relay is close to the destination and the performance of the decode-forward strategy when the relay is close to the source, respectively. We see that as the relay-destination distance d_{23} diminishes, or equivalently, $d_{12} \rightarrow 1$, the rate achievable by the compress-forward strategy approaches the capacity. This corresponds to case (6). When the source-the-relay's distance d_{12} diminishes, or equivalently, $d_{23} \rightarrow 1$, the rate achievable by the decode-forward strategy approaches the capacity. This corresponds to case (4). These happen because $\lambda_{12} \rightarrow \infty$ as $d_{12} \rightarrow 0$, and $\lambda_{23} \rightarrow \infty$ as $d_{23} \rightarrow 0$ in the SPL model. Later, we will see that these strategies do not approach the capacity with low P_2 in the MPL model.

Figs. 6 and 7 show that the compress-forward strategy approaches the capacity regardless of the relay's position as long as P_2 grows large. These correspond to cases (1a), (2), and (3).

7.6 On the MPL Model

Table 2 summarizes the results for the last two cases, i.e., those using the MPL model. In the MPL model, λ_{12} and λ_{23} do not approach infinity even if the source-relay distance or the relay-destination distance approaches zero. Here, we see that when the relay's power is limited, none of the strategies here achieves the capacity regardless of the relay's position. However, when the relay's power grows, the compress-forward strategy achieves the capacity. The proof of the results can be found in Appendix B. Simulation results can be found in Figs. 8–11. In Figs. 8 and 9, we show that the compress-forward strategy does not achieve the capacity in the MPL model even when the relay-destination distance goes to zero, and the decode-forward strategies do not

Table 2

The effect of the relay's distance and power on achievable rates and the capacity \mathcal{C} in the MPL model.

	$d_{12} \rightarrow 0$	$d_{12} \approx 0.5$	$d_{23} \rightarrow 0$
$P_2 \rightarrow \infty$	case (7) $R_{DF} < \mathcal{C}$ $R_{CF} \rightarrow \mathcal{C}(P_2 \uparrow)$		
$P_2 < \infty$	case (8) $R_{DF} < R_{CS}$ $R_{CF} < R_{CS}$		

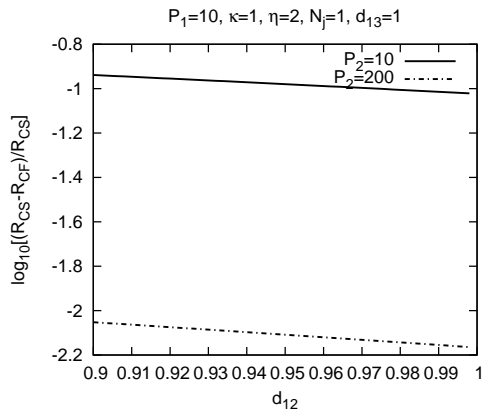


Fig. 8. The performance of the compress-forward strategy in the MPL model when the relay is close to the destination.

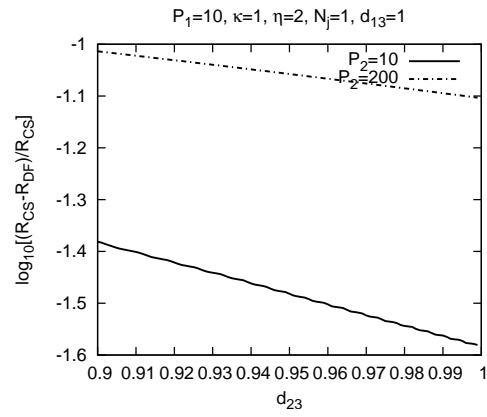


Fig. 9. The performance of the decode-forward strategy in the MPL model when the relay is close to the source.

achieve the capacity in the MPL model when the source-the relay's distance goes to zero.

However, we see in Fig. 10 that the rate achievable by the compress-forward strategy approaches the capacity as the relay's power grows. In addition, the graphs show that it is not necessary for the relay to transmit at infinitely large power for the rate of the compress-forward strategy to get near the capacity. At $d_{23} = 0.05$, the compress-forward rate gets to 97% of the capacity when the relay transmits at power five times the source's power; at $d_{23} = 0.3$, the compress-forward rate gets to 97% of the capacity when the relay transmits at power nine times the source's power.

Fig. 11 shows the performance of the TDMA multihop strategy. Comparing the curves in Fig. 11 and in Fig. 10, we find that the performance of the TDMA multihop strategy is far below that of the compress-forward strategy.

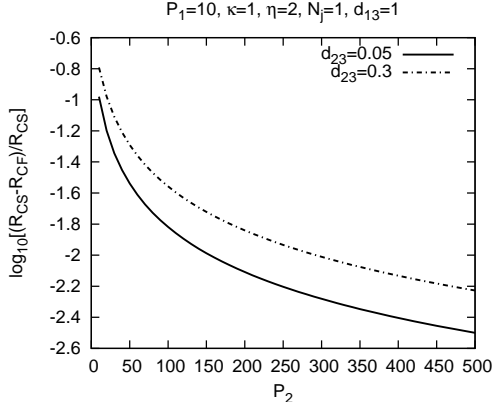


Fig. 10. The performance of the compress-forward strategy in the MPL model when the relay's power is high.

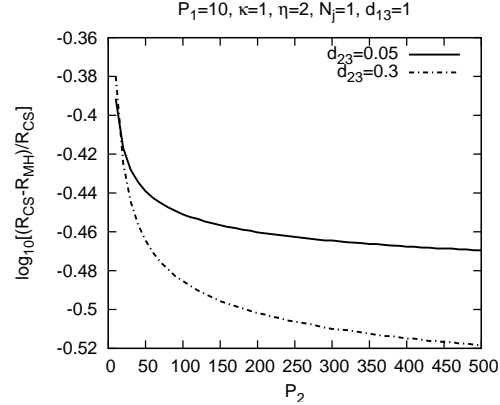


Fig. 11. The performance of the TDMA multihop strategy in the MPL model when the relay's power is high.

8 Conclusions

In this paper, we derived the capacity of the SSMRSD mesh network. We showed that the compress-forward strategy achieves the capacity asymptotically as the relays' powers increase. We performed numerical analyses on the single relay channel using two signal propagation models, i.e, the SPL model and the MPL model. We considered three special cases, i.e., when the relay is close to the source, when the relay is close to the destination, and when the relay's power is high. In the SPL model, we showed that when the relay's power is high or the relay is close to the destination, the compress-forward strategy approaches the capacity but not the decode-forward strategy; when the relay is placed close to the source, the decode-forward strategy approaches the capacity. However, in the MPL model, the compress-forward strategy only approaches the capacity when the relay's power gets large. In other words, none of the strategies achieves the capacity with low relay's power regardless of the relay's position. We show that the performance of the compress-forward strategy gets close to the capacity when the relay's power is a few times higher than the source's power, meaning that we do not need infinite relay's power to get close the capacity.

We can view a multiple source multiple destination mesh network as a collection of many SSMRSD mesh networks. What our results say for the SSMRSD mesh network design is that cooperative relaying (via the compress-forward strategy) by mesh routers can facilitate communication between mesh clients. Recent work [24] indicates how one can design practical coding schemes to implement cooperative (information theoretic sense) relaying. Our results can be used to determine what power the mesh routers should transmit at, and how the nodes should cooperate. The analysis of the SSMRSD network is particularly useful for a multiple source multiple destination network in which

the channels of all source-destination pairs (with the help of relays) in the network are orthogonal, e.g., using TDMA or FDMA. Under such conditions, each source-destination pair, together with their assigned relays, operates as it would in a SSMRSD network. This is different from TDMA multihop as cooperation among the source, the relays, and the destination is still possible. Combining this with network layer algorithms will lead to an efficient protocol stack for mesh networks, on which a variety of user applications can be built.

References

- [1] E. C. Van-Der-Meulen, Three-terminal communication channels, *Adv. Appl. Prob.* 3 (1971) 120–154.
- [2] H. Liao, Multiple access channel, Ph.D. thesis, Univ. Hawaii Honolulu, HI (1972).
- [3] R. Ahlswede, The capacity of a channel with two senders and two receivers, *Ann. Probab.* 2 (1974) 805–814.
- [4] T. M. Cover, A. A. El-Gamal, Capacity theorems for the relay channel, *IEEE Trans. Inform. Theory* IT-25 (5) (1979) 572–584.
- [5] P. P. Bergmans, Random coding theorem for broadcast channels with degraded components, *IEEE Trans. Inform. Theory* IT-19 (2) (1973) 197–207.
- [6] I. F. Akyildiz, X. Wang, W. Wang, Wireless mesh networks: A survey, *Computer Networks Journal* (Elsevier), Mar. 2005.
- [7] J. Hauser, D. Baker, W. S. Conner, Draft par for ieee 802.11 ess mesh, 2004. URL <http://www.ieee802.org/11/PARs/11-04-0054-02-0mes-par-ieee-802-11-ess-mesh.doc>
- [8] L. Xie, P. R. Kumar, An achievable rate for the multiple level relay channel, *IEEE Trans. Inform. Theory* 51 (4) (2005) 1348–1358.
- [9] G. Kramer, M. Gastpar, P. Gupta, Cooperative strategies and capacity theorems for relay networks, *IEEE Trans. Inform. Theory* 51 (9) (2005) 3037–3063.
- [10] L. Ong, M. Motani, Myopic coding in wireless networks, *Proceedings of the 39th Conference on Information Sciences and Systems (CISS 2005)*, John Hopkins University, Baltimore, MD.
- [11] L. Ong, M. Motani, Myopic coding in multiple relay channels, *Proceedings of the 2005 IEEE International Symposium on Information Theory (ISIT 2005)*, Adelaide Convention Centre, Adelaide, Australia.
- [12] H. F. Chong, M. Motani, H. K. Garg, New coding strategies for the relay channel, *2005 IEEE International Symposium on Information Theory (ISIT 2005)*, Adelaide Convention Centre, Adelaide, Australia.

- [13] P. Gupta, P. R. Kumar, The capacity of wireless networks, *IEEE Trans. Inform. Theory* 46 (2) (2000) 388–404.
- [14] J. Jun, M. L. Sichitiu, The nominal capacity of wireless mesh networks, *IEEE Wireless Comms.* 10 (5) (2003) 8–14.
- [15] J. Jun, Capacity of wireless mesh networks, Master’s Thesis (2002).
- [16] M. Kodialam, T. Nandagopal, Characterizing the capacity region in multi-radio multi-channel wireless mesh networks, *MobiCom ’05: Proceedings of the 11th annual international conference on Mobile computing and networking* (2005) 73–87.
- [17] S. Roy, A. Das, R. Vijayakumar, H. M. H. Alazemi and, E. Alotaibi, Capacity scaling with multiple radios and multiple channels in wireless mesh networks, *The First IEEE Workshop on Wireless Mesh Networks (WiMesh)* (2005).
- [18] T. M. Cover, J. A. Thomas, *Elements of Information Theory*, John Wiley and Sons, 1991.
- [19] T. Berger, Multiterminal source coding, in: *Lecture notes presented at the 1977 CISM Summer School, Udine, Italy, 1977*, pp. 171–231.
- [20] S. Toumpis, A. J. Goldsmith, Capacity regions for wireless ad hoc networks, *IEEE Trans. Wireless Comms.* 2 (4) (2003) 736–748.
- [21] M. Gatspar, M. Vetterli, On the capacity of large gaussian relay networks, *IEEE Trans. Inform. Theory* 51 (3) (2005) 765–779.
- [22] K. Ravindra, A. D. Sarma, Modelling path loss in the near field region for cm and mm wave mobile communication, *Proceedings of the International Conference on Electromagnetic Interference and Compatibility ’99* (1999) 79–82.
- [23] M. Gastpar, G. Kramer, P. Gupta, The multiple-relay channel: coding and antenna clustering capacity, *2002 IEEE International Symposium on Information Theory (ISIT 2002)*, Lausanne, Switzerland.
- [24] P. Razaghi, W. Yu, Bilayer ldpc codes for the relay channel, *IEEE International Conference on Communications (ICC)*, Istanbul, Turkey (2006).

A Proof of Table 1

Case (1a): $P_2 \rightarrow \infty, d_{12} \rightarrow 0, \frac{P_2}{d_{12}^\eta} \rightarrow \infty$ or equivalently $\lambda_{12} \rightarrow \infty, \frac{P_2}{\lambda_{12}} \rightarrow \infty$.

Setting $\alpha_{CS} = 0$ maximizes the cut-set rate.

$$R_{CS} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2} \right) \tag{A.1a}$$

$$\rightarrow L \left(\frac{P_1 \lambda_{12}}{N_2} \right) \text{ as } d_{12} \rightarrow 0. \tag{A.1b}$$

Using the decode-forward strategy, we set $\alpha_{DF} = 0$. The rate achievable is

$$R_{DF} = L \left(\frac{P_1 \lambda_{12}}{N_2} \right) \quad (\text{A.2a})$$

$$R_{DF} \rightarrow R_{CS} \quad \text{as } d_{12} \text{ decreases} \quad (\text{A.2b})$$

$$R_{DF} \approx R_{CS} \quad \text{as } P_2 \text{ increases.} \quad (\text{A.2c})$$

So, for case (1a), the decode-forward strategy approaches the capacity as the distance d_{12} goes smaller. It does not approaches the capacity, but stay close to the capacity, when the relay's power increases.

Using the compress-forward strategy, noting that $\frac{\lambda_{12}}{P_2} \rightarrow 0$,

$$R_{CF} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 P_2 \lambda_{12} \lambda_{23}}{P_1 (\lambda_{13} N_2 + \lambda_{12} N_3) + P_2 \lambda_{23} N_2 + N_2 N_3} \right) \quad (\text{A.3a})$$

$$= L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2 + \frac{1}{P_2} \frac{P_1 \lambda_{13} N_2}{\lambda_{23}} + \frac{\lambda_{12}}{P_2} \frac{P_1 N_3}{\lambda_{23}} + \frac{1}{P_2} \frac{N_2 N_3}{\lambda_{23}}} \right) \quad (\text{A.3b})$$

$$\rightarrow R_{CS} \quad \text{as either } P_2 \text{ increases or } d_{12} \text{ decreases.} \quad (\text{A.3c})$$

We note that the while only decreasing d_{12} drives rate of the decode-forward rate to the capacity, increasing P_2 or decreasing d_{12} can both drive the compress-forward rate to the capacity.

Case (1b): $P_2 \rightarrow \infty, d_{12} \rightarrow 0, \frac{P_2}{d_{12}^\eta} \approx 1$ or equivalently $\lambda_{12} \rightarrow \infty, \frac{P_2}{\lambda_{12}} \approx 1$.

Under this condition, the cut-set bound is given R_{CS} that satisfies the conditions below, for some $0 < \alpha_{CS} < 1$.

$$R_{CS} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{CS} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3} \right) \quad (\text{A.4a})$$

$$= L \left(\frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{CS} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3} \right)^+, \quad (\text{A.4b})$$

$$R_{CS} = L \left(\left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2} \right) (1 - \alpha_{CS}) \right) \quad (\text{A.4c})$$

$$= L \left(\frac{P_1 \lambda_{12}}{N_2} (1 - \alpha_{CS}) \right)^+, \quad (\text{A.4d})$$

The decode-forward strategy achieves the rate up to R_{DF} that satisfies the

following conditions, for some $0 < \alpha_{DF} < \alpha_{CS}$. We note that $\alpha_{DF} \approx \alpha_{CS}$.

$$R_{DF} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{DF} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3} \right) \quad (\text{A.5a})$$

$$= L \left(\frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{DF} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3} \right)^+ \quad (\text{A.5b})$$

$$\Rightarrow R_{CS} \quad \text{as } P_2 \text{ increases because } \alpha_{DF} P_2 \ll \alpha_{CS} P_2, \quad (\text{A.5c})$$

$$R_{DF} = L \left(\frac{P_1 \lambda_{12}}{N_2} (1 - \alpha_{DF}) \right) \quad (\text{A.5d})$$

$$\rightarrow R_{CS} \quad \text{as } d_{12} \text{ decreases because } (1 - \alpha_{DF}) \approx (1 - \alpha_{CS}), \quad (\text{A.5e})$$

Using the compress-forward strategy, the following rate is achievable.

$$R_{CF} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 P_2 \lambda_{12} \lambda_{23}}{P_1 (\lambda_{13} N_2 + \lambda_{12} N_3) + P_2 \lambda_{23} N_2 + N_2 N_3} \right) \quad (\text{A.6a})$$

$$= L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2 + \frac{1}{P_2} \frac{P_1 \lambda_{13} N_2}{\lambda_{23}} + \frac{\lambda_{12}}{P_2} \frac{P_1 N_3}{\lambda_{23}} + \frac{1}{P_2} \frac{N_2 N_3}{\lambda_{23}}} \right) \quad (\text{A.6b})$$

$$\approx L \left(\frac{P_1 \lambda_{12}}{N_2 + \frac{\lambda_{12}}{P_2} \frac{P_1 N_3}{\lambda_{23}}} \right). \quad (\text{A.6c})$$

It is not clear if R_{CF} approaches R_{CS} .

Case (1c): $P_2 \rightarrow \infty, d_{12} \rightarrow 0, \frac{P_2}{d_{12}^\eta} \rightarrow 0$ or equivalently $\lambda_{12} \rightarrow \infty, \frac{P_2}{\lambda_{12}} \rightarrow 0$.

The optimal power split is $\alpha_{CS} \rightarrow 1$ and $\alpha_{DF} \rightarrow 1$ but $\alpha_{DF} < \alpha_{CS}$. Using the same reasoning as that in case (1b),

$$R_{DF} \Rightarrow R_{CS} \quad , \text{ as } P_2 \text{ increases because } \alpha_{DF} P_2 \ll \alpha_{CS} P_2, \quad (\text{A.7a})$$

$$R_{DF} \rightarrow R_{CS} \quad , \text{ as } d_{12} \text{ decreases because } (1 - \alpha_{DF}) \approx (1 - \alpha_{CS}). \quad (\text{A.7b})$$

The cut set bound is

$$R_{CS} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{CS} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3} \right) \quad (\text{A.8a})$$

$$= L \left(\frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{CS} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3} \right)^+. \quad (\text{A.8b})$$

Note that α_{CS} increases as d_{12} decreases.

The compress-forward strategy achieves the following rate.

$$R_{CF} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 P_2 \lambda_{12} \lambda_{23}}{P_1 (\lambda_{13} N_2 + \lambda_{12} N_3) + P_2 \lambda_{23} N_2 + N_2 N_3} \right) \quad (\text{A.9a})$$

$$= L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 P_2 \lambda_{23}}{P_1 N_3 + \frac{P_2}{\lambda_{12}} \lambda_{23} N_2 + \frac{1}{\lambda_{12}} (P_1 \lambda_{13} N_2 + N_2 N_3)} \right), \quad (\text{A.9b})$$

$$= L \left(\frac{P_2 \lambda_{23}}{N_3} \right)^+ \quad (\text{A.9c})$$

$$R_{CF} \nearrow R_{CS} \quad \text{as } d_{12} \text{ decreases because } \alpha_{CS} \text{ increases,} \quad (\text{A.9d})$$

$$R_{CF} \rightarrow R_{CS} \quad \text{as } P_2 \text{ increases because } P_2 \text{ increases faster than } \sqrt{P_2}. \quad (\text{A.9e})$$

Cases (2) and (3): Relay power $P_2 \rightarrow \infty$. Source-relay distance $d_{12} = K_2$ and $d_{23} \rightarrow 0$ for cases (2) and (3) respectively.

The cut-set bound is

$$R_{CS} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2} \right). \quad (\text{A.10})$$

The decode-forward strategy is optimized by setting $\alpha_{DF} = 0$. It achieves the following rate.

$$R_{DF} = L \left(\frac{P_1 \lambda_{12}}{N_2} \right) < R_{CS}. \quad (\text{A.11})$$

The compress-forward strategy achieves the rate up to

$$R_{CF} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 P_2 \lambda_{12} \lambda_{23}}{P_1 (\lambda_{13} N_2 + \lambda_{12} N_3) + P_2 \lambda_{23} N_2 + N_2 N_3} \right) \quad (\text{A.12a})$$

$$= L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2 + \frac{1}{P_2 \lambda_{23}} (P_1 \lambda_{13} N_2 + P_1 \lambda_{12} N_3 + N_2 N_3)} \right) \quad (\text{A.12b})$$

$$= L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2} \right)^- \quad \text{as } P_2 \text{ increases or } d_{23} \text{ decreases} \quad (\text{A.12c})$$

$$\rightarrow R_{CS} \quad \text{as } P_2 \text{ increases or } d_{23} \text{ decreases.} \quad (\text{A.12d})$$

Case (4): $P_2 = K_1, d_{12} \rightarrow 0$ or $\lambda_{12} = \kappa d_{12}^{-\eta} \rightarrow \infty$.

The cut-set bound is obtained by setting α_{CS} such that

$$\left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2} \right) (1 - \alpha_{CS}) = \frac{P_1 \lambda_{13}}{N_3} + \frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{CS} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3}. \quad (\text{A.13})$$

The cut set bound is thus

$$R_{CS} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{CS} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3} \right), \quad (\text{A.14})$$

for some $\alpha_{CS} \rightarrow 1$ as $d_{12} \rightarrow 0$.

Using the decode-forward strategy, we choose α_{DF} such that

$$\frac{P_1 \lambda_{12}}{N_2} (1 - \alpha_{DF}) = \frac{P_1 \lambda_{13}}{N_3} + \frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{DF} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3}. \quad (\text{A.15})$$

Note that $\alpha_{DF} \rightarrow 1$ but $\alpha_{DF} < \alpha_{CS}$. As $d_{12} \rightarrow 0$, $\alpha_{DF} \rightarrow \alpha_{CS}$. Hence, we get

$$R_{DF} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{DF} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3} \right) \quad (\text{A.16a})$$

$$\rightarrow R_{CS} \quad \text{as } d_{12} \rightarrow 0. \quad (\text{A.16b})$$

Using the compress-forward strategy,

$$Q = \frac{(\lambda_{13} N_2 + \lambda_{12} N_3) P_1 + N_2 N_3}{P_2 \lambda_{23}} \rightarrow \infty \quad (\text{A.17a})$$

$$R_{CF} = L \left(\frac{P_1 \lambda_{13}}{N_3} \right) < R_{CS}. \quad (\text{A.17b})$$

Case (5): $P_2 = K_1, d_{12} = K_2$ or $\lambda_{12} = K_4$ for some finite K_4 .

The cut-set bound is

$$R_{CS} = L \left(\left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2} \right) (1 - \alpha_{CS}) \right) \quad (\text{A.18a})$$

$$= L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{CS} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3} \right), \quad (\text{A.18b})$$

for some α_{CS} or

$$R_{CS} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2} \right), \quad (\text{A.19})$$

if the optimal $\alpha_{CS} = 0$.

Using the decode-forward strategy,

$$R_{DF} = L \left(\frac{P_1 \lambda_{12}}{N_2} (1 - \alpha_{DF}) \right) \quad (\text{A.20a})$$

$$= L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_2 \lambda_{23}}{N_3} + \frac{2\sqrt{\alpha_{DF} \lambda_{13} \lambda_{23} P_1 P_2}}{N_3} \right) \quad (\text{A.20b})$$

$$< R_{CS}, \quad (\text{A.20c})$$

for some $\alpha_{DF} < \alpha_{CS}$ or

$$R_{DF} = L \left(\frac{P_1 \lambda_{12}}{N_2} \right) < R_{CS}, \quad (\text{A.21})$$

if the optimal $\alpha_{DF} = 0$.

Using the compress-forward strategy,

$$R_{CF} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 P_2 \lambda_{12} \lambda_{23}}{P_1 (\lambda_{13} N_2 + \lambda_{12} N_3) + P_2 \lambda_{23} N_2 + N_2 N_3} \right). \quad (\text{A.22})$$

It is not clear how the compress-forward strategy performs compared to the cut-set bound. However, an example [9] shows that it can be “far” below the cut-set bound.

Case (6): $P_2 = K_1, d_{23} \rightarrow 0$ or $\lambda_{23} = \kappa d_{23}^{-\eta} \rightarrow \infty$.

Under this condition, setting $\alpha_{CS} = 0$ maximizes the cut-set rate.

$$R_{CS} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2} \right). \quad (\text{A.23})$$

Setting $\alpha_{DF} = 0$ maximizes the achievability of the decode-forward strategy.

$$R_{DF} = L \left(\frac{P_1 \lambda_{13}}{N_3} \right) < R_{CS}. \quad (\text{A.24})$$

Using the compress-forward strategy,

$$R_{CF} = L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 P_2 \lambda_{12} \lambda_{23}}{P_1 (\lambda_{13} N_2 + \lambda_{12} N_3) + P_2 \lambda_{23} N_2 + N_2 N_3} \right) \quad (\text{A.25a})$$

$$= L \left(\frac{P_1 \lambda_{13}}{N_3} + \frac{P_1 \lambda_{12}}{N_2 + \frac{1}{\lambda_{23}} \left(\frac{P_1 \lambda_{13} N_2 + P_1 \lambda_{12} N_3 + N_2 N_3}{P_2} \right)} \right) \quad (\text{A.25b})$$

$$\rightarrow R_{CS} \quad \text{as } d_{23} \rightarrow 0. \quad (\text{A.25c})$$

B Proof of Table 2

Case (7): The proof is similar to that for case (2). The compress-forward strategy achieves the capacity but not the decode-forward strategy.

Case (8): The proof is similar to that for case (5), in which P_2, λ_{12} , and λ_{23} are finite. Neither the decode-forward strategy nor the compress-forward strategy achieves the capacity.