The Capacity Region of a Class of Semideterministic Interference Channels

Hon-Fah Chong, *Member, IEEE*, and Mehul Motani, *Member, IEEE*

***Abstract*—The capacity region of a class of discrete memoryless interference channels with common information is established. The setup is similar to the class of deterministic interference channels without common information studied by El Gamal and Costa, which was later extended to the class of deterministic interference channels with common information. In this paper, certain conditions that were originally imposed by El Gamal and Costa are relaxed and it is shown, by a specific example, that this new class of interference channels is strictly larger than the class of deterministic interference channels previously studied. In fact, the result of this paper is obtained by combining the class of deterministic interference channels with the class of discrete memoryless interference channels with strong interference. Hence, it also includes the capacity region of the class of discrete memoryless interference channels with strong interference as a special case.**

***Index Terms*—Capacity region, interference channel, semideterministic.**

I. Introduction

An interference channel (IC) models the situation where unrelated transmitters transmit separate information to different receivers via a common channel. There is no cooperation between any of the transmitters or any of the receivers. Hence, transmission of information from each transmitter to its intended receiver interferes with the communication between the other transmitter–receiver pairs.

In this paper, we shall restrict our attention to the two-user IC. In [1], we derived a simplified description of the Han–Kobayashi rate region, which is the best rate region to date for the general IC. However, the capacity region of the general IC has yet to be found. The capacity region of the IC has only been determined for the following cases: the capacity of the Gaussian IC with strong interference [2]–[4]; the capacity of the discrete memoryless IC with strong interference [5], which was recently also extended to the strong IC with common information [6]; the capacity of a class of discrete degraded ICs [7], which includes the discrete additive degraded IC studied by Benzel [8]; the capacity of a class of deterministic ICs [9], which was recently extended to the IC with common information [10]; the frequency-selective Gaussian IC under strong interference [11].

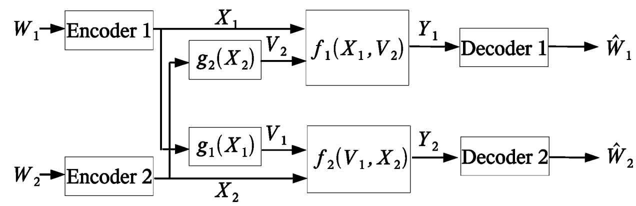

Fig. 1. The class of deterministic ICs studied by El Gamal and Costa.

We first consider a class of deterministic ICs (without common information) as shown in Fig. 1. The outputs Y_1 and Y_2, and the interferences V_1 and V_2 are deterministic functions of the inputs X_1 and X_2:

$$Y_1 = f_1(X_1, V_2) \tag{1}$$
$$Y_2 = f_2(V_1, X_2) \tag{2}$$
$$V_1 = g_1(X_1) \tag{3}$$
$$V_2 = g_2(X_2). \tag{4}$$

In addition, for this class of deterministic ICs, Y_1 and X_1 must uniquely determine V_2, while Y_2 and X_2 must uniquely determine V_1. Hence, there exist functions k_1 and k_2 such that

$$V_1 = k_2(X_2, Y_2) \tag{5}$$
$$V_2 = k_1(X_1, Y_1). \tag{6}$$

El Gamal and Costa determined the capacity region of the IC shown in Fig. 1 and satisfying conditions (5) and (6) in [9, Theorem 1]. This was later extended to the deterministic IC with common information to both transmitters in [10, Theorem 6].

In this paper, we relax the constraint, imposed by El Gamal and Costa, such that the outputs Y_1 and Y_2 are deterministic functions of the inputs X_1 and X_2. Hence, we effectively remove conditions (1) and (2). In addition, we relax conditions (5) and (6) to include the case of strong interference. This essentially combines the result of the deterministic IC together with the result of the IC with strong interference.

II. Channel Model

Consider the class of ICs shown in Fig. 2. The channel itself consists of four finite alphabets $\mathcal{X}_1 = \{1, 2, \ldots, \|\mathcal{X}_1\|\}$, $\mathcal{X}_2 = \{1, 2, \ldots, \|\mathcal{X}_2\|\}$, $\mathcal{Y}_1 = \{1, 2, \ldots, \|\mathcal{Y}_1\|\}$, and $\mathcal{Y}_2 =$

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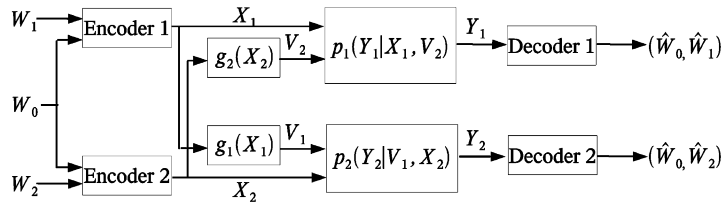

Fig. 2. The class of ICs under investigation.

$\{1, 2, \ldots, \|\mathcal{Y}_2\|\}$, two deterministic functions in agreement with (3) and (4), and two conditional marginal distributions given by

$$\sum_{y_2 \in \mathcal{Y}_2} p(y_1 y_2 | x_1 x_2) = p_1(y_1|x_1 x_2) = p_1(y_1|x_1 v_2) \quad (7)$$

$$\sum_{y_1 \in \mathcal{Y}_1} p(y_1 y_2 | x_1 x_2) = p_2(y_2|x_1 x_2) = p_2(y_2|v_1 x_2). \quad (8)$$

Equalities (7) and (8) follow from the fact that v_1 and v_2 are deterministic functions of x_1 and x_2, respectively, and from the assumptions of the channel model, which are $p_1(y_1|x_1x_2) = p_1(y_1|x_1v_2)$ and $p_2(y_2|x_1x_2) = p_2(y_2|v_1x_2)$. Since there is no cooperation between the receivers, the capacity region of the IC depends only on the conditional marginal distributions. We assume that the channel is memoryless. A $\left(2^{NR_0}, 2^{NR_1}, 2^{NR_2}, N\right)$ code for this channel consists of two encoding functions

$$e_1 : \left\{1, \ldots, 2^{NR_0}\right\} \times \left\{1, \ldots, 2^{NR_1}\right\} \to \mathcal{X}_1^N$$
$$e_2 : \left\{1, \ldots, 2^{NR_0}\right\} \times \left\{1, \ldots, 2^{NR_2}\right\} \to \mathcal{X}_2^N$$

and two decoding functions

$$d_1 : \mathcal{Y}_1^N \to \left\{1, \ldots, 2^{NR_0}\right\} \times \left\{1, \ldots, 2^{NR_1}\right\}$$
$$d_2 : \mathcal{Y}_2^N \to \left\{1, \ldots, 2^{NR_0}\right\} \times \left\{1, \ldots, 2^{NR_2}\right\}.$$

The average probability of error is defined as follows:

$$P_e^{(N)} = \Pr\left(d_1\left(Y_1^N\right) \neq (W_0, W_1) \text{ or } d_2\left(Y_2^N\right) \neq (W_0, W_2)\right)$$

where (W_0, W_1, W_2) is assumed to be uniformly distributed over $\{1, 2, \ldots, 2^{NR_0}\} \times \{1, 2, \ldots, 2^{NR_1}\} \times \{1, 2, \ldots, 2^{NR_2}\}$. A rate triplet (R_0, R_1, R_2) is said to be achievable for the IC if there exists a sequence of $\left(2^{NR_0}, 2^{NR_1}, 2^{NR_2}, N\right)$ codes with $P_e^{(N)} \to 0$.

We will take a look at the capacity region of this class of ICs. In addition, we require that, $\forall N \in \{1, 2, \ldots\}$, the following two conditions:

$$I\left(V_1^N; Y_2^N | X_2^N W_0\right) \geq I\left(V_1^N; Y_1^N | X_2^N W_0\right) \quad (9)$$
$$I\left(V_2^N; Y_1^N | X_1^N W_0\right) \geq I\left(V_2^N; Y_2^N | X_1^N W_0\right) \quad (10)$$

hold for all input probability distributions of the form

$$p\left(W_0 X_1^N X_2^N\right) = p(W_0)\, p\left(X_1^N | W_0\right) p\left(X_2^N | W_0\right).$$

We give the following sufficient conditions for single-letter constraints so that conditions (9) and (10) are satisfied.

- Let us consider $V_1 \triangleq X_1$. From [6, Theorem 2], we note that for the strong interference condition, i.e., if $I(X_1; Y_2|X_2U) \geq I(X_1; Y_1|X_2U)$ is satisfied for all input probability distributions on $\mathcal{U} \times \mathcal{X}_1 \times \mathcal{X}_2$ such that $p(u, x_1, x_2) = p(u)\, p(x_1|u)\, p(x_2|u)$, condition (9) is satisfied. Similarly, condition (10) is satisfied if $I(X_2; Y_1|X_1U) \geq I(X_2; Y_2|X_1U)$ is satisfied for all input probability distributions on $\mathcal{U} \times \mathcal{X}_1 \times \mathcal{X}_2$ such that $p(u, x_1, x_2) = p(u)\, p(x_1|u)\, p(x_2|u)$.
- If a function k_1 exists such that (6) is satisfied, we see that condition (10) will be satisfied. Similarly, if a function k_2 exists such that (5) is satisfied, we see that condition (9) will be satisfied.

Remark 1: Even though we found sufficient conditions for single-letter constraints to satisfy conditions (9) and (10), we were unable to prove necessary conditions for single-letter constraints. We describe our larger class of ICs in terms of the block-level constraints (9) and (10) instead of the single-letter constraints described as they may not be exhaustive. This also simplifies our proof and will include the sufficient single-letter constraints described as special cases. In fact, this larger class of ICs includes the class of strong ICs with common information as a special case. It also includes the class of deterministic ICs without common information [9] and the class of deterministic ICs with common information [10] as special cases.

Remark 2: We note that if a function k_1 exists such that (6) is satisfied, receiver 1 will always be able to decode V_2 naturally once it has decoded X_1. Replacing constraint (6) with a more relaxed constraint (10) basically uses the idea of a stronger channel where decoding V_2 at receiver 1 is easier than decoding V_2 at receiver 2.

III. Determination of the Capacity Region

Theorem 1: The capacity region of the IC shown in Fig. 2 satisfying conditions (9) and (10) is the union of all rate triplets (R_0, R_1, R_2) satisfying

$$R_1 \leq I(X_1; Y_1|V_0V_2) \quad (11)$$
$$R_2 \leq I(X_2; Y_2|V_0V_1) \quad (12)$$
$$R_1 + R_2 \leq I(X_1V_2; Y_1|V_0) + I(X_2; Y_2|V_0V_1V_2) \quad (13)$$
$$R_1 + R_2 \leq I(X_1; Y_1|V_0V_1V_2) + I(X_2V_1; Y_2|V_0) \quad (14)$$
$$R_1 + R_2 \leq I(X_1V_2; Y_1|V_0V_1) + I(X_2V_1; Y_2|V_0V_2) \quad (15)$$
$$2R_1 + R_2 \leq I(X_1V_2; Y_1|V_0) + I(X_1; Y_1|V_0V_1V_2) + I(X_2V_1; Y_2|V_0V_2) \quad (16)$$

$$R_1+2R_2 \le I(X_2;Y_2|V_0V_1V_2)+I(X_2V_1;Y_2|V_0) + I(X_1V_2;Y_1|V_0V_1) \quad (17)$$

$$R_0+R_1 \le I(V_2X_1;Y_1) \quad (18)$$

$$R_0+R_2 \le I(V_1X_2;Y_2) \quad (19)$$

$$R_0+R_1+R_2 \le I(V_2X_1;Y_1)+I(X_2;Y_2|V_0V_1V_2) \quad (20)$$

$$R_0+R_1+R_2 \le I(X_1;Y_1|V_0V_1V_2)+I(V_1X_2;Y_2) \quad (21)$$

$$R_0+2R_1+R_2 \le I(V_2X_1;Y_1)+I(X_1;Y_1|V_1V_2V_0) + I(V_1X_2;Y_2|V_2V_0) \quad (22)$$

$$R_0+R_1+2R_2 \le I(X_2;Y_2|V_1V_2V_0)+I(V_1X_2;Y_2) + I(X_1V_2;Y_1|V_1V_0) \quad (23)$$

for all input distributions $p(v_0)p(x_1|v_0)p(x_2|v_0)$. Furthermore, the region remains invariant if we impose the following constraint: $\|\mathcal{V}_0\| \le \|\mathcal{X}_1\|\,\|\mathcal{X}_2\|+7$.

We first give two examples of new ICs for which Theorem 1 gives the capacity region before going on to the proof.

- The following example is a channel that satisfies (3)–(6) but does not satisfy (1)–(2). Hence, it is not covered by the class of deterministic ICs considered by El Gamal and Costa.

Example 1: We consider a symmetric IC (without common information) with the following alphabets: $\mathcal{X}_1=\{0,1,2\}$, $\mathcal{V}_1=\{0,1\}$, $\mathcal{Y}_1=\{0,1,2,3\}$, $\mathcal{X}_2=\{0,1,2\}$, $\mathcal{V}_2=\{0,1\}$, and $\mathcal{Y}_2=\{0,1,2,3\}$. The functions g_1 and g_2 are given by

$$g_1(X_1=0)=0, \quad g_1(X_1=1)=g_1(X_1=2)=1 \quad (24)$$

$$g_2(X_2=0)=0, \quad g_2(X_2=1)=g_2(X_2=2)=1. \quad (25)$$

Consider the probability transition matrices shown in Table I. One can easily check that it is not in the class of deterministic ICs studied by El Gamal and Costa since $Y_1 \ne f_1(X_1,V_2)$ and $Y_2 \ne f_2(X_2,V_1)$. One can also easily verify that it is not a discrete memoryless IC under strong interference. Set $p(X_1=1)=p(X_1=2)=\frac{1}{2}$, $p(X_2=0)=\frac{1}{2}$, $p(X_2=1)=\frac{1}{4}$, $p(X_2=2)=\frac{1}{4}$, $p=\frac{1}{10}$, and $p_1=\frac{1}{10}$. We then have $I(X_1;Y_2|X_2)=0$ and $I(X_1;Y_1|X_2)=\frac{1}{2}$. Hence, $I(X_1;Y_2|X_2) \ge I(X_1;Y_1|X_2)$ is not satisfied for all input product distributions $p(x_1)p(x_2)$.

- A semideterministic strong IC with common information: If $I(X_1;Y_2|X_2U) \ge I(X_1;Y_1|X_2U)$ is satisfied for all product distributions on $\mathcal{U}\times\mathcal{X}_1\times\mathcal{X}_2$ such that $p(u,x_1,x_2)=p(u)p(x_1|u)p(x_2|u)$ (hence, $V_1 \triangleq X_1$), and there exists a function k_1 such that (6) is satisfied, we readily see that the conditions (9) and (10) will be satisfied for all input distributions of the form $p(W_0X_1^NX_2^N)=p(W_0)p(X_1^N|W_0)p(X_2^N|W_0)$. This class of ICs is a mixture of the IC with strong interference and the class of deterministic ICs introduced by El Gamal and Costa and is as shown in Fig. 3.

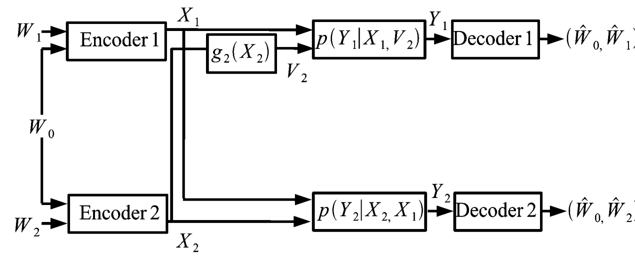

Fig. 3. Asymmetric IC.

TABLE I
PROBABILITY TRANSITION MATRICES

$p(Y_1\|X_1,V_2=0)$			
	$X_1=0$	$X_1=1$	$X_1=2$
$Y_1=0$	p	0	0
$Y_1=1$	$1-p$	0	0
$Y_1=2$	0	0	0
$Y_1=3$	0	1	1
$p(Y_1\|X_1,V_2=1)$			
	$X_1=0$	$X_1=1$	$X_1=2$
$Y_1=0$	0	0	1
$Y_1=1$	0	p_1	0
$Y_1=2$	1	$1-p_1$	0
$Y_1=3$	0	0	0
$p(Y_2\|X_2,V_1=0)$			
	$X_2=0$	$X_2=1$	$X_2=2$
$Y_2=0$	p	0	0
$Y_2=1$	$1-p$	0	0
$Y_2=2$	0	0	0
$Y_2=3$	0	1	1
$p(Y_2\|X_2,V_1=1)$			
	$X_2=0$	$X_2=1$	$X_2=2$
$Y_2=0$	0	0	1
$Y_2=1$	0	p_1	0
$Y_2=2$	1	$1-p_1$	0
$Y_2=3$	0	0	0

IV. Proof of the Capacity Region

1) Achievability: This follows directly by applying Fourier–Motzkin elimination to the conditions of [10, Theorem 1] and by substituting $V_0 \triangleq U_0$, $V_1 \triangleq U_1$, and $V_2 \triangleq U_2$. There are two additional constraints given by

$$R_1 \le I(X_1;Y_1|V_0V_1V_2)+I(V_1X_2;Y_2|V_0V_2) \quad (26)$$

$$R_2 \le I(X_2;Y_2|V_0V_1V_2)+I(V_2X_1;Y_1|V_0V_1). \quad (27)$$

However, we note that (26) is redundant due to our imposed constraint (9) as follows:

$$\begin{aligned} I(X_1;Y_1|V_0V_1V_2)&+I(V_1X_2;Y_2|V_0V_2)\\ &\ge I(X_1;Y_1|V_0V_1V_2)+I(V_1;Y_2|V_0X_2)\\ &\ge I(X_1;Y_1|V_0V_1V_2)+I(V_1;Y_1|V_0X_2)\\ &= I(X_1;Y_1|V_0V_2). \end{aligned} \quad (28)$$

Similarly, (27) is redundant due to our imposed constraint (10). The assertion about the cardinality of $\mathcal{V}_0$ follows from the application of Fenchel–Eggleston strengthening of the Caratheodory's theorem.

2) Converse: We will make use of the following facts in our derivations below: (a) The independence of W_0, W_1, and W_2. (b) The conditional independence of (W_1,V_1^N,X_1^N) and (W_2,V_2^N,X_2^N) given W_0. (c) $W_1 \to W_0X_1^N \to Y_1^N$ and $W_2 \to W_0X_2^N \to Y_2^N$ form Markov chains. (d) Y_{1i} depends only on (X_{1i},V_{2i}) and Y_{2i} depends only on (V_{1i},X_{2i}). (e) Conditions (9) and (10). (f) Conditioning reduces entropy. In addition, from Fano's inequalities, we obtain $H(W_1|Y_1^N) \le N\epsilon_{1N}$, $H(W_2|Y_2^N) \le N\epsilon_{2N}$,

$H\left(W_0|Y_1^N\right) \leq N\epsilon_{3N}$, and $H\left(W_0|Y_2^N\right) \leq N\epsilon_{4N}$. First, let us consider

$$\begin{aligned}NR_1 &\stackrel{(a)}{=} H\left(W_1|W_0\right)\\ &\leq I\left(W_1;Y_1^N|W_0\right) + N\epsilon_{1N}\\ &\stackrel{(c)}{\leq} I\left(X_1^N;Y_1^N|W_0\right) + N\epsilon_{1N}\\ &\stackrel{(b)}{\leq} I\left(X_1^N;Y_1^N|V_2^N W_0\right) + N\epsilon_{1N}\\ &\stackrel{(d)(f)}{\leq} \sum_{i=1}^{N}\left[H\left(Y_{1i}|V_{2i}W_0\right) - H\left(Y_{1i}|X_{1i}V_{2i}W_0\right)\right] + N\epsilon_{1N}\\ &= \sum_{i=1}^{N} I\left(X_{1i};Y_{1i}|V_{2i}W_0\right) + N\epsilon_{1N}.\end{aligned} \tag{29}$$

Analogously, we may derive an expression for R_2 similar in form to (12). Next, let us consider

$$\begin{aligned}&N\left(R_1+R_2\right)\\ &\stackrel{(a)}{=} H\left(W_1|W_0\right) + H\left(W_2|W_0\right)\\ &\leq I\left(W_1;Y_1^N|W_0\right) + I\left(W_2;Y_2^N|W_0\right) + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(c)}{\leq} I\left(X_1^N;Y_1^N|W_0\right) + I\left(X_2^N;Y_2^N|W_0\right) + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(b)}{\leq} I\left(X_1^N;Y_1^N|W_0\right) + I\left(X_2^N;Y_2^N|X_1^N W_0\right)\\ &\quad + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &= I\left(X_1^N;Y_1^N|W_0\right) + I\left(V_2^N;Y_2^N|X_1^N W_0\right)\\ &\quad + I\left(X_2^N;Y_2^N|X_1^N V_2^N W_0\right) + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(e)}{\leq} I\left(X_1^N;Y_1^N|W_0\right) + I\left(V_2^N;Y_1^N|X_1^N W_0\right)\\ &\qquad + I\left(X_2^N;Y_2^N|X_1^N V_2^N W_0\right) + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &\quad = I\left(V_2^N X_1^N;Y_1^N|W_0\right) + I\left(X_2^N;Y_2^N|X_1^N V_1^N V_2^N W_0\right)\\ &\quad + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(d)(f)}{\leq} \sum_{i=1}^{N}\left[H\left(Y_{1i}|W_0\right) - H\left(Y_{1i}|W_0 V_{2i} X_{1i}\right)\right]\\ &\quad + \sum_{i=1}^{N}\left[H\left(Y_{2i}|W_0 V_{1i} V_{2i}\right) - H\left(Y_{2i}|W_0 V_{1i} V_{2i} X_{2i}\right)\right]\\ &\quad + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &= \sum_{i=1}^{N}\left[I\left(V_{2i}X_{1i};Y_{1i}|W_0\right) + I\left(X_{2i};Y_{2i}|W_0 V_{1i} V_{2i}\right)\right]\\ &\quad + N\left(\epsilon_{1N}+\epsilon_{2N}\right).\end{aligned} \tag{30}$$

Analogously, we may derive an expression for $R_1 + R_2$ similar in form to (14). Next, let us also consider

$$\begin{aligned}&N\left(R_1+R_2\right)\\ &\stackrel{(a)}{=} H\left(W_1|W_0\right) + H\left(W_2|W_0\right)\\ &\leq I\left(W_1;Y_1^N|W_0\right) + I\left(W_2;Y_2^N|W_0\right) + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(c)}{\leq} I\left(X_1^N;Y_1^N|W_0\right) + I\left(X_2^N;Y_2^N|W_0\right) + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &= I\left(V_1^N;Y_1^N|W_0\right) + I\left(X_1^N;Y_1^N|V_1^N W_0\right)\\ &\quad + I\left(V_2^N;Y_2^N|W_0\right) + I\left(X_2^N;Y_2^N|V_2^N W_0\right)\\ &\quad + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(b)}{\leq} I\left(V_1^N;Y_1^N|X_2^N W_0\right) + I\left(X_1^N;Y_1^N|V_1^N W_0\right)\\ &\quad + I\left(V_2^N;Y_2^N|X_1^N W_0\right) + I\left(X_2^N;Y_2^N|V_2^N W_0\right)\\ &\quad + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(e)}{\leq} I\left(V_1^N;Y_2^N|X_2^N W_0\right) + I\left(X_1^N;Y_1^N|V_1^N W_0\right)\\ &\quad + I\left(V_2^N;Y_1^N|X_1^N W_0\right) + I\left(X_2^N;Y_2^N|V_2^N W_0\right)\\ &\quad + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &= I\left(V_2^N X_1^N;Y_1^N|V_1^N W_0\right) + I\left(V_1^N X_2^N;Y_2^N|V_2^N W_0\right)\\ &\quad + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(d)(f)}{\leq} \sum_{i=1}^{N}\left[H\left(Y_{1i}|W_0 V_{1i}\right) - H\left(Y_{1i}|W_0 V_{1i} V_{2i} X_{1i}\right)\right]\\ &\quad + \sum_{i=1}^{N}\left[H\left(Y_{2i}|W_0 V_{2i}\right) - H\left(Y_{2i}|W_0 V_{1i} V_{2i} X_{2i}\right)\right]\\ &\quad + N\left(\epsilon_{1N}+\epsilon_{2N}\right)\\ &= \sum_{i=1}^{N}\left[I\left(X_{1i}V_{2i};Y_{1i}|W_0 V_{1i}\right) + I\left(X_{2i}V_{1i};Y_{2i}|W_0 V_{2i}\right)\right]\\ &\quad + N\left(\epsilon_{1N}+\epsilon_{2N}\right).\end{aligned} \tag{31}$$

Next, let us consider

$$\begin{aligned}&N\left(2R_1+R_2\right)\\ &\stackrel{(a)}{=} 2H\left(W_1|W_0\right) + H\left(W_2|W_0\right)\\ &\leq 2I\left(W_1;Y_1^N|W_0\right) + I\left(W_2;Y_2^N|W_0\right) + N\left(2\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(c)}{\leq} 2I\left(X_1^N;Y_1^N|W_0\right) + I\left(X_2^N;Y_2^N|W_0\right) + N\left(2\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(b)}{\leq} I\left(X_1^N;Y_1^N|W_0\right) + I\left(X_1^N;Y_1^N|X_2^N W_0\right)\\ &\quad + I\left(X_2^N;Y_2^N|W_0\right) + N\left(2\epsilon_{1N}+\epsilon_{2N}\right)\\ &= I\left(X_1^N;Y_1^N|W_0\right) + I\left(V_1^N;Y_1^N|X_2^N W_0\right)\\ &\quad + I\left(X_1^N;Y_1^N|X_2^N V_1^N W_0\right) + I\left(V_2^N;Y_2^N|W_0\right)\\ &\quad + I\left(X_2^N;Y_2^N|V_2^N W_0\right) + N\left(2\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(b)}{\leq} I\left(X_1^N;Y_1^N|W_0\right) + I\left(V_1^N;Y_1^N|X_2^N W_0\right)\\ &\quad + I\left(X_1^N;Y_1^N|X_2^N V_1^N W_0\right) + I\left(V_2^N;Y_2^N|X_1^N W_0\right)\\ &\quad + I\left(X_2^N;Y_2^N|V_2^N W_0\right) + N\left(2\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(e)}{\leq} I\left(X_1^N;Y_1^N|W_0\right) + I\left(V_1^N;Y_2^N|X_2^N W_0\right)\\ &\quad + I\left(X_1^N;Y_1^N|X_2^N V_1^N W_0\right) + I\left(V_2^N;Y_1^N|X_1^N W_0\right)\\ &\quad + I\left(X_2^N;Y_2^N|V_2^N W_0\right) + N\left(2\epsilon_{1N}+\epsilon_{2N}\right)\\ &= I\left(V_2^N X_1^N;Y_1^N|W_0\right) + I\left(X_2^N V_1^N;Y_2^N|V_2^N W_0\right)\\ &\quad + I\left(X_1^N;Y_1^N|V_1^N V_2^N X_2^N W_0\right) + N\left(2\epsilon_{1N}+\epsilon_{2N}\right)\\ &\stackrel{(d)(f)}{\leq} \sum_{i=1}^{N}\left[H\left(Y_{1i}|W_0\right) - H\left(Y_{1i}|W_0 V_{2i} X_{1i}\right)\right]\end{aligned}$$

$$
\begin{aligned}
&+\sum_{i=1}^{N}\left[H\left(Y_{2i}|V_{2i}W_0\right)-H\left(Y_{2i}|W_0V_{1i}V_{2i}X_{2i}\right)\right]\\
&+\sum_{i=1}^{N}\left[H\left(Y_{1i}|V_{1i}V_{2i}W_0\right)-H\left(Y_{1i}|V_{1i}V_{2i}X_{1i}W_0\right)\right]\\
&+N\left(2\epsilon_{1N}+\epsilon_{2N}\right)\\
=&\sum_{i=1}^{N}\left[I\left(V_{2i}X_{1i};Y_{1i}|W_0\right)+I\left(X_{2i}V_{1i};Y_{2i}|V_{2i}W_0\right)\right]\\
&+\sum_{i=1}^{N}\left[I\left(X_{1i};Y_{1i}|V_{1i}V_{2i}W_0\right)\right]+N\left(2\epsilon_{1N}+\epsilon_{2N}\right). \qquad (32)
\end{aligned}
$$

Analogously, we may derive an expression for R_1+2R_2 similar in form to (17). Next, let us consider

$$
\begin{aligned}
N&\left(R_0+R_1\right)\\
&=H\left(W_0W_1\right)\\
&\le I\left(W_0W_1;Y_1^N\right)+N\left(\epsilon_{1N}+\epsilon_{3N}\right)\\
&\overset{(c)}{\le} I\left(W_0X_1^N;Y_1^N\right)+N\left(\epsilon_{1N}+\epsilon_{3N}\right)\\
&\overset{(d)(f)}{\le}\sum_{i=1}^{N}\left[H\left(Y_{1i}\right)-H\left(Y_{1i}|V_{2i}X_{1i}\right)\right]+N\left(\epsilon_{1N}+\epsilon_{3N}\right)\\
&=\sum_{i=1}^{N}I\left(X_{1i}V_{2i};Y_{1i}\right)+N\left(\epsilon_{1N}+\epsilon_{3N}\right). \qquad (33)
\end{aligned}
$$

Analogously, we may derive an expression for R_0+R_2 similar in form to (19). Next, let us consider

$$
\begin{aligned}
N&\left(R_0+R_1+R_2\right)\\
&\overset{(a)}{=}H\left(W_1W_0\right)+H\left(W_2|W_0\right)\\
&\le I\left(W_1W_0;Y_1^N\right)+I\left(W_2;Y_2^N|W_0\right)\\
&\quad+N\left(\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&\overset{(b)(c)}{\le} I\left(W_0X_1^N;Y_1^N\right)+I\left(X_2^N;Y_2^N|X_1^NW_0\right)\\
&\quad+N\left(\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&=I\left(W_0X_1^N;Y_1^N\right)+I\left(V_2^N;Y_2^N|X_1^NW_0\right)\\
&\quad+I\left(X_2^N;Y_2^N|X_1^NV_2^NW_0\right)+N\left(\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&\overset{(e)}{\le} I\left(W_0X_1^N;Y_1^N\right)+I\left(V_2^N;Y_1^N|X_1^NW_0\right)\\
&\quad+I\left(X_2^N;Y_2^N|X_1^NV_2^NW_0\right)+N\left(\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&=I\left(W_0V_2^NX_1^N;Y_1^N\right)+I\left(X_2^N;Y_2^N|V_1^NX_1^NV_2^NW_0\right)\\
&\quad+N\left(\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&\overset{(d)(f)}{\le}\sum_{i=1}^{N}\left[H\left(Y_{1i}\right)-H\left(Y_{1i}|V_{2i}X_{1i}\right)\right]\\
&\quad+\sum_{i=1}^{N}\left[H\left(Y_{2i}|V_{1i}V_{2i}W_0\right)-H\left(Y_{2i}|V_{1i}V_{2i}X_{2i}W_0\right)\right]\\
&\quad+N\left(\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&=\sum_{i=1}^{N}\left[I\left(V_{2i}X_{1i};Y_{1i}\right)+I\left(X_{2i};Y_{2i}|V_{1i}V_{2i}W_0\right)\right]\\
&\quad+N\left(\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right). \qquad (34)
\end{aligned}
$$

We may analogously derive an expression for $R_0+R_1+R_2$ similar in form to the expression (21). Finally, let us consider

$$
\begin{aligned}
N&\left(R_0+2R_1+R_2\right)\\
&\overset{(a)}{=}H\left(W_1W_0\right)+H\left(W_1|W_0\right)+H\left(W_2|W_0\right)\\
&\le I\left(W_1W_0;Y_1^N\right)+I\left(W_1;Y_1^N|W_0\right)\\
&\quad+I\left(W_2;Y_2^N|W_0\right)+N\left(2\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&\overset{(b)(c)}{\le} I\left(W_0X_1^N;Y_1^N\right)+I\left(X_1^N;Y_1^N|X_2^NW_0\right)\\
&\quad+I\left(X_2^N;Y_2^N|W_0\right)+N\left(2\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&=I\left(W_0X_1^N;Y_1^N\right)+I\left(V_1^N;Y_1^N|X_2^NW_0\right)\\
&\quad+I\left(X_1^N;Y_1^N|X_2^NV_1^NW_0\right)+I\left(V_2^N;Y_2^N|W_0\right)\\
&\quad+I\left(X_2^N;Y_2^N|V_2^NW_0\right)+N\left(2\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&\overset{(b)}{\le}\left(W_0X_1^N;Y_1^N\right)+I\left(V_1^N;Y_1^N|X_2^NW_0\right)\\
&\quad+I\left(X_1^N;Y_1^N|X_2^NV_1^NW_0\right)+I\left(V_2^N;Y_2^N|X_1^NW_0\right)\\
&\quad+I\left(X_2^N;Y_2^N|V_2^NW_0\right)+N\left(2\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&\overset{(e)}{\le} I\left(W_0X_1^N;Y_1^N\right)+I\left(V_1^N;Y_2^N|X_2^NW_0\right)\\
&\quad+I\left(X_1^N;Y_1^N|X_2^NV_1^NW_0\right)+I\left(V_2^N;Y_1^N|X_1^NW_0\right)\\
&\quad+I\left(X_2^N;Y_2^N|V_2^NW_0\right)+N\left(2\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&=I\left(W_0V_2^NX_1^N;Y_1^N\right)+I\left(X_2^NV_1^N;Y_2^N|V_2^NW_0\right)\\
&\quad+I\left(X_1^N;Y_1^N|V_1^NV_2^NX_2^NW_0\right)+N\left(2\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&\overset{(d)(f)}{\le}\sum_{i=1}^{N}\left[H\left(Y_{1i}\right)-H\left(Y_{1i}|V_{2i}X_{1i}\right)\right]\\
&\quad+\sum_{i=1}^{N}\left[H\left(Y_{2i}|V_{2i}W_0\right)-H\left(Y_{2i}|V_{1i}V_{2i}X_{2i}W_0\right)\right]\\
&\quad+\sum_{i=1}^{N}\left[H\left(Y_{1i}|V_{1i}V_{2i}W_0\right)-H\left(Y_{1i}|V_{1i}V_{2i}X_{1i}W_0\right)\right]\\
&\quad+N\left(2\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right)\\
&=\sum_{i=1}^{N}\left[I\left(V_{2i}X_{1i};Y_{1i}\right)+I\left(X_{2i}V_{1i};Y_{2i}|V_{2i}W_0\right)\right]\\
&\quad+\sum_{i=1}^{N}I\left(X_{1i};Y_{1i}|V_{1i}V_{2i}W_0\right)+N\left(2\epsilon_{1N}+\epsilon_{2N}+\epsilon_{3N}\right).
\end{aligned}
\qquad (35)
$$

Analogously, we may derive an expression for $R_0+R_1+2R_2$ similar in form to (23). Finally, we define $V_0 \triangleq W_0$ and allowing $N\to\infty$, we obtain the conditions in Theorem 1. For Theorem1 , a time-sharing random parameter Q is unnecessary as we may set $V_0 \triangleq (V_0', Q)$.

V. Conclusion

In this paper, we determined the capacity region of a new class of deterministic ICs with common information. The capacity region includes as special cases the capacity region of the deterministic IC of El Gamal and Costa with and without common information, as well as the discrete memoryless strong IC with and without common information. We show by a specific example that this is strictly a new class of ICs.

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