On the Diversity-Multiplexing Tradeoff of Amplify-and-Forward Half-Duplex Relaying

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***Abstract*—In this paper, an amplify-and-forward (AF) two-path half-duplex relaying scheme is considered in which one of the relays additionally performs inter-relay interference cancellation. We first generalize lower bounds for the diversity-multiplexing tradeoff (DMT) of an arbitrary block lower triangular channel matrix. We then characterize the diversity-multiplexing tradeoff for the AF two-path relaying scheme and show that the DMT achieves the multiple-input single-output (MISO) upper bound. The analysis also demonstrates that, with a careful choice of the coding strategy, the DMT of this scheme is achievable for finite codeword lengths. We then propose using an equivalent linear space time code at the source, which does not require any form of channel state information, as a simple and effective coding strategy to achieve the full DMT of the scheme. From the DMT perspective, the proposed AF two-path relaying with the equivalent linear space time coding outperforms existing schemes. Our analysis is then extended to the slotted-amplify-and-forward (SAF) scheme with multiple relays, where we provide a stronger result by deriving the DMT while taking into account inter-relay interference.**

***Index Terms*—Amplify-and-forward, half-duplex relaying, two-path relaying, diversity-multiplexing tradeoff (DMT).**

I. INTRODUCTION

COOPERATIVE relaying [1], [2] has gained much attention in recent years as a technique to exploit spatial diversity to combat fading. A fundamental performance measure to evaluate different cooperative schemes is the diversity-multiplexing tradeoff (DMT) introduced in [3] in the high signal-to-noise (SNR) regime. It is a tool to characterize the tradeoff between the reliability and the number of degrees of freedom of a communication system. Reliability is measured by diversity gain, defined as the rate of decay of the error probability as the SNR increases, while the degrees of freedom is measured in terms of spatial multiplexing gain, defined as the rate of increase in the transmission rate with increasing SNR.

For a two-relay topology, also known as the diamond relay channel, it was shown in [4], [5] that a two-path relaying scheme can recover a significant portion of the spatial multiplexing loss in conventional half-duplex relaying. This multiplexing loss is due to the fact that the transmission of one data symbol from the source to destination, via a relay, occupies two-channel uses which leads to a loss in multiplexing gain. Thus, to overcome this half-duplex constraint, two relays are used to alternate between transmission and reception.

In [5], decode-and-forward (DF) two-path relaying, also called successive digital relaying, is studied. Achievable rates and capacity bounds for successive digital relaying are analyzed in [5]–[7] and [6], [7] respectively. DMT analysis of this successive digital relaying is given in [5] under the assumption that the relays are always able to correctly decode the signals. The DMT curve derived is a straight line with maximum diversity of two and a multiplexing gain that approaches one as the source transmission length goes to infinity.

Since two-path relaying protocol alternates between transmission and reception, inter-relay interference will be present, a point which has been highlighted in [4]. Unlike DF two-path relaying, AF two-path relaying further suffers from the accumulation of inter-relay interference and noise. It was shown in [4] that this accumulation of inter-relay interference degrades the performance of the system as the inter-relay channel gain increases. Successive decoding at the destination with partial or full cancellation [4] was shown to perform well only for weak to moderate inter-relay channel gains. In [7], dirty paper coding (DPC) based on interference pre-subtraction at the source transmitter is proposed to solve the inter-relay interference problem. The DPC in [7] assumes full knowledge of all channel state information (CSI) at the source transmitter, which is usually very difficult to achieve in practice [8], [9].

Further extension of two-path relaying to the multiple relay scenario is given in [10], [11]. In [10], a class of AF two-path relaying for multiple relays called slotted amplify-and-forward (SAF) is analyzed. From the DMT analysis, it was shown that SAF performs very well, especially in the high spectral efficiency and large network size regime, under the assumption that the relays are isolated, i.e., inter-relay interference does not exist. An extension of DF two-path relaying protocol to a multiple relay scenario, called multihop relay selection (MHRS), was proposed in [11]. MHRS utilizes relay selection methods with limited feedback to recover the multiplexing loss in DF half-duplex relay networks. Inter-relay interference was considered and the DMT of MHRS was analyzed in [11]. MHRS performs well for a large number of relays, but is not suitable for the two-relay scenario since a relay will not be able to contribute to the diversity gain once it has been selected for transmission.

In this paper, we consider an AF two-path relaying with inter-relay interference cancellation where the cancellation is

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performed at *one of the relays*. Inter-relay interference cancellation prevents an accumulation of inter-relay interference and it can be achieved easily because the relay has perfect knowledge of the signal it transmitted in the previous time slot. The direct link between source and destination is also considered. This scheme was first proposed in [12]. In [13], it was shown that this scheme can significantly reduce the degradation in performance of AF two-path relaying due to inter-relay interference. It was also indicated in [12], [13] that only a diversity order of two can be achieved although theoretically a diversity order of three is possible as there are three independent spatial paths from S to D. Recently, we have found that the full diversity gain of three can indeed be achieved while maintaining a multiplexing gain of one by using precoding at the source [14], which can be viewed as an equivalent linear space time code. The precoding matrix is arbitrarily chosen from a set of matrices, which are not dependent on any form of CSI. In this paper, we will prove a stronger result that the same space time coding technique proposed in [14] can also be used as a practical strategy to achieve the full DMT of this scheme.

Our main contributions are summarized as follows:

- We first generalize the result in [15, Theorem 2.12] and obtain lower bounds for the DMT of an arbitrary block lower triangular channel matrix. In [15], lower bounds can only be found by using the main and the last sub-diagonal matrix. However, with the generalized results, we are able to lower bound the DMT by using any sub-diagonal matrix.
- We then derive the DMT of AF two-path relaying scheme, while taking into account inter-relay interference in the analysis. Inter-relay interference was not considered in [5], [10]. We prove that the DMT of the AF two-path relaying scheme with inter-relay interference cancellation at one of the relays in a slow fading channel, where the channel coefficients remain constant for $L+2$ time slots, is given by

$$d(r) = 3\left(1 - \frac{L+2}{L}r\right)^{+}. \tag{1}$$

  This indicates that a full diversity order of three can be achieved while maintaining a spatial multiplexing gain of one for $L \to \infty$, where L is the frame length. It can also be seen that the DMT of this scheme asymptotically achieves the multiple-input single-output (MISO) upper bound as $L \to \infty$.
- For the multiple-relay case, we provide a stronger result than [10] and prove that the DMT of the N-relay M-slot SAF scheme in [10] is indeed given as

$$d_{\text{SAF}}(r) = (1-r)^{+} + N\left(1 - \frac{M}{M-1}r\right)^{+} \tag{2}$$

  without the assumption that the relays are isolated, i.e. inter-relay interference is taken into consideration.
- In [14], we have discovered that a full diversity order of three can be achieved easily by using precoding, which can be viewed as an equivalent linear space time code. Our analysis in this paper demonstrates that the same space time coding technique can be used as a simple and yet effective finite codeword length coding strategy to achieve the full DMT of this scheme. The precoding matrix for this linear space time code does not require any form of CSI. From the DMT perspective, the proposed AF two-path relaying with the equivalent linear space time coding is optimal and outperforms existing schemes in [4], [5], [12], [13].

Throughout the paper, we use bold upper and lower case letters to denote matrices and vectors respectively. $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote transposition and conjugate transposition respectively. $\mathbf{I}_m$ denotes a $m \times m$ identity matrix. $\mathbb{E}\{\cdot\}$ denotes the statistical expectation operator. A complex Gaussian random variable z with mean μ and variance σ^2 is denoted as $z \sim \mathcal{CN}(\mu, \sigma^2)$. The dot equal operator $\doteq$ denotes asymptotic equality in the high signal-to-noise ratio (SNR) regime i.e., $p_1 \doteq p_2$ denotes

$$\lim_{\text{SNR}\to\infty} \frac{\log p_1}{\log \text{SNR}} = \lim_{\text{SNR}\to\infty} \frac{\log p_2}{\log \text{SNR}}$$

and $\dot{\leq}, \dot{\geq}$ are similarly defined. $(x)^{+}$ denotes $\max(0, x)$. $\mathbb{R}^{N+}$ denotes the set of real non-negative N-tuples. For any set $O \subset \mathbb{R}^N$, $O^{+} = O \cap \mathbb{R}^{N+}$. $\lambda_i[\mathbf{J}]$ denotes the i-th eigenvalue of matrix $\mathbf{J}$. $\mathbf{Q}(p,q)$ denotes the element at p-th row and q-th column of matrix $\mathbf{Q}$. $\operatorname{tr}(\mathbf{P})$ denotes the trace of matrix $\mathbf{P}$. A diagonal matrix $\mathbf{A}$ with diagonal elements given by the vector $\mathbf{a}$ is denoted as $\mathbf{A} = \operatorname{diag}(\mathbf{a})$.

II. System Model

Consider a cooperative network system with one source node (S), two relay nodes (R1, R2) and one destination node (D). We assume that all nodes have a single antenna. The source transmission is divided into frames, each containing L symbols denoted as $x(l)$, $l \in \{1, 2, \ldots, L\}$. The destination will perform detection on a frame-by-frame basis. Without loss of generality we presume L is even. We will also presume that the channel remains static within a frame.

A. Transmission Protocol

The transmission protocol of the AF two-path relaying scheme in two consecutive time slots is depicted in Fig. 1. $h_{s,d} \sim \mathcal{CN}(0, \gamma_{s,d}^2)$, $h_{s,rj} \sim \mathcal{CN}(0, \gamma_{s,rj}^2)$, $h_{rj,d} \sim \mathcal{CN}(0, \gamma_{rj,d}^2)$ and $h_r \sim \mathcal{CN}(0, \gamma_r^2)$ are the channel coefficients from S to D, S to j-th relay, j-th relay to D and R1 (R2) to R2 (R1) respectively where $j \in \{1, 2\}$. We have implicitly presumed that the inter-relay channel h_r is reciprocal.

The protocol is described in detail in [13], [14], but a summary will be provided here for ease of reference. In time slot 1, $x(1)$ is sent from S. R1 and D receive $x(1)$. In time slot 2, $x(2)$ is sent from S and $x_{r1}(2)$ is sent from R1, where $x_{rj}(i)$ is the AF transmitted signal from j-th relay in i-th time slot. Both R2 and D receive a linear combination of $x_{r1}(2)$ and $x(2)$. In time slot 3, $x(3)$ is sent from S. $x_{r2}(3)$ is sent from R2. Both R1 and D receive a linear combination of $x_{r2}(3)$ and $x(3)$. Taking note that this is an AF protocol, we realize that $x_{r2}(3)$ contains $x_{r1}(2)$ which is known perfectly at R1 as $x_{r1}(2)$ is the transmitted signal from R1 in time slot 2, and thus it can be cancelled off at R1. These transmission steps are continuously repeated, with R1 performing the cancellation,

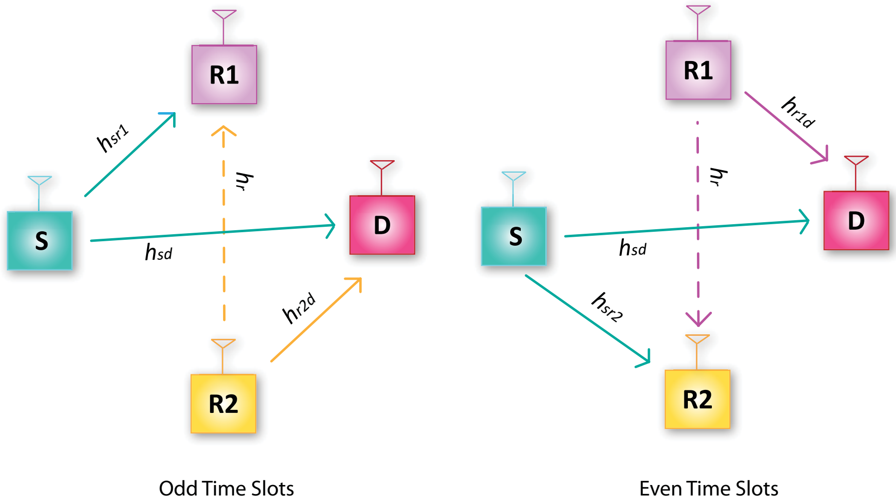

Fig. 1. Transmission protocol for AF two-path relaying.

TABLE I
TRANSMISSION PROTOCOL OF AF TWO-PATH RELAYING

Node \ Time	1	2	3	4	5	$\cdots$
S	Tx : $x(1)$	Tx : $x(2)$	Tx : $x(3)$	Tx : $x(4)$	Tx : $x(5)$	$\cdots\cdots\cdots$
R1	Rx : $x(1)$	Tx : $x_{r1}(2)$	Rx : $x(3)$, $x_{r2}(3)$	Tx : $x_{r1}(4)$	Rx : $x(5)$, $x_{r2}(5)$	$\cdots\cdots\cdots$
R2		Rx : $x(2)$, $x_{r1}(2)$	Tx : $x_{r2}(3)$	Rx : $x(4)$, $x_{r1}(4)$	Tx : $x_{r2}(5)$	$\cdots\cdots\cdots$
D	Rx : $x(1)$	Rx : $x(2)$, $x_{r1}(2)$	Rx : $x(3)$, $x_{r2}(3)$	Rx : $x(4)$, $x_{r1}(4)$	Rx : $x(5)$, $x_{r2}(5)$	$\cdots\cdots\cdots$

until L symbols have been transmitted from the source. This transmission protocol is summarized in Table I.

In practice, two additional time slots are required at the end to complete the transmission and thus a multiplexing gain of $\frac{L}{L+2}$ is achieved. This slight overhead is also reported in [5] where the multiplexing gain is given by $\frac{L}{L+1}$. However, this overhead asymptotically goes to zero as L increases. It is also necessary for D to wait for all $L+2$ transmissions to occur before performing decoding.

The transmitted signals at R1 and R2 in i-th time slot is generalized in [13], [14] and given respectively as

$$x_{r1}(i) = \begin{cases} g_{r1}(i)y_{r1}(i-1) & \text{for} \quad i=2, \\ g_{r1}(i)\Big[y_{r1}(i-1) - h_r g_{r2}(i-1)h_r x_{r1}(i-2)\Big] \\ \qquad\qquad \text{for} \quad i = 4,6,8,\ldots,L, \end{cases}$$

$$x_{r2}(i) = g_{r2}(i)y_{r2}(i-1) \quad \text{for} \quad i = 3,5,7,\ldots,L-1,$$

where the scaling factors

$$g_{r1}(i) = \begin{cases} \sqrt{\frac{\rho}{\gamma_{s,r1}^2\rho+\sigma^2}} & \text{for} \quad i=2, \\ \sqrt{\frac{\rho}{\gamma_{s,r1}^2\rho+\gamma_r^2 g_{r2}^2(i-1)\left(\gamma_{s,r2}^2\rho+\sigma^2\right)+\sigma^2}} \\ \qquad\qquad \text{for} \quad i=4,6,8,\ldots,L, \end{cases}$$

$$g_{r2}(i) = \sqrt{\frac{\rho}{\gamma_{s,r2}^2\rho+\gamma_r^2\rho+\sigma^2}} \quad \text{for} \quad i=3,5,7,\ldots,L-1.$$

Here, $y_{rj}(i)$ is the received signal at j-th relay in i-th time slot and σ^2 is the noise variance which is assumed to be equal for all nodes. We presume that R1 and R2 have the same average transmit power ρ, as S. Notice that $g_{r2}(i)$ is constant for $i = 3,5,\ldots,L-1$ as we have used the average channel gains instead of the instantaneous ones for normalization [16], [17]. The use of average channel gains is more practical and its performance is comparable to that using instantaneous channel gains [15]. Furthermore, $g_{r1}(i)$ which depends on $g_{r2}(i-1)$ is also constant for $i = 4,6,\ldots,L$. Hence, we can see that both $g_{r1}(i)$ and $g_{r2}(i)$ are constants for $i \geq 3$. Only the expression for $g_{r1}(2)$ is different, but this will not affect our analysis later where we consider the steady state performance of the system. Thus, we will drop the time index i hereafter for g_{r1} and g_{r2}.

For the transmission protocol described above, we can see that a few assumptions have been made. Firstly, perfect receive CSI at all receiving nodes is assumed[1]. In practice, robust channel estimation can be achieved through the transmission of training sequences. Furthermore, by ensuring that the training sequences transmitted by S and the relays[2] (R1, R2) are orthogonal, we will be able to achieve simultaneous estimation of $h_{s,rj}$, $j \in \{1,2\}$ and h_r by R1, R2, and simultaneous estimation of $h_{rj,d}$, $j \in \{1,2\}$ and $h_{s,d}$ at D.

We have also assumed that R1 has been informed of its responsibility to perform interference cancellation and it has the knowledge of $\gamma_{s,r2}^2$. In practice, both assumptions can be easily realized through the use of control messages carried in the header of each frame.

B. Equivalent Channel Model

Stacking the received signals at D across $L+2$ time slots into a vector $\mathbf{y}_d$, the equivalent channel matrix $\mathbf{H}$, as

[1]Transmit CSI is not available at the transmitters.

[2]R1 and R2 can use the same training sequence because, according to the two-path relaying protocol, they will not transmit at the same time.

shown in Appendix A, is a banded matrix with at most 4 non-zero entries in each row. This indicates that with inter-relay interference cancellation, the accumulation of inter-relay interference is at most across 4 time slots. Then, we have

$$\mathbf{y}_d = \sqrt{\rho}\,\mathbf{H}\mathbf{x} + \mathbf{n}_d + \mathbf{B}\mathbf{n}_r \tag{3}$$

where $\mathbf{y}_d \in \mathcal{C}^{L+2}$ is the received signal vector at D, $\mathbf{x} \in \mathcal{C}^L$ is the transmitted signal vector, $\mathbf{n}_r \in \mathcal{C}^{L+2}$ and $\mathbf{n}_d \in \mathcal{C}^{L+2}$ are the additive white Gaussian noise vectors at the relays and destination respectively.

The non-zero elements of $\mathbf{H} \in \mathcal{C}^{(L+2)\times L}$ and $\mathbf{B} \in \mathcal{C}^{(L+2)\times L}$ are given as

$$\mathbf{H}(m,m) = h_{s,d}, \quad m = 1,2,3,\dots,L,$$

$$\mathbf{H}(m,m-1) = \begin{cases} h_{s,r1}g_{r1}h_{r1,d}, & m = 2,4,\dots,L \\ h_{s,r2}g_{r2}h_{r2,d}, & m = 3,5,\dots,L+1, \end{cases}$$

$$\mathbf{H}(m,m-2) = \begin{cases} h_{s,r1}g_{r1}h_r g_{r2}h_{r2,d}, & m = 3,5,\dots,L+1 \\ h_{s,r2}g_{r2}h_r g_{r1}h_{r1,d}, & m = 4,6,\dots,L+2, \end{cases}$$

$$\mathbf{H}(m,m-3) = h_{s,r2}g_{r2}^2 h_r^2 g_{r1}h_{r2,d},\ m = 5,7,\dots,L+1,$$

and,

$$\mathbf{B}(m,m-1) = \begin{cases} g_{r1}h_{r1,d}, & m = 2,4,\dots,L \\ g_{r2}h_{r2,d}, & m = 3,5,\dots,L+1, \end{cases}$$

$$\mathbf{B}(m,m-2) = \begin{cases} g_{r1}h_r g_{r2}h_{r2,d}, & m = 3,5,\dots,L+1 \\ g_{r2}h_r g_{r1}h_{r1,d}, & m = 4,6,\dots,L+2, \end{cases}$$

$$\mathbf{B}(m,m-3) = g_{r2}^2 h_r^2 g_{r1}h_{r2,d},\ m = 5,7,\dots,L+1.$$

Notice that $\mathbf{B}\mathbf{n}_r$ is the forwarded noise from the relays and it is not white in general.

We can further express (3) as

$$\mathbf{y}_d = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{B}'\mathbf{n} \tag{4}$$

where $\mathbf{B}' = [\mathbf{B}\ \mathbf{I}_{L+2}]$, and $\mathbf{n} = \left[\mathbf{n}_r^{\mathrm{T}}\ \mathbf{n}_d^{\mathrm{T}}\right]^{\mathrm{T}}$.

From the canonical MIMO channel model in (4), we obtain the mutual information

$$\begin{aligned} I(\mathbf{x};\mathbf{y}_d) &= \log\det\left(\mathbf{I}_{L+2} + \rho\mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}}\left(\mathbf{B}'\mathbf{R}_{\mathbf{n}}\mathbf{B}'^{\mathrm{H}}\right)^{-1}\right) \\ &= \log\det\left(\mathbf{I}_{L+2} + \frac{\rho}{\sigma^2}\mathbf{H}\mathbf{H}^{\mathrm{H}}\left(\mathbf{B}\mathbf{B}^{\mathrm{H}} + \mathbf{I}_{L+2}\right)^{-1}\right) \end{aligned} \tag{5}$$

where $\mathbf{R}_{\mathbf{x}} = \mathbb{E}\left\{\mathbf{x}\mathbf{x}^{\mathrm{H}}\right\} = \mathbf{I}_L$, $\mathbf{R}_{\mathbf{n}} = \mathbb{E}\left\{\mathbf{n}\mathbf{n}^{\mathrm{H}}\right\} = \sigma^2\mathbf{I}_{2L+4}$. The derivation for the mutual information with correlated noise in (5) can be found in [18].

C. Motivation

In [12], [13], the bit error rate of AF two-path relaying scheme with inter-relay interference cancellation only shows a diversity of two, even though physically there are three independent paths from S to D. This observation motivates us to look into the DMT to analyze the potential diversity gains and investigate what kind of strategies will enable us to achieve the full DMT offered by the scheme.

III. Diversity-Multiplexing Tradeoff

A coding scheme $\{\mathcal{C}(\mathrm{SNR})\}$ is said to achieve multiplexing gain r and diversity gain d if

$$\lim_{\mathrm{SNR}\to\infty} \frac{R(\mathrm{SNR})}{\log \mathrm{SNR}} = r \tag{6}$$

and

$$\lim_{\mathrm{SNR}\to\infty} \frac{P_e(\mathrm{SNR})}{\log \mathrm{SNR}} = -d \tag{7}$$

where $P_e(\mathrm{SNR})$ is the average error probability with a maximum-likelihood (ML) receiver. $R(\mathrm{SNR})$ is the data rate measured by bits per channel use [3]. We will use SNR and $\frac{\rho}{\sigma^2}$ interchangeably in the analysis hereafter.

A. DMT Lower Bound for an Arbitrary Block Lower Triangular Channel Matrix

We first generalize the result in [15, Theorem 2.12] to find lower bounds for the DMT of an arbitrary block lower triangular channel matrix.

Definition 1: Let $\mathbf{A}$ be a block lower triangular matrix comprising of block matrices $\mathbf{A}_{ij} \in \mathcal{C}^{N_i\times N_j}$ where $i \geq j$. The l-th sub-diagonal matrix $\mathbf{A}^{(l)}$ of matrix $\mathbf{A}$ is defined as the matrix where

$$\mathbf{A}_{ij}^{(l)} = \begin{cases} \mathbf{A}_{ij} & \text{if} \quad i-j = l, \\ \mathbf{0} & \text{otherwise.} \end{cases} \tag{8}$$

The last sub-diagonal matrix is defined as $\mathbf{A}^{(k)}$ where k is the largest integer for which $\mathbf{A}^{(k)}$ is a non-zero matrix. We denote the DMT of channel matrix $\mathbf{A}$ as $d_A(r)$.

Theorem 3.1: Consider a random block lower triangular channel matrix $\mathbf{H}$ comprising of block matrices $\mathbf{H}_{ij} \in \mathcal{C}^{N_i\times N_j}$ where $i \geq j$ and $i,j \in \{1,2,\dots,N\}$. Then, we have

$$d_H(r) \geq d_{H^{(l)}}(r) \tag{9}$$

for $0 \leq l \leq k$, where $\mathbf{H}^{(k)}$ is the last sub-diagonal matrix.

Proof: In [15], the special cases where $l=0$ and $l=k$ have been proven. Here, we will use a similar method to prove (9) for any $0 \leq l \leq k$.

In general, the channel model is given by $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$. Without loss of generality, we assume that $\mathbf{w} \sim \mathcal{CN}(0,\mathbf{I})$ since the noise is white in the scale of interest [15, Theorem 2.3]. Let $\mathbf{x} = [\mathbf{x}_1^T\ \mathbf{x}_2^T\ \dots\ \mathbf{x}_N^T]^T$, $\mathbf{y} = [\mathbf{y}_1^T\ \mathbf{y}_2^T\ \dots\ \mathbf{y}_N^T]^T$, and $\mathbf{w} = [\mathbf{w}_1^T\ \mathbf{w}_2^T\ \dots\ \mathbf{w}_N^T]^T$ such that $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$ is expressed as (10). We further define $\mathbf{x}_i^N \triangleq [\mathbf{x}_i^T\ \mathbf{x}_{i+1}^T\ \dots\ \mathbf{x}_N^T]^T$.

For any instantaneous channel matrix H, the outage probability exponent is given by

$$\mathrm{SNR}^{-d_H(r)} \doteq \Pr\left\{I(\mathbf{x};\mathbf{y}:\mathbf{H}=H) \leq r\log\mathrm{SNR}\right\}. \tag{11}$$

Hence, to obtain the lower bound on the exponent, we will begin by identifying the lower bound for the mutual information.

[3]For ease of presentation, we sometimes denote $R(\mathrm{SNR})$ as R.

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{pmatrix} = \begin{pmatrix} \mathbf{H}_{11} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{H}_{21} & \mathbf{H}_{22} & \ddots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{H}_{N1} & \mathbf{H}_{N1} & \dots & \mathbf{H}_{NN} \end{pmatrix} \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{pmatrix} + \begin{pmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_N \end{pmatrix} \tag{10}$$

Thus,

$$\begin{aligned} I(\mathbf{x};\mathbf{y}|\mathbf{H}=H) &\overset{(a)}{=} \sum_{i=1}^{N} I\left(\mathbf{x}_i;\mathbf{y}|\mathbf{H}=H,\mathbf{x}_{i+1}^N\right) \\ &\overset{(b)}{\geq} \sum_{i=1}^{N-l} I\left(\mathbf{x}_i;\mathbf{y}|\mathbf{H}=H,\mathbf{x}_{i+1}^N\right) \\ &= \sum_{i=1}^{N-l} I\left(\mathbf{x}_i;\mathbf{H}\mathbf{x}+\mathbf{w}|\mathbf{H}=H,\mathbf{x}_{i+1}^N\right) \end{aligned} \tag{12}$$

where (a) is from chain rule and (b) is from the fact that $I \geq 0$.

Let $\widehat{\mathbf{H}} = \mathbf{H}^{(l)} + \mathbf{H}^{(l+1)} + \ldots + \mathbf{H}^{(k)}$; $\widetilde{\mathbf{H}}$ is formed by removing the first $\sum_{i=1}^{l} N_i$ all-zero rows and the last $\sum_{i=N-l+1}^{N} N_i$ all-zero columns of $\widehat{\mathbf{H}}$, $\widetilde{\mathbf{w}} = \mathbf{w}_{l+1}^N$, and $\widetilde{\mathbf{x}} = \mathbf{x}_1^{N-l}$. Then from (12),

$$\begin{aligned} I(\mathbf{x};\mathbf{y}|\mathbf{H}=H) &\geq \sum_{i=1}^{N-l} I\left(\mathbf{x}_i;\mathbf{H}\mathbf{x}+\mathbf{w}|\mathbf{H}=H,\mathbf{x}_{i+1}^N\right) \\ &\overset{(c)}{\geq} \sum_{i=1}^{N-l} I\left(\mathbf{x}_i;\widehat{\mathbf{H}}\mathbf{x}+\mathbf{w}|\mathbf{H}=H,\mathbf{x}_{i+1}^N\right) \\ &\overset{(d)}{=} I\left(\widetilde{\mathbf{x}};\widetilde{\mathbf{H}}\widetilde{\mathbf{x}}+\widetilde{\mathbf{w}}_i|\widetilde{\mathbf{H}}=\widetilde{H}\right) \\ &\overset{(e)}{\geq} I\left(\widetilde{\mathbf{x}};\widetilde{\mathbf{H}}^{(0)}\widetilde{\mathbf{x}}+\widetilde{\mathbf{w}}_i|\widetilde{\mathbf{H}}^{(0)}=\widetilde{H}^{(0)}\right) \\ &\overset{(f)}{=} I\left(\mathbf{x};\mathbf{H}^{(l)}\mathbf{x}+\mathbf{w}_i|\mathbf{H}^{(l)}=H^{(l)}\right) \end{aligned} \tag{13}$$

where (c) is from the fact that $\widehat{\mathbf{H}}\mathbf{x}$ contains less information than $\mathbf{H}\mathbf{x}$, (d) is from chain rule, (e) is from [15, Theorem 2.12], and (f) is obvious since the non-zero block matrices of $\widetilde{\mathbf{H}}^{(0)}$ are the same as $\mathbf{H}^{(l)}$.

Hence,

$$\begin{aligned} &\mathrm{SNR}^{-d_H(r)} \\ &\quad \doteq \Pr\left\{I(\mathbf{x};\mathbf{y}|\mathbf{H}=H) \leq r\log\mathrm{SNR}\right\} \\ &\quad \leq \Pr\left\{I\left(\mathbf{x};\mathbf{H}^{(l)}\mathbf{x}+\mathbf{w}_i|\mathbf{H}^{(l)}=H^{(l)}\right) \leq r\log\mathrm{SNR}\right\} \\ &\quad \doteq \mathrm{SNR}^{-d_{H^{(l)}}(r)} \\ &\Longrightarrow d_H(r) \geq d_{H^{(l)}}(r). \end{aligned} \tag{14}$$

■

Remark 1: Although Theorem 3.1 is only derived for block lower triangular matrices, we can also apply the theorem to arbitrary banded matrices which are not square in general. This can be proven by adding an appropriate number of all-zero rows to the top and all-zero columns to the right of the matrix.

Proposition 3.1: If the entries of $\mathbf{H}^{(l)}$ and $\mathbf{H}^{(l')}$ are independent where $l \neq l'$, $0 \leq l \leq k$, and $0 \leq l' \leq k$, then we have

$$d_H(r) \geq d_{H^{(l)}}(r) + d_{H^{(l')}}(r). \tag{15}$$

Proof: The proof is similar to the one in [15] for the case $l=0$ and $l'=k$. Since the entries of $\mathbf{H}^{(l)}$ and $\mathbf{H}^{(l')}$ are independent, we have

$$\begin{aligned} &\mathrm{SNR}^{-d_H(r)} \\ &\quad \doteq \Pr\left\{I(\mathbf{x};\mathbf{y}|\mathbf{H}=H) \leq r\log\mathrm{SNR}\right\} \\ &\quad \leq \Pr\Big\{\max\Big(I\left(\mathbf{x};\mathbf{H}^{(l)}\mathbf{x}+\mathbf{w}|\mathbf{H}=H\right), \\ &\qquad\quad I\left(\mathbf{x};\mathbf{H}^{(l')}\mathbf{x}+\mathbf{w}|\mathbf{H}=H\right)\Big) \leq r\log\mathrm{SNR}\Big\} \\ &\quad = \Pr\left\{I\left(\mathbf{x};\mathbf{H}^{(l)}\mathbf{x}+\mathbf{w}|\mathbf{H}=H\right) \leq r\log\mathrm{SNR}\right\} \\ &\qquad \times \Pr\left\{I\left(\mathbf{x};\mathbf{H}^{(l')}\mathbf{x}+\mathbf{w}|\mathbf{H}=H\right) \leq r\log\mathrm{SNR}\right\} \\ &\quad \doteq \mathrm{SNR}^{-d_{H^{(l)}}(r)}\mathrm{SNR}^{-d_{H^{(l')}}(r)}. \end{aligned} \tag{16}$$

Thus, $d(r) \geq d_{H^{(l)}}(r) + d_{H^{(l')}}(r)$. ■

B. DMT of AF Two-Path Relaying with Inter-Relay Self Interference Cancellation

From (5), the outage probability can be expressed as

$$\begin{aligned} &P_{\text{out}}(R) \\ &\quad = P_r\left\{I(\mathbf{x};\mathbf{y}_d) < (L+2)R\right\} \\ &\quad = P_r\Big\{\log\det\left(\mathbf{I}_{L+2} + \frac{\rho}{\sigma^2}\mathbf{H}\mathbf{H}^{\mathrm{H}}(\mathbf{B}\mathbf{B}^{\mathrm{H}}+\mathbf{I}_{L+2})^{-1}\right) \\ &\qquad < (L+2)R\Big\} \\ &\quad \doteq P_r\left\{\log\det\left(\mathbf{I}_{L+2} + \frac{\rho}{\sigma^2}\mathbf{H}\mathbf{H}^{\mathrm{H}}\right) < (L+2)R\right\} \\ &\quad \doteq \mathrm{SNR}^{-d(r)} \end{aligned} \tag{17}$$

since we are only interested in the exponent function of $P_{\text{out}}(R)$ and the DMT is white in the scale of interest[4][15]. The factor $L+2$ is to account for the fact that $L+2$ transmission slots are required to transmit L symbols.

Theorem 3.2: The diversity-multiplexing tradeoff for the AF two-path relaying scheme with one of the relays additionally performing inter-relay interference cancellation in a slow fading channel, where the channel coefficients remain constant for $L+2$ time slots, is given by

$$d(r) = 3\left(1 - \frac{L+2}{L}r\right)^+. \tag{18}$$

Proof: We prove by showing in Appendix B that RHS of (18) is both an upper and a lower bound for the DMT of the AF two-path relaying scheme, and thus the DMT is given by (18). ■

[4]The colored noise encountered in cooperative networks can be treated as a white noise for DMT computations.

From DMT expression in (18), it is clear to see that AF two-path relaying with inter-relay self interference cancellation achieves maximum diversity gain of three and maximum multiplexing gain of $\frac{L}{L+2}$.

For a N-relay cooperative diversity scheme, the DMT is upper bounded by the MISO transmit diversity upper bound which is given by [2]

$$d^*(r) = (N+1)(1-r)^+ . \tag{19}$$

Thus, we can see that for $N = 2$, AF two-path relaying scheme asymptotically achieves this upper bound as $L \to \infty$.

C. Achievability of the DMT

The outage formulation used to derive the DMT is based on the existence of *universal codes* [19] that can achieve arbitrarily small error probability whenever the channel is not in outage, i.e. $P_e \approx P_{out}$. However, to achieve such performance, arbitrarily long block lengths and powerful codes are usually required. In this subsection, we will show that the DMT of the AF two-path relaying scheme with inter-relay interference cancellation in Theorem 3.2 can indeed be achieved with a finite codeword length. Practical coding strategies to achieve the DMT have not been discussed before in previous works [5], [10], [11].

In [12], [13], it was shown that AF two-path relaying with inter-relay interference cancellation only achieves a maximum diversity order of two. However, Theorem 3.2 suggests that potentially, a diversity order of three can be achieved while maintaining a spatial multiplexing gain of one for large L. To provide insights to this phenomenon, we will first express (3) as

$$\mathbf{y}_d = \sqrt{\rho}\mathbf{X}\mathbf{h} + \mathbf{w} \tag{20}$$

where

$$\mathbf{X} = \begin{bmatrix} x(1) & 0 & 0 & \cdots & \cdots & 0 \\ x(2) & x(1) & 0 & 0 & \ddots & \vdots \\ x(3) & 0 & x(1) & x(2) & 0 & \vdots \\ x(4) & x(3) & 0 & 0 & x(2) & 0 \\ x(5) & 0 & x(3) & x(4) & 0 & x(2) \\ \vdots & \ddots & \cdots & \cdots & \ddots & \ddots \\ x(L) & \cdots & \cdots & \cdots & \cdots & \vdots \end{bmatrix}, \tag{21}$$

$$\mathbf{h} = \begin{bmatrix} h_{s,d} \\ h_{r1,d}\, g_{r1}\, h_{s,r1} \\ h_{r2,d}\, g_{r2}\, h_r\, g_{r1}\, h_{s,r1} \\ h_{r2,d}\, g_{r2}\, h_{s,r2} \\ h_{r1,d}\, g_{r1}\, h_r\, g_{r2}\, h_{s,r2} \\ h_{r2,d}\, g_{r2}\, h_r\, g_{r1}\, h_r\, g_{r2}\, h_{s,r2} \end{bmatrix}, \tag{22}$$

and $\mathbf{w} \triangleq \mathbf{B}\mathbf{n}_r + \mathbf{n}_d$, conditioned on $\mathbf{h}$, is the correlated Gaussian noise with covariance matrix $\mathbf{\Sigma}_W = \sigma^2\left(\mathbf{B}\mathbf{B}^H + \mathbf{I}\right)$. To be precise, the non-zero elements of $\mathbf{X}$ are given as

$$\begin{aligned} &\mathbf{X}(m,1) = \mathbf{X}(m+1,2) = \mathbf{X}(m+2,3) = x(m) \\ &\quad \text{for} \quad m = 1,3,5,\ldots,L-1, \\ &\mathbf{X}(m,1) = \mathbf{X}(m+1,4) = \mathbf{X}(m+2,5) \\ &\quad = \mathbf{X}(m+3,6) = x(m) \ \text{ for } \ m = 2,4,6,\ldots,L. \end{aligned} \tag{23}$$

Thus, we can view $\mathbf{X}$ as the equivalent space time code for $\mathbf{x}$. With this equivalent space time code expression, the reason for failing to achieve the full diversity order of three can then be attributed to the limitations in the code design resulting from the transmission scheme. However, by using a simple linear coding strategy at S, we shall show that the full DMT of the AF two-path relaying scheme can indeed be achieved.

Consider at data rate $R = r\log\text{SNR}$ (bits/symbol), for finite codeword lengths we have[5] [3],

$$\begin{aligned} &P_e(\text{SNR}) \\ &\quad = P_{\text{out}}(R)P(\text{error}|\text{outage}) + P(\text{error}, \text{no outage}) \\ &\quad \le P_{\text{out}}(R) + P(\text{error}, \text{no outage}) \\ &\quad \approx P_{\text{out}}(R) + P^*(\text{error}, \text{no outage}) \\ &\quad \le P_{\text{out}}(R) + P^*(\text{error}) \\ &\quad \dot{\le} \text{SNR}^{-d(r)} + \text{SNR}^{-d_{\text{G}}(r)} \end{aligned} \tag{24}$$

where $P^*(\text{error}) \doteq \text{SNR}^{-d_{\text{G}}(r)}$ is the probability of the dominant error in the high SNR regime. This dominant error event occurs when only one data symbol is different between two space time codewords.

$P^*(\text{error})$ can be determined from the pairwise error probability (PEP) in [13], [14], where we have

$$\begin{aligned} &P\left(\mathbf{X}_k \to \mathbf{X}_l\right) \\ &\le \mathbb{E}_{\mathbf{h}}\left\{e^{-\frac{\rho}{4\,\text{tr}(\mathbf{\Sigma}_W)}\mathbf{h}^{\text{H}}(\mathbf{X}_k-\mathbf{X}_l)^{\text{H}}(\mathbf{X}_k-\mathbf{X}_l)\mathbf{h}}\right\} \\ &= \mathbb{E}_{\mathbf{T}}\left\{\int \frac{1}{\pi^3} e^{-\frac{\rho}{4\,\text{tr}(\mathbf{\Sigma}_W)}\mathbf{v}^{\text{H}}\mathbf{C}^{\text{H}}\mathbf{T}^{\text{H}}(\mathbf{X}_k-\mathbf{X}_l)^{\text{H}}(\mathbf{X}_k-\mathbf{X}_l)\mathbf{T}\mathbf{C}\mathbf{v}} \right. \\ &\qquad \left. \times\ e^{-\mathbf{v}^{\text{H}}\mathbf{v}}\text{d}\mathbf{v}\right\} \\ &= \mathbb{E}_{\mathbf{T}}\left\{\det{}^{-1}\left(\mathbf{I} + \frac{\rho}{4\,\text{tr}(\mathbf{\Sigma}_W)}\mathbf{A}^{\text{H}}\mathbf{A}\right)\right\} \\ &\le \mathbb{E}_{\mathbf{T}}\left\{\left(\prod_{i=1}^{\alpha}\lambda_i[\mathbf{A}^{\text{H}}\mathbf{A}]\right)^{-1}\left(\frac{\rho}{4\,\text{tr}\left(\mathbf{\Sigma}_W\right)}\right)^{-\alpha}\right\} \\ &\doteq \text{SNR}^{-\min\limits_{\mathbf{X}_k-\mathbf{X}_l}(\text{rank}(\mathbf{A}))}. \end{aligned} \tag{26}$$

Here, $\mathbf{h} = \mathbf{T}\mathbf{C}\mathbf{v}$, $\mathbf{v} = [h_{s,d}, h_{s,r1}, h_{s,r2}]^{\text{T}}$, $\mathbf{T} = \text{diag}\left(\begin{bmatrix} 1 \\ h_{r1,d}\ g_{r1} \\ h_{r2,d}\ h_r\ g_{r1}\ g_{r2} \\ h_{r2,d}\ g_{r2} \\ h_{r1,d}\ h_r\ g_{r1}\ g_{r2} \\ h_{r2,d}\ h_r^2\ g_{r1}\ g_{r2}^2 \end{bmatrix}\right)$ is a full rank matrix, $\mathbf{C} = \begin{bmatrix} 1&0&0&0&0&0 \\ 0&1&1&0&0&0 \\ 0&0&0&1&1&1 \end{bmatrix}^{\text{T}}$ is a rank 3 matrix, $\mathbf{A} = (\mathbf{X}_k - \mathbf{X}_l)\,\mathbf{T}\mathbf{C}$, and α is the minimum rank of $\mathbf{A}$.

From (26) and by taking the union bound across all space time codewords which differ by one symbol, we have

$$\begin{aligned} P^*(\text{error}) &\dot{\le} \left(L\,2^{R_s} - 1\right)P\left(\mathbf{X}_k \to \mathbf{X}_l\right) \\ &\dot{\le} 2^{R_s}P\left(\mathbf{X}_k \to \mathbf{X}_l\right) \\ &\dot{\le} \text{SNR}^{\frac{L+2}{L}r}\text{SNR}^{-\min\limits_{\mathbf{X}_k-\mathbf{X}_l}(\text{rank}(\mathbf{A}))} \\ &\doteq \text{SNR}^{-\left(\min\limits_{\mathbf{X}_k-\mathbf{X}_l}(\text{rank}(\mathbf{A})) - \frac{L+2}{L}r\right)} \end{aligned} \tag{27}$$

where each $x(l)$ is transmitted with data rate $R_s = \frac{L+2}{L}R$ bits in each transmission slot.

[5] $P(\text{error}, \text{no outage})$ cannot be ignored for finite codeword lengths.

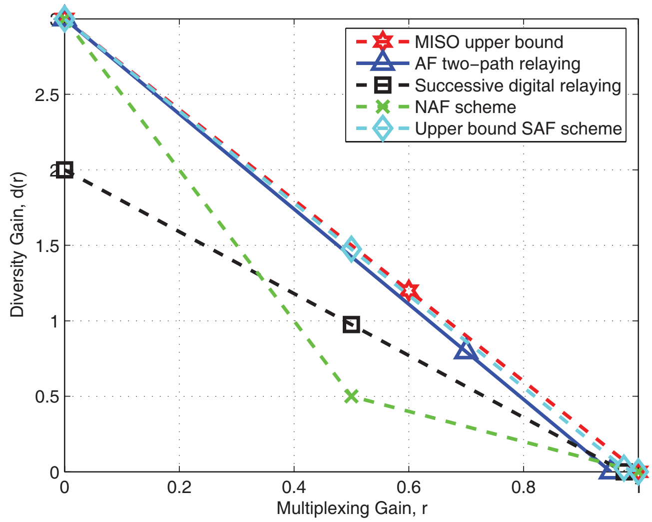

Fig. 2. Diversity-multiplexing tradeoff for two-relay topology. Here, $L = M = 40$.

Thus, we obtain

$$d_{\mathrm{G}}(r) = \left(\min_{\mathbf{X}_k - \mathbf{X}_l} (\mathrm{rank}\,(\mathbf{A})) - \frac{L+2}{L} r\right)^+ \tag{28}$$

and from (24) we have

$$\begin{aligned} P_e(\mathrm{SNR}) &\dot{\le} \mathrm{SNR}^{-d(r)} + \mathrm{SNR}^{-d_{\mathrm{G}}(r)} \\ &= \mathrm{SNR}^{-d(r)} + \mathrm{SNR}^{-\left(\min_{\mathbf{X}_k - \mathbf{X}_l} (\mathrm{rank}(\mathbf{A})) - \frac{L+2}{L} r\right)^+}. \end{aligned} \tag{29}$$

As explained in [12], [13], the transmission protocol causes $\min_{\mathbf{X}_k - \mathbf{X}_l} (\mathrm{rank}\,(\mathbf{A}))$ in (29) to be limited to 2, which will cause $\mathrm{SNR}^{-d_G(r)}$ to dominate over $\mathrm{SNR}^{-d(r)}$. Thus, only a maximum diversity order of two is observed as shown in [12], [13].

In [14, Theorem 1], which is reproduced in Appendix C for ease of reference, we have shown that $\min_{\mathbf{X}_k - \mathbf{X}_l} (\mathrm{rank}\,(\mathbf{A}))$ can be improved to 3 by using an equivalent linear space time code at S. With this linear space time coding, (3) and (20) can be expressed as

$$\mathbf{y}_d = \sqrt{\rho}\mathbf{H}\mathbf{U}\mathbf{x} + \mathbf{w} = \sqrt{\rho}\widehat{\mathbf{X}}\mathbf{h} + \mathbf{w} \tag{30}$$

where $\mathbf{U}$ is the precoding matrix and $\widehat{\mathbf{X}}$ is the corresponding equivalent space time codeword. The precoding matrix $\mathbf{U}$ only has to satisfy the following two conditions:

- It is an arbitrary complex unitary matrix.
- It must satisfy [14, Theorem 1] in order to ensure $\min_{\mathbf{X}_k - \mathbf{X}_l} (\mathrm{rank}\,(\mathbf{A})) = 3$.

By using this linear space time code at S [14], we have $d_{\mathrm{G}}(r) = \left(3 - \frac{L+2}{L} r\right)^+$ and

$$\begin{aligned} P_e\,(\mathrm{SNR}) &\dot{\le} \mathrm{SNR}^{-d(r)} + \mathrm{SNR}^{-d_{\mathrm{G}}(r)} \\ &\dot{\le} \mathrm{SNR}^{-d(r)} \end{aligned} \tag{31}$$

as $d_{\mathrm{G}}(r) \ge d(r)$. Hence, the full DMT of the AF two-path relaying scheme in Theorem 3.2 is achieved.

D. Discussion

1) In Fig. 2, we compare the DMT of the proposed AF two-path relaying scheme with other cooperative schemes, such as successive digital relaying [5], Non-Orthogonal Amplify-and-Forward (NAF) scheme [20], and DMT upper bound of SAF scheme [10] for the two-relay topology. For the two-relay case, DMT for NAF scheme is given in [20] as

$$d_{\mathrm{NAF}}(r) = (1-r) + 2\,(1-2r)^+. \tag{32}$$

It can be observed that AF two-path relaying outperforms both successive digital relaying and NAF scheme. AF two-path relaying is also very close to the DMT upper bound of SAF scheme. The slight difference is due to the fact that we have an additional two time slots for the finite codeword length design. For large values of L, the DMT of AF two-path relaying will approach the MISO upper bound.

2) *Proposition 3.2:* The DMT of AF two-path relaying *without* interference cancellation at one of the relays is also given by

$$d(r) = 3\left(1 - \frac{L+2}{L} r\right)^+. \tag{33}$$

Proof: Please refer to Appendix D. ■

Both two-path relaying with and without interference cancellation achieve the same DMT. This result is intuitively satisfying since DMT is computed in the high SNR regime and thus the effects of inter-relay interference is negligible[6]. However, this does not mean that inter-relay interference cancellation is not necessary. With practical SNR values, the accumulation of interference will degrade the overall performance of two-path relaying without cancellation, as shown in [13], [14]. Interference cancellation at one of the relays is required to reduce the accumulation of inter-relay interference at the destination.

3) AF two-path relaying *without* interference cancellation at one of the relays can also be considered as the two-relay case of SAF scheme in [10]. We can extend the result in Proposition 3.2 to the multiple-relay case considered in [10].

Proposition 3.3: For the multiple-relay case, the DMT of N-relay M-slot SAF scheme in [10], is indeed given as

$$d_{\mathrm{SAF}}(r) = (1-r)^+ + N\left(1 - \frac{M}{M-1} r\right)^+. \tag{34}$$

Proof: Please see Appendix E. ■

In [10], the DMT of SAF scheme is derived under the assumption that the relays are isolated, i.e. there is no inter-relay interference. By using Theorem 3.1, we have proven a stronger result in Proposition 3.3 that the DMT of SAF scheme meets the genie-aided upper bound in [10], while taking into account the inter-relay interference. The assumption of relay isolation in [10] is not necessary to achieve the same DMT performance.

[6] Unlike decode-and-forward protocol, in amplify-and-forward protocol the relay does not treat the interference as 'interference'.

IV. Conclusion

In this paper, an amplify-and-forward (AF) two-path relaying scheme is considered in which one of the relays additionally performs inter-relay interference cancellation. We first generalized lower bounds for the diversity multiplexing tradeoff (DMT) of an arbitrary block lower triangular channel matrix. The DMT was then derived and it was shown that the DMT of this scheme is achievable for finite codeword lengths by using a simple equivalent linear space time code at the source. The precoding matrix of this linear space time code does not require any form of channel state information. From the DMT perspective, we have shown that the proposed AF two-path relaying scheme is optimal as the DMT achieves the MISO upper bound. We then extended our analysis to the slotted-amplify-and-forward (SAF) scheme [8] with multiple relays, where we provided a stronger result by deriving the DMT while taking into account inter-relay interference.

Appendix A
Equivalent Channel Matrix

For $L = 6$, the channel matrix $\mathbf{H}$ is expressed as

$$\mathbf{H} = \begin{pmatrix} h_1 & 0 & 0 & 0 & 0 & 0 \\ h_2 & h_1 & 0 & 0 & 0 & 0 \\ h_3 & h_4 & h_1 & 0 & 0 & 0 \\ 0 & h_5 & h_2 & h_1 & 0 & 0 \\ 0 & h_6 & h_3 & h_4 & h_1 & 0 \\ 0 & 0 & 0 & h_5 & h_2 & h_1 \\ 0 & 0 & 0 & h_6 & h_3 & h_4 \\ 0 & 0 & 0 & 0 & 0 & h_5 \end{pmatrix} \tag{35}$$

where

$$\begin{aligned} h_1 &= h_{s,d}, \\ h_2 &= h_{s,r1} g_{r1} h_{r1,d}, \\ h_3 &= h_{s,r1} g_{r2} h_r g_{r1} h_{r2,d}, \\ h_4 &= h_{s,r2} g_{r2} h_{r2,d}, \\ h_5 &= h_{s,r2} g_{r2} h_r g_{r1} h_{r1,d}, \\ h_6 &= h_{s,r2} g_{r2}^2 h_r^2 g_{r1} h_{r2,d}. \end{aligned} \tag{36}$$

Appendix B
Proof of Theorem 3.2

Upper Bound for the DMT

Lemma B.1: [20, Definition 5] Let $v = -\lim_{\rho\to\infty} \frac{\log(|g|^2)}{\log(\rho)}$ be the exponential order of $1/|g|^2$, where $g \sim \mathcal{CN}(0,1)$. Then the probability density function (pdf) of v can be shown to be

$$\begin{aligned} p_v &= \lim_{\rho\to\infty} \ln(\rho)\rho^{-v}\exp(-\rho^{-v}) \\ &\doteq \begin{cases} \rho^{-\infty} = 0 & \text{for } \; v < 0 \\ \rho^{-v} & \text{for } \; v \geq 0 \end{cases}. \end{aligned} \tag{37}$$

Thus, the probability P_O that $(v_1, v_2, \ldots, v_N)$ belongs to set O can be characterized by

$$P_O \doteq \rho^{-d_o}, \quad \text{for } \; d_o = \inf_{(v_1,v_2,\ldots,v_N)\in O^+} \sum_{i=1}^{N} v_i \tag{38}$$

where $\{v_i\}_{i=1}^N$ are independent and identically distributed random variables and provided that O^+ is not empty.

Proof: This is true because $\{v_i\}_{i=1}^N$ are independent and the outage exponent will be determined by the largest SNR exponent. ■

Lemma B.2: Given a positive semidefinite matrix $\mathbf{H}^{\mathrm{H}}\mathbf{H}$, we have

$$\det\left(\frac{\rho}{\sigma^2}\mathbf{H}^{\mathrm{H}}\mathbf{H}\right) \leq \det\left(\frac{\rho}{\sigma^2}\mathrm{tr}\left(\mathbf{H}^{\mathrm{H}}\mathbf{H}\right)\mathbf{I}_L\right). \tag{39}$$

Proof: The proof is straightforward and is thus omitted. ■

From (17), we have

$$P_{\text{out}}(R) \doteq P_r\left\{\log\det\left(\mathbf{I}_L + \frac{\rho}{\sigma^2}\mathbf{H}^{\mathrm{H}}\mathbf{H}\right) < (L+2)R\right\}$$

since $\det\left(\mathbf{I}_{L+2} + \frac{\rho}{\sigma^2}\mathbf{H}\mathbf{H}^{\mathrm{H}}\right) = \det\left(\mathbf{I}_L + \frac{\rho}{\sigma^2}\mathbf{H}^{\mathrm{H}}\mathbf{H}\right)$.

As $\rho \to \infty$,

$$P_{\text{out}}(R) \doteq P_r\left\{\log\det\left(\frac{\rho}{\sigma^2}\mathbf{H}^{\mathrm{H}}\mathbf{H}\right) < (L+2)R\right\}.$$

Using Lemma B.2, we obtain

$$\begin{aligned} P_{\text{out}}(R) &\doteq \text{SNR}^{-d(r)} \\ &\doteq P_r\left\{\log\det\left(\frac{\rho}{\sigma^2}\mathbf{H}^{\mathrm{H}}\mathbf{H}\right) < (L+2)R\right\} \\ &\geq P_r\left\{\log\det\left(\frac{\rho}{\sigma^2}\mathrm{tr}\left(\mathbf{H}^{\mathrm{H}}\mathbf{H}\right)\mathbf{I}_L\right) < (L+2)R\right\} \\ &\doteq \text{SNR}^{-d_{\text{UB}}(r)} \end{aligned} \tag{40}$$

where $d_{\text{UB}}(r)$ is the DMT upper bound.

We denote h_i, $1 \leq i \leq 6$, as the elements of $\mathbf{H}$, as shown in Appendix A. Continuing from the RHS of (40), we have

$$\begin{aligned} &P_r\left\{\log\det\left(\frac{\rho}{\sigma^2}\mathrm{tr}\left(\mathbf{H}^{\mathrm{H}}\mathbf{H}\right)\mathbf{I}_L\right) < (L+2)R\right\} \\ &\approx P_r\left\{\log\left(\frac{\rho L}{2\sigma^2}\left(|h_1|^2 + \sum_{i=1}^{6}|h_i|^2\right)\right)^L \right. \\ &\qquad \left. < (L+2)R\right\} \\ &\doteq P_r\left\{\log\left(\text{SNR}\sum_{i=1}^{6}|h_i|^2\right) < \frac{L+2}{L} r \log \text{SNR}\right\} \\ &\doteq \text{SNR}^{-d_{\text{UB}}(r)}. \end{aligned} \tag{41}$$

Thus for large SNR, the outage events will be dominated by (42) where v_x is the exponential order of the corresponding channel coefficient.

From Lemma B.1, the DMT upper bound can be expressed as

$$d_{\text{UB}}(r) = \inf_{O^+}\left(v_{s,d} + v_{s,r_1} + v_{s,r_2} + v_{r_1,d} + v_{r_2,d} + v_r\right) \tag{43}$$

which can be solved by summing the first five inequalities[7] in (42). By simplifying the expression, we obtain

$$d_{\text{UB}}(r) = 3\left(1 - \frac{L+2}{L} r\right)^+. \tag{44}$$

Hence, we have

$$d(r) \leq d_{\text{UB}}(r) = 3\left(1 - \frac{L+2}{L} r\right)^+. \tag{45}$$

[7] $\max(a,b) < c \Longrightarrow a < c, b < c$

$$O^{+}=\left\{\left(\begin{array}{c}v_{s,d},v_{s,r_1},v_{s,r_2},\\ v_{r_1,d},v_{r_2,d},v_r\end{array}\right)\in\mathbb{R}^{6+}\,\middle|\,\max\left(\begin{array}{c}(1-v_{s,d})^{+},\\ (1-v_{r_1,d}-v_{s,r_1})^{+},\\ (1-v_{r_2,d}-v_r-v_{s,r_1})^{+},\\ (1-v_{r_2,d}-v_{s,r_2})^{+},\\ (1-v_{r_1,d}-v_r-v_{s,r_2})^{+},\\ (1-v_{r_2,d}-2v_r-v_{s,r_2})^{+}\end{array}\right)<\frac{L+2}{L}r\right\} \tag{42}$$

Lower Bound for the DMT

For AF two-path relaying scheme with inter-relay interference cancellation, we can add two all-zero columns to the right of the channel matrix $\mathbf{H}$ to form a block lower triangular matrix. From Theorem 3.1 and Proposition 3.1, we obtain $d(r)\geq d_{H^{(0)}}(r)+d_{H^{(1)}}(r)=d_{\mathrm{LB}}(r)$ where $d_{\mathrm{LB}}(r)$ is the DMT lower bound. The DMT of the matrices $\mathbf{H}^{(0)}$ and $\mathbf{H}^{(1)}$ can be easily derived as $d_{H^{(0)}}(r)=(1-\frac{L+2}{L}r)^{+}$ and $d_{H^{(1)}}(r)=2(1-\frac{L+2}{L}r)^{+}$ respectively. Thus, we obtain

$$d_{\mathrm{LB}}(r)=3\left(1-\frac{L+2}{L}r\right)^{+}=d_{\mathrm{UB}}(r) \tag{46}$$

and this completes the proof for Theorem 3.2.

Appendix C
Theorem 1 in [14]

AF two-path relaying with inter-relay interference cancellation with an precoding matrix $\mathbf{U}$ achieves full diversity order of three, if the space time code difference matrix $\Delta\widehat{\mathbf{X}}=(\widehat{\mathbf{X}}_k-\widehat{\mathbf{X}}_l)$ is a full rank matrix for all $\widehat{\mathbf{X}}_k$ and $\widehat{\mathbf{X}}_l$. In other words, it is sufficient that (47), where $\Delta\mathbf{x}=\mathbf{x}_k-\mathbf{x}_l$, is full rank in order to achieve the full diversity of three.

Appendix D
Proof of Proposition 3.2

Let $\bar{\mathbf{H}}\in\mathcal{C}^{(L+2)\times L}$ be the equivalent MIMO channel matrix of AF two-path relaying *without* interference cancellation at one of the relays. First, notice that the elements in the columns of $\bar{\mathbf{H}}$ are just the repeated elements in the first two columns. The first two columns of $\bar{\mathbf{H}}$ can be shown as

$$\bar{\mathbf{H}}=\begin{pmatrix} h_1=h_{s,d} & 0 & \cdots\\ h_2=h_{s,r1}h_{r1,d} & h_1 & \cdots\\ h_3=h_{s,r1}h_r h_{r2,d} & h_4=h_{s,r2}g_{r2}h_{r2,d} & \cdots\\ h_7=h_{s,r1}h_r^2 h_{r1,d} & h_5=h_{s,r2}h_r h_{r1,d} & \cdots\\ h_8=h_{s,r1}h_r^3 h_{r2,d} & h_6=h_{s,r2}h_r^2 h_{r2,d} & \cdots\\ h_9=h_{s,r1}h_r^4 h_{r1,d} & h_{11}=h_{s,r2}h_r^3 h_{r1,d} & \cdots\\ h_{10}=h_{s,r1}h_r^5 h_{r2,d} & h_{12}=h_{s,r2}h_r^4 h_{r2,d} & \cdots\\ \vdots & \vdots & \cdots \end{pmatrix}$$

where g_{r1} and g_{r2} terms are ignored in the above matrix because they are constants and will not affect the exponential order analysis i.e. $\lim_{\mathrm{SNR}\to\infty}\frac{\log g_{ri}}{\log\mathrm{SNR}}=0,\ i\in\{1,2\}$.

The DMT can be derived easily by following closely the DMT derivation in Appendix B. The upper bound of the outage probability can be derived as $\mathrm{SNR}^{-3\left(1-\frac{L+2}{L}r\right)^{+}}$ by observing that the exponent of $\mathrm{SNR}\sum_{i=1}^{6}|h_i|^2$ will dominate in $\mathrm{SNRtr}\left\{\bar{\mathbf{H}}^H\bar{\mathbf{H}}\right\}$, i.e.

$$\frac{\log\left\{\mathrm{SNRtr}\left\{\bar{\mathbf{H}}^H\bar{\mathbf{H}}\right\}\right\}}{\log\mathrm{SNR}}\doteq\frac{\log\left\{\mathrm{SNR}\sum_{i=1}^{6}|h_i|^2\right\}}{\log\mathrm{SNR}}$$

which is a similar step involved in (41), Appendix B.

Notice that the main and the first sub-diagonal elements of $\bar{\mathbf{H}}$ is exactly the same as $\mathbf{H}$, which corresponds to the case with interference cancellation. Thus, for the lower bound, we have

$$\begin{aligned} d_{\bar{H}}(r)&\geq d_{\bar{H}^{(0)}}(r)+d_{\bar{H}^{(1)}}(r)=d_{H^{(0)}}(r)+d_{H^{(1)}}(r)\\ &=3\left(1-\frac{L+2}{L}r\right)^{+}. \end{aligned} \tag{48}$$

Hence, the DMT is given by $3\left(1-\frac{L+2}{L}r\right)^{+}$.

Appendix E
Proof of Proposition 3.3

First, consider $\check{\mathbf{H}}$ as the channel matrix of the SAF scheme in [10]. With the assumption that the relays are isolated, $\check{\mathbf{H}}$ will have non-zero elements only at the main and first sub-diagonal. With the relay isolation assumption, [10] has shown that the DMT in (34) is achievable. Now, if we remove the assumption of relay isolation, i.e. inter-relay interference is considered, the only change in $\check{\mathbf{H}}$ is that all the entries in the lower triangular part of $\check{\mathbf{H}}$ will be non-zeros. However, the main and first sub-diagonal elements will remain the same as the case with the relay isolation assumption.

Hence, by using Theorem 3.1, the lower bounds for the DMT are given by the DMT of $\check{\mathbf{H}}^{(0)}$ and $\check{\mathbf{H}}^{(1)}$, which are taken from the main and the first-sub diagonal of $\check{\mathbf{H}}$ respectively. It is easy to compute that $d_{\check{H}^{(0)}}(r)=(1-r)^{+}$ and $d_{\check{H}^{(1)}}(r)=N\left(1-\frac{M}{M-1}r\right)^{+}$. Since $\check{\mathbf{H}}^{(0)}$ and $\check{\mathbf{H}}^{(1)}$ are independent, from Proposition 3.1, we can see that the sum of the lower bounds $d_{\check{H}^{(0)}}(r)+d_{\check{H}^{(1)}}(r)$ coincides with the SAF DMT upper bound in [10], and this completes the proof.

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$$\Delta\widehat{\mathbf{X}} = \begin{bmatrix} \mathbf{U}(1,:)\Delta\mathbf{x} & 0 & 0 & \cdots & \cdots & 0 \\ \mathbf{U}(2,:)\Delta\mathbf{x} & \mathbf{U}(1,:)\Delta\mathbf{x} & 0 & 0 & \ddots & \vdots \\ \mathbf{U}(3,:)\Delta\mathbf{x} & 0 & \mathbf{U}(1,:)\Delta\mathbf{x} & \mathbf{U}(2,:)\Delta\mathbf{x} & 0 & \vdots \\ \mathbf{U}(4,:)\Delta\mathbf{x} & \mathbf{U}(3,:)\Delta\mathbf{x} & 0 & 0 & \mathbf{U}(2,:)\Delta\mathbf{x} & 0 \\ \mathbf{U}(5,:)\Delta\mathbf{x} & 0 & \mathbf{U}(3,:)\Delta\mathbf{x} & \mathbf{U}(4,:)\Delta\mathbf{x} & 0 & \mathbf{U}(2,:)\Delta\mathbf{x} \\ \vdots & \ddots & \cdots & \cdots & \ddots & \ddots \\ \mathbf{U}(L,:)\Delta\mathbf{x} & \cdots & \cdots & \cdots & \cdots & \vdots \end{bmatrix} \quad (47)$$

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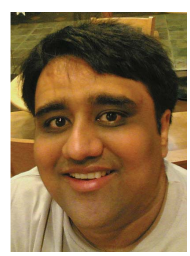

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