Optimal Energy Allocation for Wireless Communications Powered by Energy Harvesters

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***Abstract*—We consider the use of energy harvesters, in place of conventional batteries with fixed energy storage, for point-to-point wireless communications. In addition to the challenge of transmitting in a channel with time selective fading, energy harvesters provide a perpetual but unreliable energy source. In this paper, we consider the problem of energy allocation over a finite horizon, taking into account a time varying channel and energy source, so as to maximize the throughput. Two types of side information are assumed to be available: causal side information of the immediate past channel condition and harvested energy, and full side information. We obtain structural results for the optimal energy allocation, via the use of dynamic programming and convex optimization techniques. In particular, if unlimited energy can be stored in the battery with harvested energy, we prove the optimality of a water-filling energy allocation solution with multiple, non-decreasing water levels.**

I. Introduction

In conventional wireless communications, transmissions are limited by power constraints for safety reasons, or by energy constraints to prolong operating time for battery-powered devices. Energy harvesters provide viable energy sources for low-power sensors to wirelessly communicate their data to a sink node. For transmitters that are powered by energy harvesters, the energy that can potentially be harvested is unlimited, and hence the sum energy constraint does not directly apply. Another difference with traditional communication system is that the energy available for each transmission varies over time: energy is consumed for transmission but is replenished by the harvested energy.

Several contributions in the literature have considered using energy harvester as an energy source, in particular based on the technique of dynamic programming [1]. In [2], the problem of maximizing a reward that is linear with the energy used is studied. In [3], the discounted throughput is maximized over an infinite horizon, where queuing for data is also considered. In [4], adaptive duty cycling is employed for throughput maximization. Several solutions are implemented in practical systems, which are benchmarked against the optimal one.

In this work, we consider the problem of maximizing the throughput via energy allocation over a finite horizon of $K < \infty$ slots. The channel SNR and the energy harvested changes over different slots, where the variation is modeled by a first-order Markov model. Our aim is to study the structure of the maximum throughput and the corresponding optimal energy allocation solution, such as concavity and monotonicity. These results may be useful for developing heuristic solutions, since the optimal solutions are often complex to obtain in practice. We consider two types of side information (SI) available to the transmitter:

- *causal SI*, consisting of past channel conditions, in terms of SNR, and past amount of energy harvested, or
- *full SI*, consisting of past, present and future channel conditions and amount of energy harvested.

The case of full SI may be justified if the environment is highly predictable, e.g., the energy is harvested from the vibration of motors that are turned on only during fixed operating hours and line-of-sight is available for communications. Given causal SI, we obtain the optimal energy allocation solution by dynamic programming and obtain structural results to characterize the optimal solution. Given full SI, we obtain a closed-form solution for $K = 2$ slots. We also obtain the structure of this optimal solution for arbitrary K with unlimited energy storage. The optimal solution then has a water-filling interpretation, as in [5]. However, instead of a single water level, there are multiple water levels that are non-decreasing over time.

This paper is organized as follows. Section II gives the system model. Then, Section III considers availability of causal side information of the SNR and harvested energy, while Section IV considers availability of full side information. Finally, Section V concludes the paper.

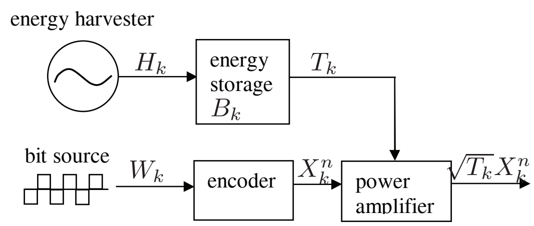

Fig. 1. Block diagram of a transmitter powered by a energy harvester. Energy is replenished by an energy harvester but is drawn for transmission.

II. System Model

For simplicity, each packet transmission is performed in one time slot. Each time slot allows n symbols to be transmitted, where n is assumed to be very large. We index time by the slot index $k \in \mathcal{K} \triangleq \{1, \cdots, K\}$.

We consider a point-to-point, flat-fading, single-antenna communication system. As shown in Fig.1, the transmitter consists of an energy harvester with energy as input, an energy storage, an encoder, and a power amplifier. Energy is measured on a per symbol (or channel use) basis, hence we use the terms energy and power interchangeably.

Consider slot $k \in \mathcal{K}$. At time instant k^-, which denotes the time instant just before slot k, the battery has $B_k \geq 0$ stored energy per symbol available for transmitting message W_k. The message is first encoded as data symbols[1] X_k^n of length n. In slot k, the transmitter transmits packet k as $\sqrt{T_k} X_k^n$, where $0 \leq T_k \leq B_k$ is the energy per symbol used by the power amplifier. Except for transmission, we assume the other circuits in the transmitter consume negligible energy. We assume that there is always back-logged data available for transmission.

1) Mutual Information: Given a channel of SNR γ_k in slot k, the maximum reliable transmission rate is given by the mutual information $I(\gamma_k, T_k) \geq 0$ in bits per symbol. In general, we assume that $I(\gamma, T)$ is concave in T given γ, and is increasing in T for all γ. For example, we may employ Gaussian signalling for transmission over a complex Gaussian channel [6], which gives

$$I(\gamma, T) = \log_2(1 + T\gamma). \tag{1}$$

2) Battery Dynamics: While transmitting packet k, the energy harvester collects an average energy of $H_k \geq 0$ per symbol, which is then stored in the battery. At time instant $(k+1)^-$, the energy stored is updated in general as

$$B_{k+1} = f\left(B^k, T^k, H^k\right), k \in \mathcal{K}.$$

The function f depends on the battery dynamics, such as the storage efficiency and memory effects. Intuitively, we expect B_{k+1} to increase (or remains the same) if B_k increases, T_k decreases, or H_k increases. As a good approximation in practice, we assume the stored energy increases and decreases linearly provided the maximum stored energy in the battery $B_{\max}$ is not exceeded, i.e.,

$$B_{k+1} = \min\{B_k - T_k + H_k, B_{\max}\}, k \in \mathcal{K}. \tag{2}$$

We assume the initial stored energy B_1 is known, where $0 \leq B_1 \leq B_{\max}$. Thus, $\{B_k\}$ follows a deterministic first-order Markov model that depends only on past random variables.

3) Channel and Harvest Dynamics: To model the unpredictable nature of energy harvesting and the wireless channel over time, we model H^K and γ^K jointly as a random process. This pdf depends on the energy harvester used and the channel environment. To yield tractable analysis, we model the variation over time with a first-order stationary Markov model. Assuming H_0, γ_0 is known, the joint pdf is modeled as

$$p_{H^K,\gamma^K}(H^K, \gamma^K | H_0, \gamma_0) = \prod_{k=1}^{K} p_{H_k}(H_k | H_{k-1}) p_{\gamma_k}(\gamma_k | \gamma_{k-1}) \tag{3}$$

where $p_{H_k}(\cdot|\cdot)$ and $p_{\gamma_k}(\cdot|\cdot)$ are independent of k. In (3), we have also assumed that the harvested energy and the SNR are independent, which is reasonable in most practical scenarios.

[1]In general, we write $X_1, \cdots, X_K$ collectively as a length-K vector X^K.

We assume that the joint distribution (3) is known, which may be obtained via long-term measurements in practice.

4) Overall Dynamics: Let us denote the *state* $\mathsf{s}_k = (\gamma_k, H_k, B_{k+1}), k \in \mathcal{K}$, or simply s if the index k is arbitrary. Let the accumulated states be $\mathsf{s}^{k-1} \triangleq (\mathsf{s}_0, \cdots, \mathsf{s}_{k-1}), k \in \mathcal{K}$.

We assume the initial state $\mathsf{s}_0 \triangleq (\gamma_0, H_0, B_1)$ to be always known at the transmitter, which may be obtained causally prior to any transmission. From (2) and (3), therefore s^K follows a first-order Markov model, i.e.,

$$p_{\mathsf{s}^K}(\mathsf{s}^K | \mathsf{s}_0) = \prod_{k=1}^{K} p_{\mathsf{s}_k}(\mathsf{s}_k | \mathsf{s}_{k-1}). \tag{4}$$

In particular, (4) includes the cases where the states are independent, or where they are deterministic rather than random.

III. Causal Side Information

A. Problem Statement

The transmitter is given knowledge[2] of s_{k-1} before packet k is transmitted, where $k \in \mathcal{K}$. In practice, for instance, the receiver feeds back γ_{k-1}, while the transmitter infers H_{k-1} and B_k from its energy storage device. We say that causal SI is available as future states are not *a priori* known. Thus, this allows us to model and treat the unpredictable nature of the wireless channel and harvesting environment.

The causal SI is used to decide the amount of energy T_k for transmitting packet k. We want to maximize the throughput, i.e., the expected mutual information summed over a finite horizon of K time slots, by choosing a power allocation policy $\pi = \{T_k(\mathsf{s}_{k-1}) \forall \mathsf{s}_{k-1}, k = 1, \cdots, K\}$. The policy can be optimized offline and implemented in real time via a lookup table that is stored in at the transmitter.

A policy is feasible if $0 \leq T_k(\mathsf{s}_{k-1}) \leq B_k$ for all possible s^k and all $k \in \mathcal{K}$; we denote the space of all feasible policies as Π. Mathematically, given s_0, the maximum throughput is

$$\mathcal{T}^\star = \max_{\pi \in \Pi} \mathcal{T}(\pi), \tag{5}$$

where

$$\mathcal{T}(\pi) = \sum_{k=1}^{K} \mathbb{E}\left[I(\gamma_k, T_k(\mathsf{s}_{k-1})) | \mathsf{s}_0, \pi\right]. \tag{6}$$

In (6), the kth summation term represents the expected throughput of packet k; its expectation is performed over all (relevant) random variables given initial state s_0 and policy π.

For example, consider $K = 2$. Then (6) simplifies as

$$\mathcal{T}(\pi) = \mathbb{E}_{\mathsf{s}_1}\left[I(\gamma_1, T_1(\mathsf{s}_0)) + \mathbb{E}_{\mathsf{s}_2}\left[I(\gamma_2, T_2(\mathsf{s}_1)) | \mathsf{s}_1, \pi\right] \middle| \mathsf{s}_0, \pi\right]$$

subject to $0 \leq T_1 \leq B_1$ for the first term and $0 \leq T_2 \leq B_2$ for the second term.

In general, the optimization of $\{T_k\}$ cannot be performed independently due to the constraints. In the above example, T_2 is constrained by B_2, which in turn depends on T_1. Instead, as will be suggested by dynamic programming, we can first optimize T_2 given all possible T_1, then optimize for T_1 with T_2 replaced by the optimized value (as a function of T_1).

[2]It can be shown that having knowledge of previous states s^{k-2} does not improve throughput, due to the Markovian property of the states in (4).

B. Optimal Solution

Given the initial state s_0, the optimization problem (5) is solved by dynamic programming in Lemma 1.

Lemma 1: Given $\mathsf{s}_0 = (\gamma, H, B)$, the maximum throughput $\mathcal{T}^\star = J_1(\gamma, H, B)$ can be computed recursively based on Bellman's equations, starting from $k = K$ until $k = 1$:

$$J_K(\gamma, H, B) = \max_{0 \le T \le B} \bar{I}(\gamma, T) = \bar{I}(\gamma, B), \tag{7a}$$

$$J_k(\gamma, H, B) = \max_{0 \le T \le B} \bar{I}(\gamma, T) + \bar{J}_{k+1}(\gamma, H, B - T) \tag{7b}$$

for $k = 1, \cdots, K-1$, where

$$\bar{I}(\gamma, T) = \mathbb{E}_{\tilde{\gamma}}[I(\tilde{\gamma}, T)|\gamma]$$

$$\bar{J}_{k+1}(\gamma, H, x) = \mathbb{E}_{\tilde{H},\tilde{\gamma}}\left[J_{k+1}(\tilde{\gamma}, \tilde{H}, \min\{B_{\max}, x + \tilde{H}\})\big|\gamma, H\right]$$

and $\tilde{H}, \tilde{\gamma}$ denote, respectively, the energy harvested and SNR in the next slot (given SNR H, γ in the present slot). An optimal policy is $\pi^\star = \{T_k^\star(\mathsf{s}_{k-1}) \forall \mathsf{s}_{k-1}, k = 1, \cdots, K\}$, where $T_k^\star(\mathsf{s}_{k-1})$ is the optimal T in (7).

Proof: The proof follows by applying Bellman's equation [1] and using (2) and (3). ■

In (7a), the optimal maximization is trivial: the interpretation is that we use all available energy for transmission in slot K. We can interpret the maximization in (7b) as a tradeoff between the present and future rewards. This is because the term $\bar{I}$ is the expected mutual information that represents the present reward, while $\bar{J}_{k+1}$, commonly known as the value function, is the expected future mutual information accumulated from slot $k+1$ until slot K.

Next, we obtain structural properties of the maximum throughput $\mathcal{T}^\star$ in (5) and the corresponding optimal policy $\pi^\star$ in Theorems 1, 2. The proofs are given in the Appendix.

Theorem 1: Suppose $I(\gamma, T)$ is concave in T given γ. Given γ and H, then

1) $J_k(\gamma, H, B)$ in (7a) is concave in B for $k \in \mathcal{K}$,
2) $\bar{J}_k(\gamma, H, x)$ in (7b) is concave in x for $k \in \mathcal{K}$,

respectively. Thus, $\mathcal{T}^\star = J_1(\gamma, H, B)$ is concave in B.

Theorem 2: Suppose $I(\gamma, T)$ is concave in T given γ. Given γ and H, then $T_k^\star(\gamma, H, B)$ is increasing in B, $k \in \mathcal{K}$.

The structural properties in Theorems 1 and 2 simplify the numerical computation of the optimal power allocation solution in Lemma 1, as shown in the next section.

C. Numerical Computations

From (7a), we get the optimal solution for slot K as $T_K^\star(\mathsf{s}_{K-1}) = B_K$. Let us fix SNR and harvested energy as γ, H, respectively, and drop these arguments. Consider the unconstrained maximization over all $T \ge 0$:

$$T_k^\dagger = \arg\max_T g(T) \tag{8}$$

where $g(T) = \bar{I}(\gamma, T) + \bar{J}_{k+1}(B - T)$. Since $\bar{I}(\gamma, T)$ is concave because expectation preserves concavity, and $\bar{J}_{k+1}(B - T)$ is concave due to Theorem 1, the objective function $g(T)$ is concave. Thus, the maximization over all T gives a unique solution $T_k^\dagger$, easily solved using numerical techniques. Also, Theorem 2 helps to reduce the search space by restricting the search to be in one direction for different B. Alternatively, if $g(T)$ is differentiable, $T_k^\dagger$ is given by solving $g'(T) = 0$. Finally, by restricting the maximization in (8) to be over $0 \le T \le B$, we get the optimal solution for (7b), given by

$$T_k^\star = \begin{cases} 0, & T_k^\dagger \le 0; \\ B, & T_k^\dagger \ge B; \\ T_k^\dagger, & 0 < T_k^\dagger < B. \end{cases} \tag{9}$$

This is because if $T_k^\dagger \le 0$, the (concave) objective function $g(T)$ must be decreasing for $T \ge 0$; if $T_k^\dagger \ge B$, the objective function must be increasing for $T \le B$.

IV. Full Side Information

The initial battery energy B_1 is always known by the transmitter. We say that full SI is available if the transmitter also has priori knowledge of the harvest power H^{K-1} and SNR γ^K before any transmission begins[3]. This corresponds to the ideal case of a predictable environment where the harvest power and channel SNR are both known in advance, and also gives an upper bound to the maximum throughput $\mathcal{T}^\star$ for any distribution (3). Moreover, it provides interesting insights that are useful for constructing practical schemes.

A. Arbitrary $B_{\max}$

First, we consider the general case where $B_{\max}$ may be finite. Corollary 1, as a consequence of Lemma 1, gives the optimal throughput $\mathcal{T}^\star$ for the same problem (5) but with full SI available.

Corollary 1: Given full SI $\{H^{K-1}, \gamma^K\}$, the maximum throughput is given by

$$J_1(B_1) = \max_{\pi \in \Pi} \sum_{k=1}^{K} I(\gamma_k, T_k), \tag{10}$$

which can be computed recursively based on Bellman's equations:

$$J_K(B) = \max_{0 \le T \le B} I(\gamma_K, T) = I(\gamma_K, B), \tag{11a}$$

$$J_k(B) = \max_{0 \le T \le B} I(\gamma_k, T) + J_{k+1}(\min\{B_{\max}, B - T + H_k\}) \tag{11b}$$

for $k = 1, \cdots, K-1$.

Proof: All side information are *a priori* known and hence the SI is deterministic rather than random. Corollary 1 thus follows immediately from Lemma 1, by replacing the pdfs in (4) by Dirac delta functions accordingly. ■

In general, power may be allocated via these modes:

- *greedy* (G): use all stored energy whenever available;
- *conservative* (C): save as much stored energy as possible (without wasting any harvested energy) to the last slot;
- *balanced* (B): stored energy is traded among slots accordingly to channel conditions.

[3] H_K is not needed in our problem, as the energy harvested in slot K affects only the throughput for slot $K+1$ onwards.

For the last slot, or if $K = 1$ where there is only one slot, from (11a) it is optimal to allocate all power for transmission. For the case $K = 2$, Corollary 2 obtains the optimal power allocation for the first slot. The proof is given in the Appendix.

Corollary 2: Consider $K = 2$ slots. Suppose the mutual information function is given by (1). Given full SI $\{B_1, H_1, \gamma_1, \gamma_2\}$, the optimal transmission energy for slot 1 is given by (corresponding to the G, B, C modes, respectively)

$$T_1^\star = \begin{cases} B_1, & a < 0 \text{ or } B_1 < b; \\ \widetilde{T}, & a \geq 0 \text{ and } |b| \leq B_1 \leq c; \\ B_1 - a, & a \geq 0 \text{ and } (B_1 > c \text{ or } B_1 < -b); \end{cases}$$

where

$$\widetilde{T} = (1/\gamma_2 - 1/\gamma_1 + B_1 + H_1)/2 \tag{12}$$

and we denote $a = B_{\max} - H_1$, $b = H_1 + 1/\gamma_2 - 1/\gamma_1$ and $c = 2B_{\max} - H_1 + 1/\gamma_2 - 1/\gamma_1$.

In Corollary 2, the power allocation is interpreted to be in G, B, or C mode. For example, consider $b > 0$. Then all modes can be active: power allocation is greedy if the energy to be harvested is large or the stored energy is small ($a < 0$ or $B_1 < b$); power allocation is conservative if the energy to be harvested is small *and* the stored energy is large ($a \geq 0$ and $B_1 > c$); otherwise, the allocation depends on the SI.

Remark 1: From Corollary 2, $T_1^\star(B_1)$ is a piece-wise linear function of B_1. We also see that $T_1^\star(B_1)$ is increasing in B_1, as predicted in Theorem 2.

Although we can derive a closed-form result for larger K, the expression becomes unwieldy and less intuitive. However, if $B_{\max} \to \infty$, for any K we can obtain a closed-form result that is a variation of the water-filling power allocation policy [5], as will be presented next.

B. *Infinite* $B_{\max}$

We consider the special case where $B_{\max} \to \infty$. From (2), the stored energy is then updated according to $B_{k+1} = B_k - T_k + H_k, k \in \mathcal{K}$, which can be expressed as

$$B_{k+1} = B_1 - \sum_{i=1}^{k} T_i + \sum_{i=1}^{k} H_i, \quad k \in \mathcal{K}. \tag{13}$$

The throughput maximization problem solved in Corollary 1 can be formulated as follows:

$$\max_{\{T_k \geq 0, k \in \mathcal{K}\}} \mathcal{T}(T^K) = \sum_{k=1}^{K} I(\gamma_k, T_k) \tag{14a}$$

subject to $T_k \leq B_k, k \in \mathcal{K}$, or equivalently, subject to

$$\sum_{i=1}^{k} T_i \leq B_1 + \sum_{i=1}^{k-1} H_i, \quad k \in \mathcal{K} \tag{14b}$$

due to (13). Theorem 3 gives the structure for the optimal power allocation solution $T_k^\star, k \in \mathcal{K}$. Let us denote $[x]^+ \triangleq \max(0, x)$.

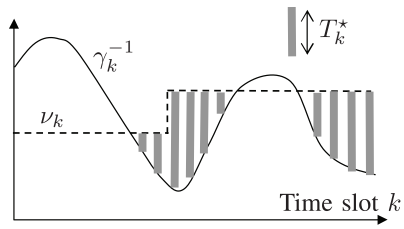

Fig. 2. Structure of optimal power allocation $T_k^\star$ with full SI and infinite $B_{\max}$. We assume two distinct water levels for ν_k, and arbitrary SNR γ_k's.

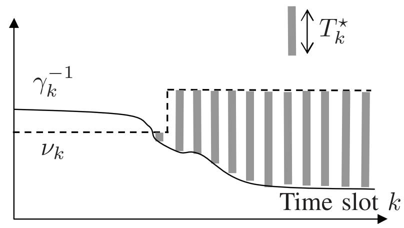

Fig. 3. Same as Fig. 2 but with increasing SNR γ_k, i.e, decreasing γ_k^{-1}, over slot k. In this case, $T_k^\star$ must increase over slot k.

Theorem 3: Suppose the mutual information function is given by (1) and $B_{\max} \to \infty$. Given full SI, the optimal power allocation is given by

$$T_k^\star = \left[\nu_k - \frac{1}{\gamma_k}\right]^+, \quad k \in \mathcal{K}, \tag{15}$$

where the *water-levels* $\{\nu_k\}$ satisfy $0 \leq \nu_1 \leq \nu_2 \leq \cdots \leq \nu_K$.

Fig. 2 gives an example of the optimal power allocation. It is possible to have multiple distinct water levels for ν_k, but the water levels are non-increasing. In conventional water-filling, however, there is only one distinct water level.

Proof: The optimization problem in (14) is convex and so can be solved by the dual problem [7]. The Lagrangian associated to the primal problem (14) is $\mathcal{L}(\lambda^K, T^K) = \mathcal{T}(T^K) - \sum_{k=1}^{K} \lambda_k \cdot \left(\sum_{i=1}^{k} T_i - \sum_{i=1}^{k-1} H_i - B_1\right) + \sum_{k=1}^{K} \mu_k T_k$, where λ_k is the Lagrangian multiplier for the kth constraint in (14b) and μ_k is the Lagrangian multiplier for the constraint $T_k \geq 0$ with $\lambda_k, \mu_k \geq 0$. Solving $\partial\mathcal{L}/\partial T_k = 0$, and with the Karush-Kuhn-Tucker conditions, we get the optimal solution

$$T_k^\star = \left[\nu_k - \frac{1}{\gamma_k}\right]^+, \quad \nu_k \triangleq \frac{1}{\ln 2 \sum_{i=k}^{K} \lambda_i}$$

for $k \in \mathcal{K}$. Since $\lambda_k \geq 0$, it follows that $\nu_k \geq 0$ and also that ν_k increases with k. ■

Remark 2: Suppose the SNR is non-decreasing over slots. Then the optimal power allocation is non-decreasing over slots, since from Theorem 3, we get $T_l^\star \leq T_k^\star$ if $\gamma_l \leq \gamma_k$ for $l < k$.

An example that illustrates Remark 2 is given in Fig. 3. The converse of Remark 2 is not true in general. In conventional water-filling, however, both Remark 2 and its converse hold.

V. Conclusion

We considered a communication system where the energy available for transmission varies from slot to slot, depending on how much energy is harvested from the environment and expended for transmission in the previous slot. We studied the problem of maximizing the throughput via power allocation over a finite horizon, given either causal SI or full SI. We obtained structural results for the optimal power allocation, which provides intuition that is useful for heuristic implementation in practice.

Appendix A
Proof of Theorem 1

Fix γ, H, so we take all functions, e.g., $J_k(\gamma, H, B)$, as functions of B only. We now prove by induction that J_k is concave in B for decreasing $k = K, \cdots, 1$.

Consider $k \in \{1, \cdots, K-1\}$. Suppose that J_{k+1} is concave in B. Clearly $J_{k+1}(\gamma, \min\{B_{\max}, x+H\})$ is concave in x, as it is the minimum of the constant $J_{k+1}(\gamma, B_{\max})$ (independent of x) and the concave function $J_{k+1}(\gamma, x+H)$. It follows that $\bar{J}_{k+1}$ is concave in x, since expectation preserves concavity. From (7b), J_k is a supremal convolution of two concave functions in B, namely $\bar{I}$ and $\bar{J}_{k+1}$ (with γ, H fixed). It follows that J_k is concave in B, since the infimal convolution of convex functions is convex [8, Theorem 5.4]. To complete the proof by induction, we need to show that J_K in (7a) is concave; this holds as expectation preserves concavity and I is concave in the second argument by assumption.

Appendix B
An Auxiliary Lemma

We need Lemma 2 to prove Theorem 2.

Lemma 2: Consider $T^\star(B) = \arg\max F(B, T)$, where the maximization is over interval $T_l(B) \le T \le T_u(B)$ that depends on B. If $T_l(B), T_u(B)$ are increasing in B, and if F has increasing differences in (B, T), i.e., $\forall T' \ge T, B' \ge B$,

$$F(B', T') - F(B, T') \ge F(B', T) - F(B, T), \quad (16)$$

then the maximal and minimal selections of $T^\star(B)$, namely $\overline{T}(B), \underline{T}(B)$, are increasing.

Proof: See proof in [9, Theorem 2]. ∎

Appendix C
Proof of Theorem 2

Fix γ, H; we drop these arguments from all functions. From (7a), we get $T_K^\star(B) = B$, which is increasing in B. We apply Lemma 2 above to establish that Theorem 2 hold for $k < K$. Let $F(B, T) = \bar{I}(T) + \bar{J}_{k+1}(B - T)$, according to (7b). Let $T_l(B) = 0$, $T_u(B) = B$, which are increasing in B. To apply Lemma 2, it is sufficient to show that each term in F has increasing differences in (B, T). Since $\bar{I}(T)$ is independent of B, trivially $\bar{I}(T)$ has increasing differences in (B, T). To show that $g(B-T) \triangleq \bar{J}_{k+1}(B-T)$ has increasing differences in (B, T), we note that $g(y+\delta) - g(y) \le g(x+\delta) - g(x)$ for $x \le y, \delta \ge 0$, since $g(x) = \bar{J}_{k+1}(x)$ is concave in x from the proof in Theorem 1. Substituting $x = B - T', y = B - T, \delta = B' - B$, we then obtain (16) with $F(B, T) = g(B - T)$. Since the objective function is concave (as shown in Appendix A), $T^\star(B)$ is unique. Thus, from Lemma 2, $T^\star(B) = \overline{T}(B) = \underline{T}(B)$ is increasing in B, $k \in \mathcal{K}$.

Appendix D
Proof of Corollary 2

Since $K = 2$ and full SI is available, from (1), (11) we get

$$J_1(\gamma_1, B_1) = \max_{0 \le T \le B_1} g(T), \quad (17)$$

$g(T) \triangleq \log_2(1+\gamma_1 T) + \log_2(1+\gamma_2 \min\{B_{\max}, B_1 - T + H_1\})$.

Suppose $H_1 > B_{\max}$. Then $\min\{B_{\max}, B_1 - T + H_1\} = B_{\max}$ if $T \le B_1$. The optimal T that solves (17) is then

$$T_1^\star = B_1 \text{ if } H > B_{\max}. \quad (18)$$

Suppose $H_1 \le B_{\max}$. Consider $T \le B_1 + H_1 - B_{\max}$. Then $\min\{B_{\max}, B_1 - T + H_1\} = B_{\max}$, which implies

$$\arg \max_{T \le B_1 + H_1 - B_{\max}} g(T) = B_1 + H_1 - B_{\max}. \quad (19)$$

This shows that the optimal T for (17) cannot be less than $B_1 + H_1 - B_{\max}$. Denote $[x]^+ = \max(0, x)$. Without loss of generality, the optimal T in (17) is therefore given by

$$T_1^\star = \arg \max_{[B_1 + H_1 - B_{\max}]^+ \le T \le B_1} g(T) \quad (20)$$

if $H_1 \le B_{\max}$. For $T \ge [B_1 + H_1 - B_{\max}]^+$, we have

$$g(T) = \log_2(1 + \gamma_1 T) + \log_2(1 + \gamma_2 (B_1 - T + H_1)),$$

which is differentiable and concave. Observe that $\widetilde{T}$ in (12) solves the equation $g'(\widetilde{T}) = 0$, i.e., $\widetilde{T}$ is the optimal solution for the *unconstrained* optimization problem $\max g(T)$. By concavity of $g(T)$, we can then obtain (20) as

$$T_1^\star = \arg \max_{[B_1 + H_1 - B_{\max}]^+ \le T \le B_1} g(T)$$
$$= \begin{cases} B_1, & \widetilde{T} > B_1; \\ \widetilde{T}, & [B_1 + H_1 - B_{\max}]^+ \le \widetilde{T} \le B_1; \\ B_1 + H_1 - B_{\max}, & \widetilde{T} < [B_1 + H_1 - B_{\max}]^+ \end{cases}$$

if $H_1 \le B_{\max}$. By re-writing the above conditions in terms of B_1 and combining the result with (18), we then obtain $T_1^\star$ as stated in Corollary 2.

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