

CREATING RIGOROUS OPEN CLOAKS

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Abstract—Based on compressing coordinate transformation (CCT) that transforms a region into a smaller region, a rigorous method for designing a cloak with controllable open window is proposed. The open cloak consists of a portion of an ideal closed cloak and an independent *system* whose position and parameters can be controlled. The *system* could be circular or diamond-shaped with coordinate transformation in radius direction and one axis of Cartesian coordinates, respectively. This cloak is able to externally hide an arbitrary-shaped object with arbitrary material property including PEC, and the object is open to the outside space. Full-wave simulations validate this rigorous cloaking concept.

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1. INTRODUCTION

In the past few years, various efforts have been made on the realization of invisibility cloaks due to their exciting property of completely hiding objects from electromagnetic (EM) detection. Scientists have developed conformal mapping method [1] and coordinate transformation method [2], which can protect the cloaked object of arbitrary shape from EM radiation. General relativity is proposed as the theoretical tools for controlling EM fields [3]. The first practical realization of a cylindrical cloak at microwave frequencies has been presented by Schurig et al. [4]. Inspired by the above pioneering work, many further investigations on cloaking have been conducted, including the petal-shaped cloak [5], elliptical cloaks [6], slab cloaks [7], toroids cloak [8], cloak with a twin cavity [9], cloaks of twisted domains [10], arbitrary shaped two-dimensional cloak [11] and three-dimensional cloak [12]. Also a different simulation method based on FDTD [13] and the sensitivity of cloaks to several factors [14] have been carefully studied.

In the approaches mentioned above, the cloaks are closed, and the object is concealed in a cloaking shell. The open cloak [15] has been investigated by an approximation approach. This cloak itself has an open window based on the approximation that the cloak is infinitesimally thin in certain areas, and such discontinuity is thus ignored resulting in an effective open window. EM scattering still exists outside the open cloak since it is an approximate approach, which implies that the window cannot be large. Based on the concept of complementary media, the cloaks that can cloak objects at a distance outside the cloaking shell have been investigated [16,17]. Complementary media can be designed under a special kind of coordinate transformation, i.e., folding a piece of space into another. Using this concept, many further investigations have been conducted, including the pathway that can block EM waves but allow the passage of other entities [18], the hidden portal that can conceal an entrance from the EM wave detection [19], new strategy to conceal an object [20], guiding waves through an invisible tunnel [21], the cloak exhibiting magnified and shifted scattering wave [22], and equiscatterer [23]. Recently, another cloaking method has been proposed to design active exterior cloaks [24], which has the disadvantage that one needs to know in advance the incoming probing wave and its phase information.

In this paper, we propose a rigorous recipe for creating open cloaks, in contrast to the approximation adopted in [15]. The proposed concept is based on the combination of transformation optics and

folded geometry to segment an original closed cloak [2] into two parts and replace one part by an independent *system*. Therefore, by assigning a few constitutions designed by the proposed method, EM invisibility can be achieved from this rigorous method without any approximation. Compared with closed cloak [2], the object cloaked by using this cloak is open to the outer space, making the matter or information exchange possible especially for moving objects. More importantly, this cloak does not rely on “anti-object” concept [16, 17] and is thus independent of the cloaked object, which greatly enhances the applicability of open cloaks.

2. THEORETICAL ANALYSIS

A schematic diagram illustrating the concept of the rigorous open cloak with a circular *system* is shown in Fig. 1. A closed cloak [1] placed at \vec{r}_0 in virtual space (x', y', z') is divided into two portions in Fig. 1(a). The blue segment is kept invariant while the red segment is replaced with a circular *system*. Then an open cloak in the real space (x, y, z) is created in Fig. 1(b). Fig. 1(c) shows a zoom-in view of the circular *system* in Fig. 1(b). It can be seen that the *system* is divided into three regions including region I ($b < r < c$) denoted by (r_1, θ_1, z_1) , region II ($a < r < b$) denoted by (r_2, θ_2, z_2) , and region III ($0 < r < a$) denoted by (r_3, θ_3, z_3) . We first compress region I, region II and region III into region III. Then the parameters of the green area (core material) and red area (*image*) can be obtained. The *image* in physical space corresponds to a portion of a closed cloak with ϵ' and μ' [2] in virtual space. Secondly, the folded geometry is employed to compensate the

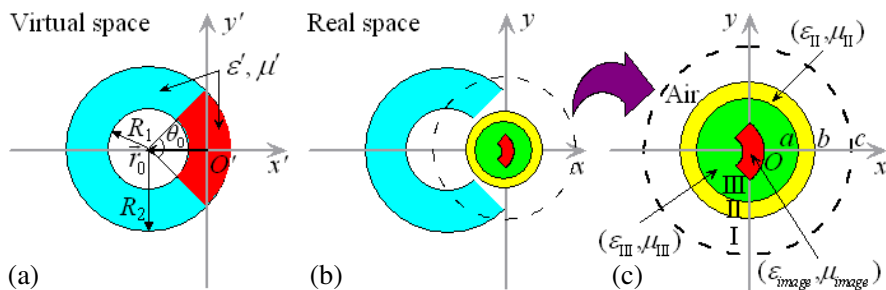


Figure 1. The scheme of designing the rigorous open cloak with a circular *system*. (a) A closed cloak [2] in virtual space is divided into two segments. (b) The red segment in (a) is replaced with a circular *system* (an open cloak is formed). (c) A magnified view of the *system* in (b).

discontinuity by using CCT. Then the parameters of the yellow area (complementary material) can be obtained. Note that the permittivity and permeability are relative parameters throughout.

The transformation equations compressing region I, region II and region III into region III are

$$r_3 = \frac{a}{c}r', \quad \theta_3 = \theta', \quad z_3 = z' \quad (1)$$

The permittivity and permeability tensors become

$$\varepsilon_{III} = \mu_{III} = \mathbf{A}^c \mathbf{A}^{cT} / \det(\mathbf{A}^c) \quad (2)$$

where $\mathbf{A}_{ij}^c = \partial x_3^i / \partial x'^j$ is the Jacobian transformation matrix from Eq. (1).

Therefore, the permittivity and permeability of the core material are

$$\varepsilon_{III} = \mu_{III} = \text{diag}[1, 1, (c/a)^2]. \quad (3)$$

Assuming a closed cloak with the inner boundary R_1 and outer boundary R_2 [Fig. 1(a)], its parameters can be expressed as $\varepsilon'_r(r') = \mu'_r(r') = \frac{|r^{\vec{r}} - \vec{r}_0| - R_1}{|r^{\vec{r}} - \vec{r}_0|}$, $\varepsilon'_\theta(r') = \mu'_\theta(r') = \frac{|r^{\vec{r}} - \vec{r}_0|}{|r^{\vec{r}} - \vec{r}_0| - R_1}$, $\varepsilon'_z(r') = \mu'_z(r') = \frac{|r^{\vec{r}} - \vec{r}_0| - R_1}{|r^{\vec{r}} - \vec{r}_0|} \left(\frac{R_2}{R_2 - R_1} \right)^2$. Then the parameters of its corresponding *image* in physical space become

$$\varepsilon_{image} = \mu_{image} = \mathbf{A}^c \cdot \varepsilon'(r_3) \cdot \mathbf{A}^{cT} / \det(\mathbf{A}^c) \quad (4)$$

where $\varepsilon'_r(r_3) = \frac{|\vec{r}_3 - a\vec{r}_0/c| - aR_1/c}{|\vec{r}_3 - a\vec{r}_0/c|}$, $\varepsilon'_\theta(r_3) = \frac{1}{\varepsilon'_r(r_3)}$, and $\varepsilon'_z(r_3) = \varepsilon'_r(r_3) \cdot \left(\frac{R_2}{R_2 - R_1} \right)^2$.

In order to compensate the discontinuity by using CCT, the folded geometry is adopted. The transformation equations folding region I into region II are

$$r_2 = \frac{a-b}{c-b}(r_1 - b) + b, \quad \theta_2 = \theta_1, \quad z_2 = z_1. \quad (5)$$

The permittivity and permeability tensors become

$$\varepsilon_{II} = \mu_{II} = \mathbf{A}^f \mathbf{A}^{fT} / \det(\mathbf{A}^f) \quad (6)$$

where $\mathbf{A}_{ij}^f = \partial x_2^i / \partial x_1^j$ is the Jacobian transformation matrix from Eq. (5).

Thus, the parameters of the complementary material in polar coordinates are

$$\varepsilon_{II} = \mu_{II} = \text{diag} \left[\frac{k_1 r_2 - k_2 b}{k_1 r_2}, \frac{k_1 r_2}{k_1 r_2 - k_2 b}, \frac{k_1 (k_1 r_2 - k_2 b)}{r_2} \right] \quad (7)$$

where $k_1 = \frac{c-b}{a-b}$ and $k_2 = \frac{c-a}{a-b}$.

The circular *system* discussed above is constructed by operating CCT along the radial direction. The *system* can also be designed by operating CCT along one axis of Cartesian coordinates [25], which is thus in the diamond shape. A schematic diagram illustrating the concept of a rigorous open cloak with diamond-shaped *system* is shown in Fig. 2. In Fig. 2(c), we can find that the *system* is divided into three regions including region I denoted by (x_1, y_1, z_1) , region II denoted by (x_2, y_2, z_2) , and region III denoted by (x_3, y_3, z_3) .

The transformation equations compressing region I, region II and region III into region III are

$$x_3 = x', \quad y_3 = \frac{a}{c}y', \quad z_3 = z'. \tag{8}$$

Therefore, the permittivity and permeability of the core material are

$$\epsilon_{III} = \mu_{III} = \text{diag}[c/a, a/c, c/a] \tag{9}$$

Assuming a closed cloak with the inner boundary R_1 and outer boundary R_2 [Fig. 2(a)], its parameters can be expressed as

$$\epsilon'(x', y') = \mu'(x', y') = \begin{pmatrix} \epsilon'_{xx} & \epsilon'_{xy} & 0 \\ \epsilon'_{yx} & \epsilon'_{yy} & 0 \\ 0 & 0 & \epsilon'_{zz} \end{pmatrix} \tag{10}$$

where $\epsilon'_{xx} = \frac{|\vec{r}' - \vec{r}_0|}{|\vec{r}' - \vec{r}_0| - R_1} + \frac{R_1(R_1 - 2|\vec{r}' - \vec{r}_0|)}{(|\vec{r}' - \vec{r}_0| - R_1)|\vec{r}' - \vec{r}_0|^3}(x' - x_0)^2$, $\epsilon'_{xy} = \epsilon'_{yx} = \frac{R_1(R_1 - 2|\vec{r}' - \vec{r}_0|)}{(|\vec{r}' - \vec{r}_0| - R_1)|\vec{r}' - \vec{r}_0|^3}(x' - x_0)(y' - y_0)$, $\epsilon'_{yy} = \frac{|\vec{r}' - \vec{r}_0|}{|\vec{r}' - \vec{r}_0| - R_1} +$

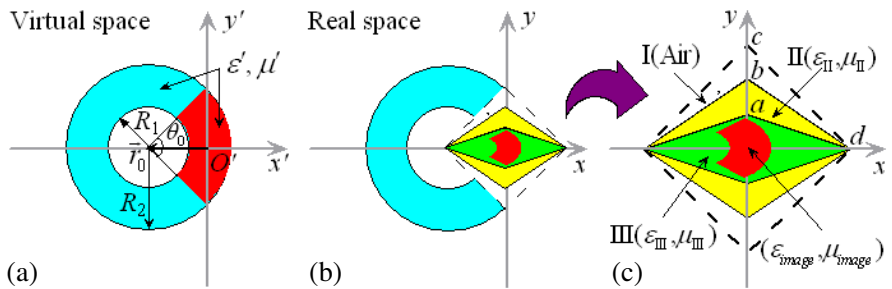


Figure 2. The scheme of designing an open cloak with a diamond-shaped *system*. (a) A closed cloak [2] in virtual space is divided into two segments. (b) The red segment in (a) is replaced with a diamond-shaped *system* to construct the open cloak. (c) The magnified view of the *system* in (b).

$$\frac{R_1(R_1 - 2|r' - \vec{r}_0|)}{(|r' - \vec{r}_0| - R_1)|r' - \vec{r}_0|^3} (y' - y_0)^2, \quad \varepsilon'_{zz} = \left(\frac{R_2}{R_2 - R_1} \right)^2 \frac{|r' - \vec{r}_0| - R_1}{|r' - \vec{r}_0|}, \quad |r'| = \sqrt{x'^2 + y'^2}, \quad \text{and} \quad |r' - \vec{r}_0| = \sqrt{(x' - x_0)^2 + (y' - y_0)^2}.$$

Then the parameters of its corresponding *image* in physical space become

$$\varepsilon_{image} = \mu_{image} = \mathbf{A} \cdot \varepsilon'(x_3, y_3) \cdot \mathbf{A}^T / \det(\mathbf{A}) \quad (11)$$

where $\mathbf{A} = \text{diag}[1, a/c, 1]$, and $\varepsilon'(x_3, y_3)$ can be obtained by substituting (8) into (10).

The transformation equations folding region I into region II are

$$x_2 = x_1, \quad y_2 = w_1(d - x_1) - w_2y_1, \quad z_2 = z_1. \quad (12)$$

where $w_1 = \frac{b(c-a)}{d(c-b)}$ and $w_2 = \frac{b-a}{c-b}$.

Thus, the parameters of the complementary material are

$$\varepsilon_{II} = \mu_{II} = \begin{pmatrix} -1/w_2 & w_1/w_2 & 0 \\ w_1/w_2 & -w_1^2/w_2 - w_2 & 0 \\ 0 & 0 & -1/w_2 \end{pmatrix} \quad (13)$$

3. SIMULATIONS AND DISCUSSION

To verify the concept of designing rigorous open cloaks, we make full-wave simulations based on the finite element method (FEM). The working frequency is 0.5 GHz under TE polarization, and we set $R_1 = 0.4$ m and $R_2 = 0.8$ m.

In Fig. 3, we consider a PEC with the radius 0.3 m covered by an open cloak with a circular *system*, and the axis of the PEC is located at $x = -0.6$ m and $y = 0$ m. Clearly, it can be observed that the open cloak is able to guide the arbitrarily incident plane waves to propagate around the center region and the EM wave fronts remain planar after the cloak. As expected, the absence of scattered waves validates the cloaking concept proposed in this paper. Note that the spatial transformation in Fig. 3 is operated along the radial direction.

We also calculated the electric fields of the open cloak with a diamond-shaped *system*, in which the spatial transformation is operated along y -axis. In Fig. 4, we consider an angular PEC with the radius 0.4 m covered by an open cloak with a diamond-shaped *system*, and the axis of the PEC is located at $x = -0.6$ m and $y = 0$ m. Fig. 4(a) corresponds to the plane wave incident from the left to the right, and Fig. 4(b) corresponds to the plane wave incident from the bottom to the top. In both cases, we can observe that the scattered waves are reduced significantly and the wave fronts are maintained in the presence of the open cloak. It is clear that perfect open cloaks can be achieved.

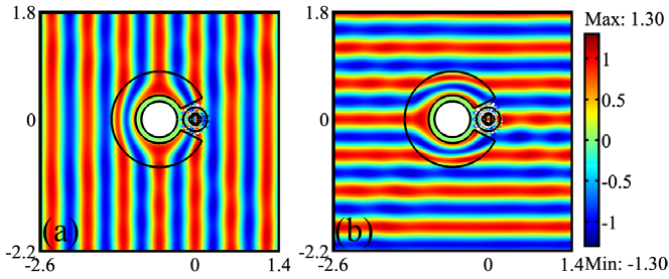


Figure 3. The snapshots of total electric fields for an open cloak with a circular *system*. The *system* follows the design of $a = 0.1$ m, $b = 0.2$ m, $c = 0.4$ m, and $\theta_0 = 90^\circ$. (a) The incident plane wave is propagating in the horizontal direction. (b) The incident plane wave is propagating in the vertical direction.

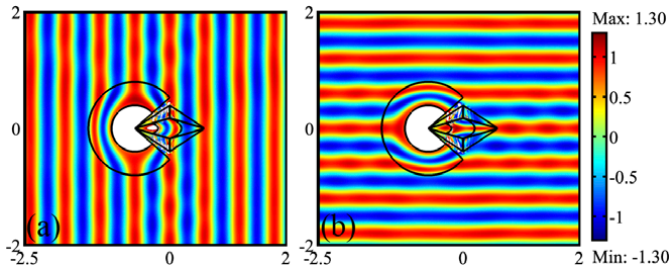


Figure 4. The snapshots of total electric fields for an open cloak with a diamond-shaped *system*. The *system* follows the design of $d = 0.6$ m, $c = \sqrt{0.8^2 - 0.6^2}$ m, $b = 3c/4$, $a = c/3$, $\theta_0 = 82.8^\circ$. (a) The plane wave is incident in the horizontal direction. (b) The plane wave is incident in the vertical direction.

4. CONCLUSION

In this paper, a rigorous method of designing open cloak is proposed. The open cloak can be designed with a circular or diamond-shaped *system*. It performs as perfectly as the original closed cloak. In contrast to the closed cloak [2], the object cloaked by such an open cloak contains multiple open windows which are controllable. Compared to the approximate open cloak [15], the proposed open cloak is designed analytically and rigorously without any assumptions. Compared to the “anti-object” cloak [16], the proposed open cloak can be repeatedly used for various objects without removing, rebuilding, or refilling different anti-objects in different

shapes into the complementary layer each time. Simulation results validate the design method, and this method can be easily extended to design open cloaks for arbitrarily-shaped closed cloaks.

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