

Homogeneous and isotropic bends to tunnel waves through multiple different/equal waveguides along arbitrary directions

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Abstract: We propose a novel optical transformation to design homogeneous isotropic bends connecting multiple waveguides of different cross sections which can ideally tunnel the wave along any directions through multiple waveguides. First, the general expressions of homogeneous and anisotropic parameters in the bend region are derived. Second, the anisotropic material can be replaced by only two kinds of isotropic materials and they can be easily arranged in planarly stratified configuration. Finally, an arbitrary bender with homogeneous and isotropic materials is constructed, which can bend electromagnetic wave to any desired directions. To achieve the utmost aim, an advanced method is proposed to design nonmagnetic, isotropic and homogeneous bends that can bend waves along arbitrary directions. More importantly, all of the proposed bender has compact shape due to all flat boundaries, while the wave can still be perfectly tunneled without mode distortion. Numerical results validate these functionalities, which make the bend much easier in fabrication and application.

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References and links

1. J. B. Pendry, D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," *Science* **312**(5781), 1780–1782 (2006).
2. J. Li and J. B. Pendry, "Hiding under the carpet: a new strategy for cloaking," *Phys. Rev. Lett.* **101**(20), 203901 (2008).
3. Y. Lai, H. Y. Chen, Z. Q. Zhang, and C. T. Chan, "Complementary media invisibility cloak that cloaks objects at a distance outside the cloaking shell," *Phys. Rev. Lett.* **102**(9), 093901 (2009).
4. H. Y. Chen, B. Hou, S. Y. Chen, X. Y. Ao, W. J. Wen, and C. T. Chan, "Design and experimental realization of a broadband transformation media field rotator at microwave frequencies," *Phys. Rev. Lett.* **102**(18), 183903 (2009).
5. M. Rahm, D. Schurig, D. A. Roberts, S. A. Cummer, D. R. Smith, and J. B. Pendry, "Design of electromagnetic cloaks and concentrators using form-invariant coordinate transformations of Maxwell's equations," *Photon. Nanostruct. Fundam. Appl.* **6**(1), 87–95 (2008).
6. J. B. Pendry, "Negative refraction makes a perfect lens," *Phys. Rev. Lett.* **85**(18), 3966–3969 (2000).
7. M. Tsang and D. Psaltis, "Magnifying perfect lens and superlens design by coordinate transformation," *Phys. Rev. B* **77**(3), 035122 (2008).
8. M. Rahm, D. A. Roberts, J. B. Pendry, and D. R. Smith, "Transformation-optical design of adaptive beam bends and beam expanders," *Opt. Express* **16**(15), 11555–11567 (2008).
9. W. X. Jiang, T. J. Cui, X. Y. Zhou, X. M. Yang, and Q. Cheng, "Arbitrary bending of electromagnetic waves using realizable inhomogeneous and anisotropic materials," *Phys. Rev. E Stat. Nonlin. Soft Matter Phys.* **78**(6), 066607 (2008).

10. J. T. Huangfu, S. Xi, F. Kong, J. Zhang, H. Chen, D. Wang, B.-I. Wu, L. Ran, and J. A. Kong, "Application of coordinate transformation in bend waveguides," *J. Appl. Phys.* **104**(1), 014502 (2008).
11. D. A. Roberts, M. Rahm, J. B. Pendry, and D. R. Smith, "Transformation-optical design of sharp waveguide bends and corners," *Appl. Phys. Lett.* **93**(25), 251111 (2008).
12. B. Vasić, G. Isic, R. Gajic, and K. Hingerl, "Coordinate transformation based design of confined metamaterial structures," *Phys. Rev. B* **79**(8), 085103 (2009).
13. X. J. Wu, Z. F. Lin, H. Y. Chen, and C. T. Chan, "Transformation optical design of a bending waveguide by use of isotropic materials," *Appl. Opt.* **48**(31), G101–G105 (2009).
14. Z. L. Mei and T. J. Cui, "Arbitrary bending of electromagnetic waves using isotropic materials," *J. Appl. Phys.* **105**(10), 104913 (2009).
15. W. Q. Ding, D. H. Tang, Y. Liu, L. X. Chen, and X. D. Sun, "Arbitrary waveguide bends using isotropic and homogeneous metamaterial," *Appl. Phys. Lett.* **96**(4), 041102 (2010).
16. P. H. Tichit, S. N. Burokur, and A. de Lustrac, "Waveguide taper engineering using coordinate transformation technology," *Opt. Express* **18**(2), 767–772 (2010).
17. K. Zhang, Q. Wu, F. Y. Meng, and L. W. Li, "Arbitrary waveguide connector based on embedded optical transformation," *Opt. Express* **18**(16), 17273–17279 (2010).
18. C. W. Qiu, L. Hu, X. F. Xu, and Y. J. Feng, "Spherical cloaking with homogeneous isotropic multilayered structures," *Phys. Rev. E Stat. Nonlin. Soft Matter Phys.* **79**(4), 047602 (2009).
19. A. Novitsky, C. W. Qiu, and S. Zouhdi, "Transformation-based spherical cloaks designed by an implicit transformation-independent method: theory and optimization," *N. J. Phys.* **11**(11), 113001 (2009).

1. Introduction

Novel microwave and optical devices are rapidly developing by manipulating the electromagnetic (EM) wave propagation based on coordinate transformation [1], such as invisibility cloaks [2,3], EM field rotators [4], concentrators [5], and imaging devices [6,7]. The coordinate transformation employs the invariance of Maxwell equations to establish an equivalence between metric transformations and changes in material parameters. This method actually reaches beyond making objects invisible. One popular application is the design of sharp waveguide bends using finite embedded transformation first proposed by Rahm et al. [8] for the design of beam shifters and splitters. This finite embedded approach is further utilized in conjunction with physical within the design process of waveguide bends [9–12], which bend EM waves to the desired directions inside a waveguide with inhomogeneous and anisotropic materials.

Besides the benders with inhomogeneous and (or) anisotropic materials [9–12], some isotropic bends are proposed more recently, including the bender by use of three kinds of isotropic but inhomogeneous materials stacked along the azimuth direction [13], benders with inhomogeneous isotropic material based on conformal transformation [14], and bender with isotropic negative refraction index material [15]. Although these benders are more advanced than previous benders [9–13], they imply that: (1) the benders can only connect two waveguides of equal width; (2) they are still difficult to realize due to the bender with inhomogeneous materials [14] or with negative materials ($\epsilon_r = \mu_r = -1$) [15]. More recently, taper is proposed to connect two different waveguides, however it cannot bend waves and its constitutive parameters are still anisotropic and inhomogeneous [16,17].

It is well understood that, if one wants to make use of the isotropic realization (either those method in Refs [9,13] or our proposed method), The initial discretization layer number N has to be large enough to ensure the validity of the effective medium theory [18]. Thus, the required types of different isotropic materials will be linearly increased if one adopts the methods in Refs [9,13], but our method is independent of the initial number N (which always requires only 2 types of isotropic materials because of the manipulated homogeneity).

In this paper, a novel transformation method is presented which demonstrates an interesting advance in the design of nonmagnetic (or magnetic), isotropic and homogeneous waveguide bends. Compared to the previous waveguide bends with inhomogeneous materials [9–14], the bends proposed in this paper are constructed with homogeneous materials while perfect wave tunneling can still be achieved. In addition to the homogeneity, isotropy is furthermore introduced into the bends, and only two kinds of isotropic materials for each region are required throughout. Compared to the total number and the positioning of different isotropic materials in [12,13], our design is much less demanding and more compact, which is

thus easier to realize. The proposed method can also enable us to shrink the area of the bending region due to all flat boundaries. Most importantly, the proposed bends can connect two waveguides of different widths and tunnel EM waves to arbitrary directions, which is more general and useful than the taper [16,17]. To remove the magnetic property in equal bend, an advanced method is proposed, which only need two kinds of homogeneous dielectrics (non-magnetic) and is much easier to realize than anti-vacuum bend [15]. Therefore our approach provides a much easier but more advantageous recipe for the practical realization of waveguide bends based on coordinate transformation.

2. Theoretical analysis

In this study, we introduce a new transformation method operated in Descartes coordinate, which is different from the previous methods in polar coordinate [9–15]. Figure 1(a) shows the scheme of designing a waveguide bend to tunnel waves between two different waveguides at arbitrary directions. It can be seen that triangles BCD and ABD in virtual space (x, y, z) are transformed into $BC'D'$ (region I) and ABD' (region II) in real space (x', y', z') , respectively. Then EM waves incident from port 1 are able to fluently flow out from port 2 without any reflection. Here the coordinates of points A, B, C, D, C', D' are constants, and can be expressed as $(x_A, y_A), (x_B, y_B), (x_C, y_C), (x_D, y_D), (x_{C'}, y_{C'}), (x_{D'}, y_{D'})$, respectively. We can freely choose the angle between AB and AD where $0 < \theta_0 \leq 90^\circ$. The transformation equations from the triangle BCD to Region I (denoted by the triangle $BC'D'$) can be expressed

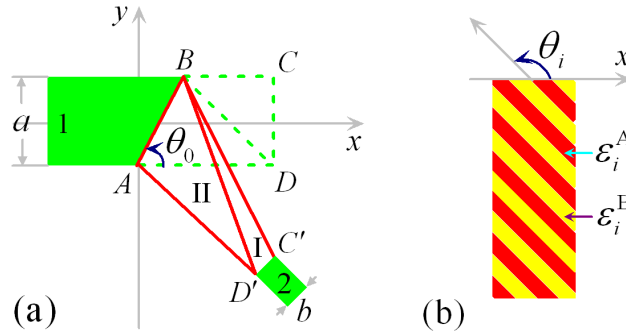


Fig. 1. (Color online). (a) The schematic illustration of the design of a homogeneous waveguide bend. (b) The layered structure includes two kinds of materials ϵ_i^A and ϵ_i^B ($i = I, II$), which is used to fill the bending region to construct the isotropic bend with all PEC (perfect electric conductor) boundaries.

$$x' = a_{11}x + b_{11}y + c_{11}, y' = a_{12}x + b_{12}y + c_{12}, z' = z \quad (1)$$

where

$$\begin{bmatrix} a_{11} \\ b_{11} \\ c_{11} \end{bmatrix} = \begin{bmatrix} x_B & y_B & 1 \\ x_C & y_C & 1 \\ x_D & y_D & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} x_B \\ x_C \\ x_{D'} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} a_{12} \\ b_{12} \\ c_{12} \end{bmatrix} = \begin{bmatrix} x_B & y_B & 1 \\ x_C & y_C & 1 \\ x_D & y_D & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} y_B \\ y_C \\ y_{D'} \end{bmatrix}.$$

According to the principles of transformation optics [1], the permittivity and permeability of Region I can be obtained

$$\epsilon_1 = \mu_1 = \Lambda_1 \cdot \Lambda_1^T / \det(\Lambda_1) \quad (2)$$

where $\Lambda_1 = [a_{11}, b_{11}, 0; a_{12}, b_{12}, 0; 0, 0, 1]$ and $\det(\Lambda_1) = a_{11}b_{12} - a_{12}b_{11}$.

Region II denoted by triangle ABD' is transformed from the triangle ABD and the transformation equations can be expressed as

$$x' = a_{21}x + b_{21}y + c_{21}, y' = a_{22}x + b_{22}y + c_{22}, z' = z \quad (3)$$

where

$$\begin{bmatrix} a_{21} \\ b_{21} \\ c_{21} \end{bmatrix} = \begin{bmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_D & y_D & 1 \end{bmatrix}^{-1} \begin{bmatrix} x_A \\ x_B \\ x_{D'} \end{bmatrix} \text{ and } \begin{bmatrix} a_{22} \\ b_{22} \\ c_{22} \end{bmatrix} = \begin{bmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_D & y_D & 1 \end{bmatrix}^{-1} \begin{bmatrix} y_A \\ y_B \\ y_{D'} \end{bmatrix}.$$

The parameters of region II thus become

$$\varepsilon_{II} = \mu_{II} = \Lambda_2 \bullet \Lambda_2^T / \det(\Lambda_2) \quad (4)$$

where $\Lambda_2 = [a_{21}, b_{21}, 0; a_{22}, b_{22}, 0; 0, 0, 1]$ and $\det(\Lambda_2) = a_{21}b_{22} - a_{22}b_{21}$.

It is interesting to note that the constitutive parameters are spatially invariant from Eq. (2) and Eq. (4). This is because no other variables are involved in the material parameters tensors. The homogeneity has been achieved, and thus we focus on achieving isotropy. Because of the symmetry of the tensors ε_I and ε_{II} , we can find a rotation transformation mapping such symmetric tensors into diagonal tensors, from which the effective isotropic media can be derived [18,19]. The permittivity tensor for Region I and Region II in the bend can be expressed as

$$\varepsilon_i = \text{diag}[\zeta_i^x, \zeta_i^y, \zeta_i^z] \quad (5)$$

where $\zeta_i^{x,y} = [\varepsilon_i^{xx} + \varepsilon_i^{yy} \pm \sqrt{(\varepsilon_i^{xx} - \varepsilon_i^{yy})^2 + (2\varepsilon_i^{xy})^2}] / 2$, and $\zeta_i^z = \varepsilon_i^{zz}$ ($i = I, \text{ or } II$).

From the rotation transformation, we can derive a relation between rotation angle and original permittivity as

$$\tan(2\theta_i) = 2\varepsilon_i^{xy} / (\varepsilon_i^{xx} - \varepsilon_i^{yy}) \quad (6)$$

From Eq. (6), the rotation angle can be uniquely determined, which is the same as the angle between the layer and x-axis, as shown in Fig. 1(b).

As shown in Fig. 1(b), we can use the layered structure to fill the bending region and finally an isotropic bend is obtained. Since the ideal anisotropic material is homogeneous in both regions, each region is only composed of 2 types of isotropic materials in a planarly layered pattern: medium-A and medium-B, while Ref [13] needs $3N$ types of isotropic materials with non-parallel interfaces (N refers to the initial number of discretized anisotropic layers in Ref [9]). In contrast, the isotropic materials in our design only need 2 types: medium-A and medium-B.

$$\varepsilon_i^{A,B} = \zeta_i^x \pm \sqrt{(\zeta_i^x)^2 - \zeta_i^x \zeta_i^y} \quad (7)$$

3. Simulation results and discussion

In order to demonstrate the feasibility of the waveguide bends proposed in this paper, full-wave simulations are performed to verify the functionality. The waveguides boundaries are assumed to be perfectly conducting (PEC). First, we choose $\theta_0 = 90^\circ$ and set the coordinates of A, B, C, D as (0, -0.05), (0, 0.05), (0.05, 0.05), (0.05, -0.05), respectively.

Let us consider a vertical bend connecting two waveguides of equal width with $a = 10$ cm, and the cutoff frequency is 1.5 GHz. The coordinates of C', D' are (0.05, -0.1) and (0.15, -0.1) respectively, and the simulation frequency is 2 GHz under transverse magnetic (TM) polarization. Figure 2 shows the magnetic field distribution of the equal vertical bender using homogeneous materials with and without anisotropy. Figure 2(a) corresponds to the bender without transformation material, and Fig. 2(b) corresponds to the bender with transformation

material. When the bend region is empty, the phase distortions caused by reflections in the bend region is clearly observed (Fig. 2(a)). If the designed transformation material is placed in the bend, it can be seen that the EM waves are tunneled via the bend region without any reflections (Fig. 2(b)). Based on the effective medium theory, the bend could be realized through an alternating layered isotropic structure. Here we choose $M = 10$ for both Region I and Region II, and the effective isotropic parameters can be found as $\varepsilon_1^A = 12.26$, $\varepsilon_1^B = 0.082$, $\mu_1 = 1/3$, and $\varepsilon_{II}^A = 5$, $\varepsilon_{II}^B = 0.2$, $\mu_{II} = 1$. Figure 2(c) illustrates the magnetic field distribution of the equal-width waveguide bender composed of isotropic alternating layers. In order to quantitatively observe the performance of the bends filled with air, ideal transformation media, and isotropic layered materials, the average power outflow at port 2 is presented in Fig. 2(d). Obviously, the waveguide bend filled with isotropic layered materials behaves as perfectly as the bend filled with ideal transformation media. Hence, the bend can now be implemented using only homogeneous and isotropic materials. We also simulated the extruded bend by extruding the 2D bend along z direction. The effective isotropic parameters of the extruded bend can be found as $\mu_1^A = 12.26$, $\mu_1^B = 0.082$, $\varepsilon_1 = 1/3$, and $\mu_{II}^A = 5$, $\mu_{II}^B = 0.2$, $\varepsilon_{II} = 1$. Figure 2(e) illustrates the electric field distribution of the extruded bend when the dominant mode TE_{10} is incident. Obviously, perfect performance can be achieved in the extruded bend.

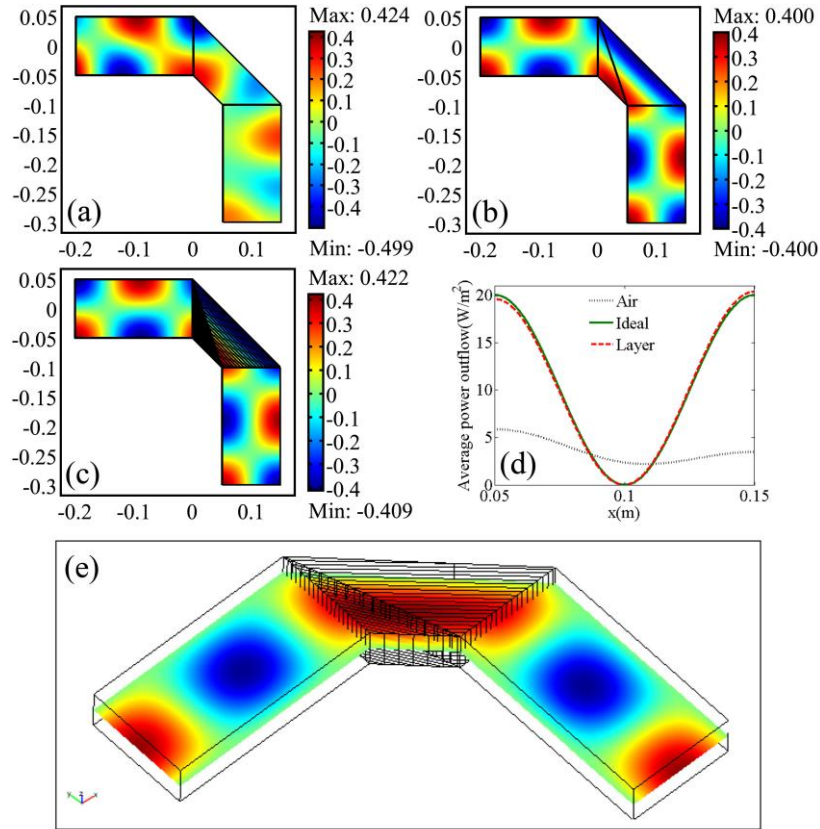


Fig. 2. (Color online). The magnetic field distribution of the designed bend using homogeneous materials for $a = 10$ cm, with or without anisotropy. (a) a vacuum bend. (b) ideal anisotropic transformation media filled-bend. (c) layered isotropic bend with $\theta_1 = 137^\circ$ and $\theta_{II} = 122^\circ$. (d) The average power outflow at port 2. (e) The electric field distribution of the dominant mode TE_{10} in the extruded bend with 0.04 m thickness along z direction.

In addition to designing bends with identical cross-sections, it is also possible to design bends connecting two waveguides of different cross-sections, e.g., $a = 10$ cm and $b = 2$ cm, and their cutoff frequency are 1.5 GHz and 7.5 GHz, respectively. The coordinates of C' , D' are (0.05, -0.1) and (0.07, -0.1) respectively, and the simulation frequency is 8 GHz under TM polarization. Figures 3(a) and 3(b) correspond to the bend with and without transformation materials, respectively. Clearly, if no transformation material is placed in the bend, we can observe that there is nearly no transmission from the wide waveguide to the narrow one (Fig. 3(a)). Nevertheless, based on the proposed method, two homogeneous mediums can be designed and embedded into respective regions of the bender as in Fig. 3(b). Then EM waves can be guided from one waveguide to the other without any reflection. Such bend can also be realized by a layered structure, and the effective isotropic parameters can be found $\epsilon_I^A = 36.5$, $\epsilon_I^B = 0.027$, $\mu_I = 5/3$, and $\epsilon_{II}^A = 5$, $\epsilon_{II}^B = 0.2$, $\mu_{II} = 1$. Both Region I and Region II are divided into 10 layers in Fig. 3(c), 20 layers in Fig. 3(d), and 40 layers in Fig. 3(e). We also calculate the average power outflow from port 2 when the bend filled with air, ideal transformation media, and isotropic layered materials, as shown in Fig. 3(f). It is obvious that the performance is quite pronounced when the discretization increases. Namely, a homogeneous and isotropic bend connecting two waveguides of different width is derived.

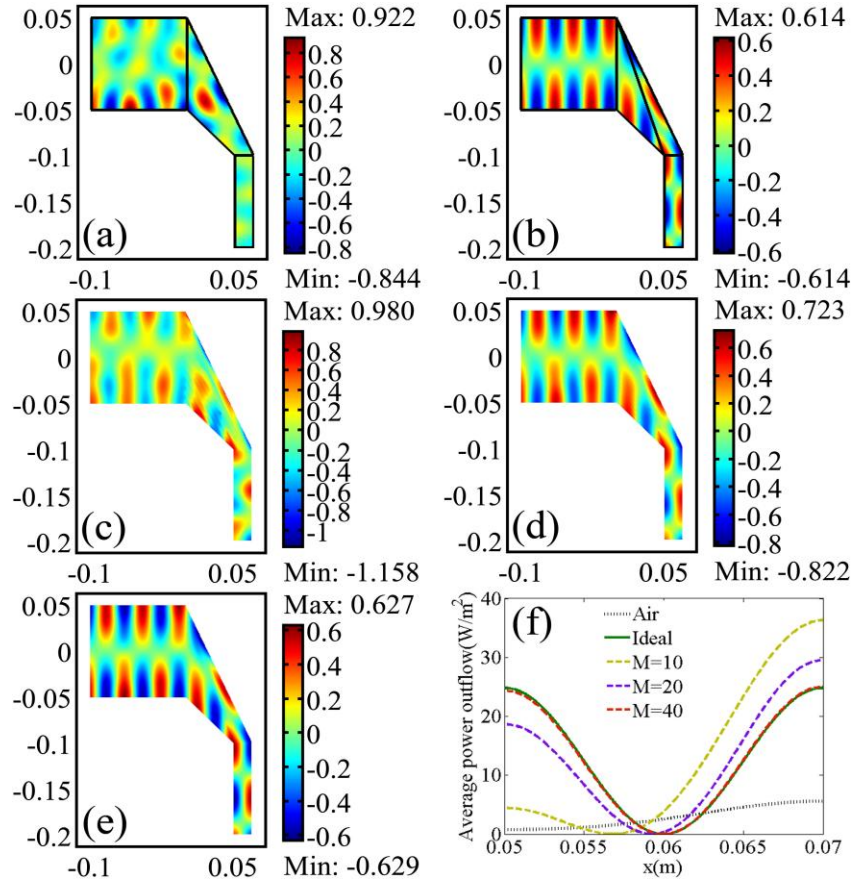


Fig. 3. (Color online). The magnetic field distribution of the bend designed using homogeneous materials for $a = 10$ cm and $b = 2$ cm, with or without anisotropy. (a) a vacuum bend. (b) ideal anisotropic transformation media filled in the bend. (c-e) Layered isotropic materials filled in the bend with $\theta_I = 115^\circ$ and $\theta_{II} = 122^\circ$. (c) $M = 10$, (d) $M = 20$, and (e) $M = 40$. (f) The average power outflow at port 2.

The optical transformation method proposed in this paper is not only used to design vertical waveguide bends (Figs. 2 and 3), it is also applied to design arbitrary bends to bend EM waves to any desired directions. The simulation frequency is 8 GHz under transverse electric (TE) polarization. Figure 4 shows the normalized electric field with the same D' as $(0.05, -0.1)$ and different C' . Figure 4(a) corresponds to the bend connecting two equal waveguides with bending angle 45° by setting C' as $(0.05+0.05\sqrt{2}, -0.1+0.05\sqrt{2})$, Fig. 4(b) corresponds to the bend connecting two equal waveguides with bending angle 135° by setting C' as $(0.05+0.05\sqrt{2}, -0.1-0.05\sqrt{2})$, Fig. 4(c) corresponds to the bend connecting two different waveguides with bending angle 45° by setting C' as $(0.05+0.01\sqrt{2}, -0.1+0.01\sqrt{2})$, and Fig. 4(d) corresponds to the bend connecting two different waveguides with bending angle 135° by setting C' as $(0.05+0.01\sqrt{2}, -0.1-0.01\sqrt{2})$. From Fig. 4, it is clear that the incident waves travel through the waveguide bends reflectionlessly and keep their original field patterns in all cases. More importantly, all the bends are constructed with homogeneous materials, which can be also realized with homogeneous and isotropic layered structure.

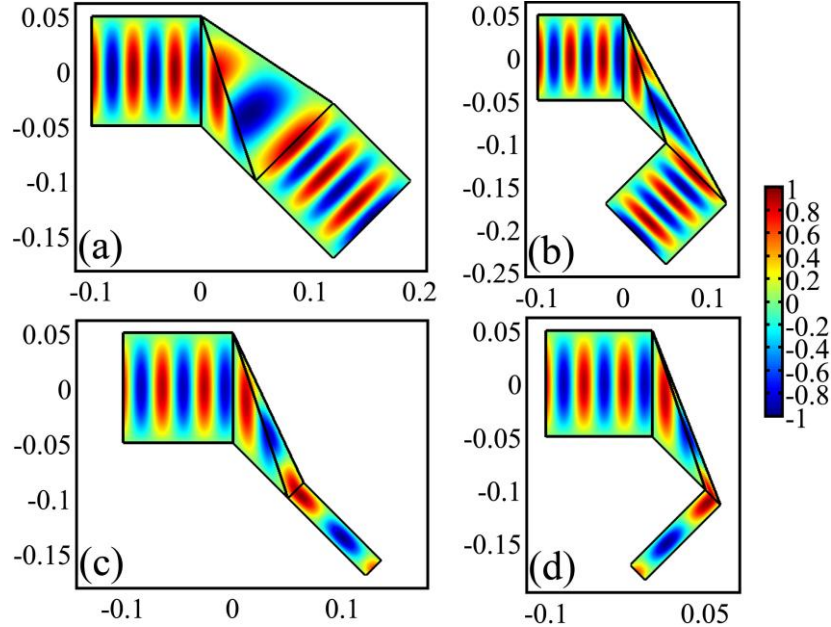


Fig. 4. (Color online). Normalized electric fields distribution inside the waveguide bends. Two waveguides of identical cross section with $a = 10$ cm: (a) bending angle 45° , (b) bending angle 135° . Two waveguides of different cross sections with $a = 10$ cm and $b = 2$ cm: (c) bending angle 45° , (d) bending angle 135° .

Next we consider a bend with bending angle 180° by setting $\theta_0 = 45^\circ$. The simulation frequency is 8 GHz under TE polarization. The coordinates of A, B, C, D, D' are $(0, -0.05), (0.1, 0.05), (0.2, 0.05), (0.2, -0.05), (0, -0.15)$, respectively. We set C' as $(0, -0.25)$ for equal bends and as $(0, -0.17)$ for non-equal bends. Figures 5(a) and 5(b) demonstrate the normalized electric fields distribution of the equal bends with and without transformation materials. Clearly, there is nearly no fields transmitted to port 2 without transformation materials in Fig. 5(a), and perfect transmission can be observed in Fig. 5(b) because of the homogeneous transformation materials in bend region. Figures 5(c) and 5(d) show the

normalized electric fields distribution of the non-equal bends with and without transformation materials. Comparing the severely phase distortions in Fig. 5(c) and the perfect performance in Fig. 5(d), the design method proposed in this paper is verified again. It is proved that the proposed bends could tunnel waves between two different waveguides at arbitrary directions.

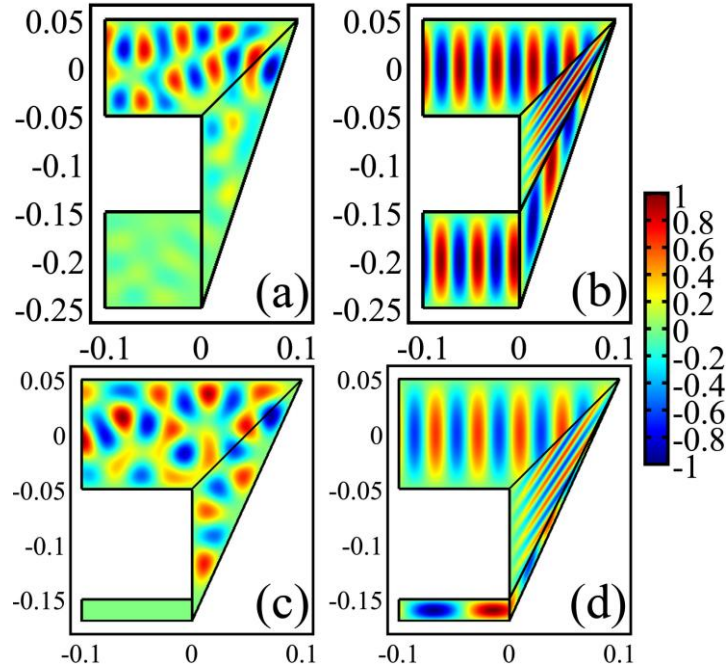


Fig. 5. (Color online). Normalized electric fields distribution inside the waveguide bends with bending angle 180° . Two waveguides of equal width with $a = 10$ cm: (a) a vacuum bend. (b) transformation media filled in the bend. Two waveguides of different width with $a = 10$ cm and $b = 2$ cm: (c) a vacuum bend. (d) transformation media filled in the bend.

When we need to construct a complicate waveguide system with many bends, the system will not compact enough if we directly use the previous bending structure as shown in Fig. 1(a). Then an advanced bend is proposed in Fig. 6(e), which is extremely compact and generated from the evolution of Fig. 1(a). The design scheme illustrates that the compact bend is obtained by transforming triangles ABC and BCD into ABC' and $BC'D'$, respectively. To verify the advanced design scheme, a collar-shaped waveguide system is constructed and the simulation frequency is 8 GHz under TE polarization. Figures 6(a) and 6(c) correspond to the normalized electric field distribution and average power flow of the system without transformation materials. Clearly, the fields are completely reflected and almost no power could reach port 2. On the contrary, based on the proposed method, when homogeneous mediums are designed and embedded into the two bending regions in Figs. 6(b) and 6(d), highly efficient transmission and well shaped mode pattern are obtained. We also calculate the average power outflow at port 2 in Fig. 6(f). Obviously, the advanced design scheme is valid and perfect transmission through multiple compact bends can still be achieved.

4. Advanced design of a nonmagnetic, isotropic and homogeneous bend

Although homogeneity and isotropy have been achieved in Fig. 2, the bends are still magnetic which translates the fabrication difficulty into magnetism. Here we propose an advanced method to design nonmagnetic, isotropic and homogeneous bend. Figure 7(a) shows the scheme of nonmagnetic bends by transforming triangles ABD and BCD into ABD' (region II) and $BC'D'$ (region I), respectively. The constitutive parameters can be obtained according to Eq. (2) and Eq. (4). Since $AB = BC = BC'$, if we select $BD' = BC/2$, the constitutive parameters

of region I and II are nonmagnetic due to the area of transformed region keeps unchanged, namely $S_{ABD} = S_{ABD'}$ and $S_{BCD} = S_{BC'D'}$.

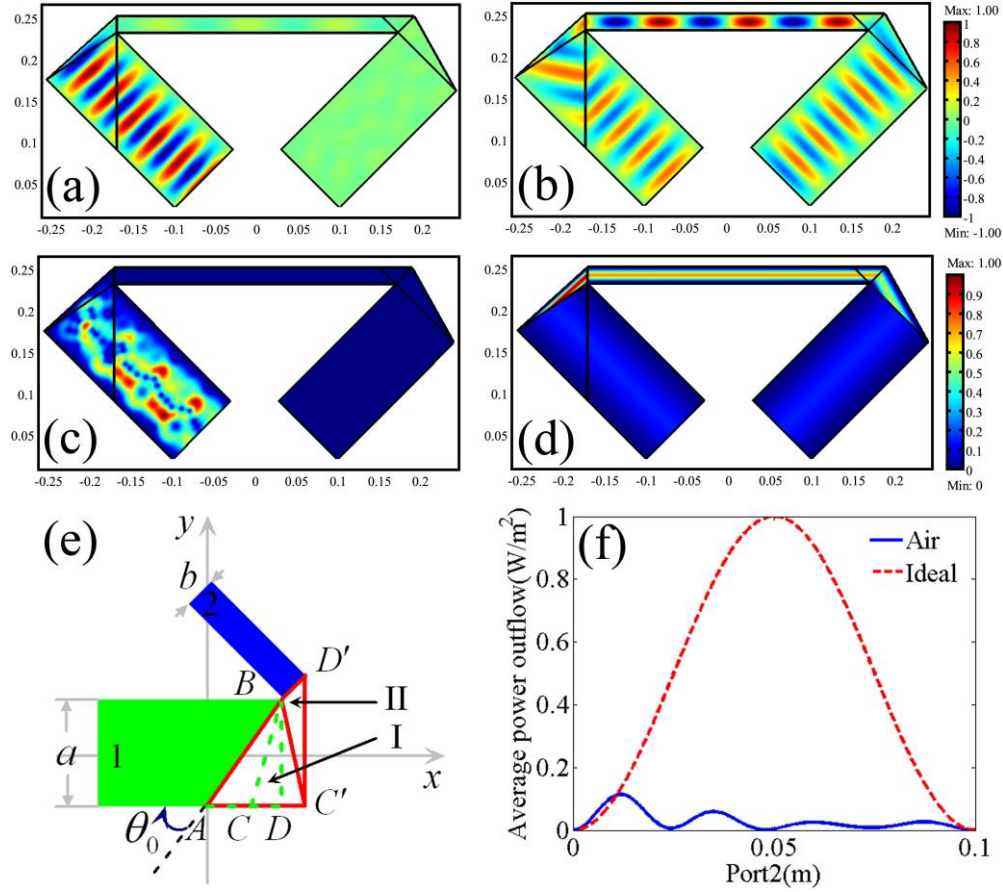


Fig. 6. (Color online). A collar-shaped waveguide bending system with two 135° compact bends when $a = 10$ cm, $b = 2$ cm and $\theta_0 = 45^\circ$. Normalized electric field distribution of the system filled with (a) air; and (b) transformation materials. Normalized average power flow of the system filled with (c) air and (d) transformation materials. (e) Schematic diagram of designing an extremely compact bending structure in (a) by transforming triangles ABC and CBD to ABC' and $C'BD'$, respectively. (f) The average outflow power at port 2 in (b).

To validate the advanced method, we consider an equal bend with bending angle $\theta = 60^\circ$ and $a = 10$ cm. The frequency is 2 GHz under TM polarization, and the wave is incident from port 1. Figure 7(b) corresponds to the nonmagnetic bender without transformation media (empty bend), in which strong reflection and severe mode distortion will be present. Figure 7(c) shows the magnetic field distribution of the nonmagnetic bender with transformation media of $\epsilon_I = \epsilon_{II} = [2.33, -1.15; -1.15, 1]$ and $\mu_I = \mu_{II} = 1$. Clearly, EM waves completely pass through the bender with homogeneous, non-magnetic and ideal anisotropic transformation media. Figure 7(d) presents the magnetic field distribution inside the homogeneous, non-magnetic and isotropic bend with $\epsilon_A = 5.83$, $\epsilon_B = 0.17$ and $\mu = 1$. The anisotropy in Fig. 7(c) has been removed by replacing the identical anisotropic material in Region I and II with two isotropic dielectrics. Obviously, the advanced bend is as perfect as the ideal case.

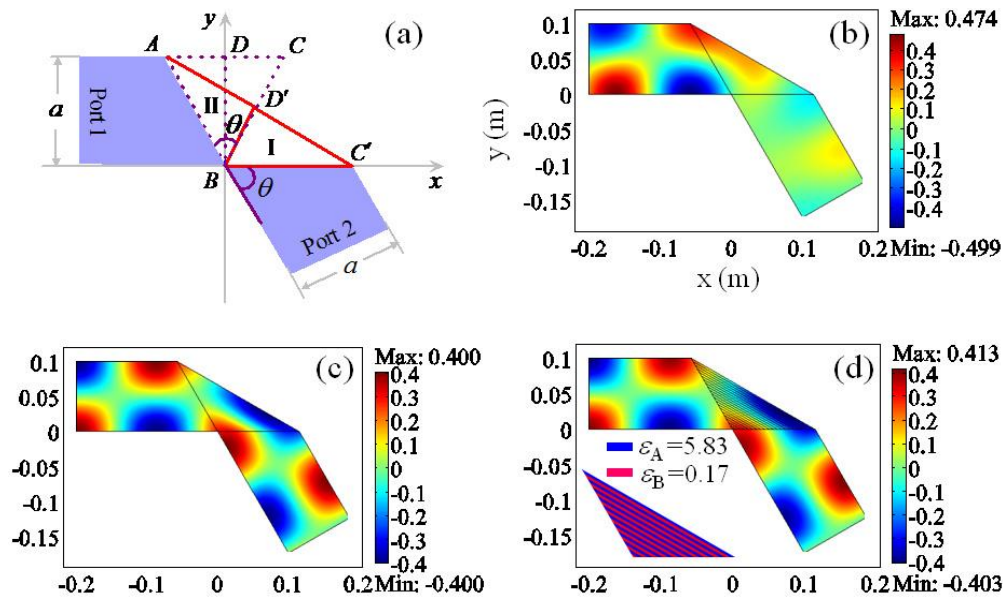


Fig. 7. (Color online). (a) The schematic illustration of coordinate transformation in the design of advanced nonmagnetic bend by transforming ABD and CBD to ABD' and $C'BD'$, respectively. (b) The magnetic field distribution of a vacuum bend with $a = 10$ cm and $\theta = 60^\circ$. (c) The magnetic field distribution of the bend in (b) filled with nonmagnetic, homogeneous and anisotropic transformation material. (d) The magnetic field distribution of the bend in (b) filled with layered isotropic dielectric ($M = 10$) and $\theta_0 = 150^\circ$. The embedded figure in (d) illustrates the distribution of constitutive parameters in transformed region.

In Fig. 7(a), the bend structure is very compact when $0 < \theta \leq 90^\circ$, but it is not compact enough when $\theta > 90^\circ$, especially when θ approaches 180° . To construct a big bending angle bender, we can cascade several small benders, i.e., an n degree bend is composed of two $n/2$ degree benders. Here we design a 120° waveguide bending system by use of only two isotropic nonmagnetic benders in Fig. 7(d). Figure 8 shows the magnetic field distribution of the system filled with air (Fig. 8(a)) as well as with dielectric materials (Fig. 8(b)). Obviously, the incident wave is perfectly guided from port 1 to port 2, and no reflection is presented when the system is filled with dielectric materials in Fig. 8(b). The inset figure in Fig. 8(b) demonstrates that the bending system is constructed with only 2 kinds of dielectrics. Namely, nonmagnetic, isotropic and homogeneous bend has been achieved, which can tunnel waves long arbitrary directions. Clearly, such bend is much easier in fabrication than previous bends [9–15].

5. Conclusion

In conclusion, we have proposed an advanced bending mechanism that uses exclusively homogeneous, isotropic, nonmagnetic (between two equal waveguides) or magnetic (between two different waveguides) materials. The benders proposed in this paper can connect multiple waveguides with different width and bend EM waves to any desired directions. They are composed of only two blocks of homogeneous materials, and can be easily realized with only two isotropic materials. This design concept may pave a realizable way to the practical applications. The full wave simulation validates the proposed design mechanism and perfect wave tunneling is demonstrated for the advanced bend with the elimination of reflection and mode distortion.

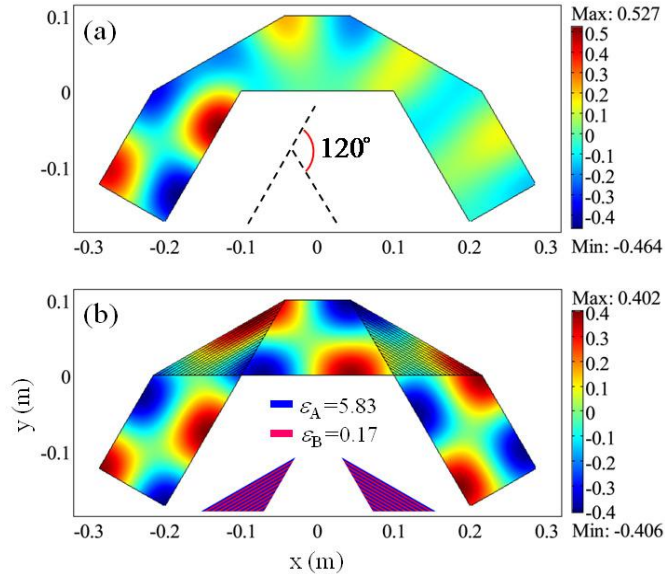


Fig. 8. (Color online). An isotropic and nonmagnetic bending system with two 60° dielectric bends in Fig. 7(d) when $a = 10$ cm. Magnetic field distribution of the system filled with (a) air; and (b) transformation materials. The embedded figure in (b) illustrates the arrangement of constitutive parameters in transformed regions.

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