Supplementary materials: Unveiling the correlation between non-diffracting tractor beam and its singularity in Poynting vector

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1. Derivation and discussion of optical forces for Bessel beam.

Firstly, we consider a magnetodielectric spherical particle of radius R in vacuum possessing only electric $\mathbf{p}$ and magnetic $\mathbf{m}$ dipole moments. Then for any incident field, the time-averaged force for a Rayleigh nanoparticle consists of electric-dipole, magnetic-dipole, and interaction-of-dipoles terms (see Eq. (2) in [1]), respectively:

$$\langle F_i\rangle = \frac{1}{2}\mathrm{Re}\left(p_j\frac{\partial E_j^*}{\partial x_i} + m_j\frac{\partial B_j^*}{\partial x_i} - \frac{k_0^4}{6\pi\varepsilon_0 c}(\mathbf{p}\times\mathbf{m}^*)_i\right), \tag{1}$$

where the sum over repeating indices ($i,\ j = 1, 2, 3.$) is assumed. Electric and magnetic dipole moments linearly depend on the fields as $\mathbf{p} = \alpha_e\mathbf{E}$ and $\mathbf{m} = \alpha_m\mathbf{B}$.

Longitudinal component of the optical force is equal to

$$\langle F_z\rangle = \frac{1}{2}\left(\mathrm{Re}(\alpha_e\mathbf{E}\frac{\partial\mathbf{E}^*}{\partial z}) + \mathrm{Re}(\alpha_m\mathbf{B}\frac{\partial\mathbf{B}^*}{\partial z})\right) - \frac{k_0^4}{12\pi\varepsilon_0 c}\mathrm{Re}(\alpha_e\alpha_m^* P_z), \tag{2}$$

where $P_z = (\mathbf{E}\times\mathbf{B}^*)_z$ and the fields are evaluated at the center of the spherical particle. For a propagation-invariant Bessel beam, the electric and magnetic field have the form $\mathbf{E} = \mathbf{e}(\mathbf{r}_\perp)\exp(i\beta k_0 z)$ and $\mathbf{B} = \mathbf{b}(\mathbf{r}_\perp)\exp(i\beta k_0 z)$, where $\mathbf{r}_\perp$ is the transverse radius-vector ($\mathbf{r} = \mathbf{r}_\perp + z\mathbf{e}_z$). Then it is not difficult to obtain $\mathrm{Re}(\alpha_e\mathbf{E}\frac{\partial\mathbf{E}^*}{\partial z}) = \mathrm{Re}(-ik_0\beta\alpha_e\mathbf{E}\mathbf{E}^*) = \mathrm{Im}(\alpha_e)k_0\beta|\mathbf{E}|^2$. Similarly, one can get $\mathrm{Re}(\alpha_m\mathbf{B}\frac{\partial\mathbf{B}^*}{\partial z}) = \mathrm{Re}(-ik_0\beta\alpha_m\mathbf{B}\mathbf{B}^*) = \mathrm{Im}(\alpha_m)k_0\beta|\mathbf{B}|^2$. Substituting above expressions into Eq. 2, one can obtain the optical force for Bessel beam in dipole approximation

$$\langle F_z\rangle = \frac{k_0\beta}{2}\left(\mathrm{Im}(\alpha_e)|\mathbf{E}|^2 + \mathrm{Im}(\alpha_m)|\mathbf{B}|^2\right) - \frac{k_0^4}{12\pi\varepsilon_0 c}\mathrm{Re}(\alpha_e\alpha_m^* P_z), \tag{3}$$

For Rayleigh particles $\mathrm{Re}(\alpha_{e,m}) \gg \mathrm{Im}(\alpha_{e,m})$, the third term in Eq. (3) can be rewritten as $\mathrm{Re}(\alpha_e\alpha_m^* P_z) \approx \mathrm{Re}(\alpha_e)\mathrm{Re}(\alpha_m)\mathrm{Re}(P_z)$. Then the optical force can be presented in the form

$$\langle F_z\rangle = \frac{k_0\beta}{2}\left(\mathrm{Im}(\alpha_e)|\mathbf{E}|^2 + \mathrm{Im}(\alpha_m)|\mathbf{B}|^2\right) - \frac{k_0^4}{12\pi\varepsilon_0 c}\mathrm{Re}(\alpha_e)\mathrm{Re}(\alpha_m)\mathrm{Re}(P_z). \tag{4}$$

Nonmagnetic lossy Rayleigh particles ($\mu = 1$, $\alpha_m = 0$ and $\mathrm{Im}(\varepsilon) > 0$) can only be pushed ($\langle F_z\rangle > 0$), but can never be pulled ($\langle F_z\rangle < 0$), because of $\mathrm{Im}(\alpha_{e,m}) > 0$. They can be negative for the gain particles, to be pulled by the light[2, 3]. For magnetodielectric particles ($\mathrm{Re}(\varepsilon) \geq 1$ and $\mathrm{Re}(\mu) \geq 1$) characterized by $\mathrm{Re}(\alpha_{e,m}) \geq 0$, the positive P_z is required to obtain the pulling force. However, when $\mathrm{Re}(\alpha_e) < 0$ and $\mathrm{Re}(\alpha_m) > 0$ or vice versa, the pulling force only occurs to the situations with $P_z < 0$, e.g., Bessel beam [4]. Inequality $\mathrm{Re}(\alpha_e) < 0$ is equivalent to $\mathrm{Re}(\alpha_e^{(0)}) < 0$ and can be realized for the Rayleigh spheres in a wide range of permittivities $-2 < \mathrm{Re}(\varepsilon) < 1$. Nevertheless, such conditions aforementioned are not sufficient conditions to have a pulling force: a quite small longitudinal wavenumber β is still required in order to reduce the positive radiation pressure corresponding to the first two terms in Eq. (4).

From Eq. (4), it is evident that positive and large Poynting power along the z axis (namely propagation direction) increases the chance of obtaining negative pulling force. It has been unambiguously demonstrated in Fig. 1 that total longitudinal Poynting component is large and positive inside the particle when negative pulling force arises.

Precise modeling of the optical force valid for arbitrary radius R can be made by using Maxwell stress tensor $\hat{\mathbf{T}} = (1/2)\mathrm{Re}[\varepsilon_0\tilde{\mathbf{E}}\otimes\tilde{\mathbf{E}}^* + \mu_0\tilde{\mathbf{H}}\otimes\tilde{\mathbf{H}}^* - (\varepsilon_0|\tilde{\mathbf{E}}|^2 + \mu_0|\tilde{\mathbf{H}}|^2)\hat{\mathbf{I}}/2]$,

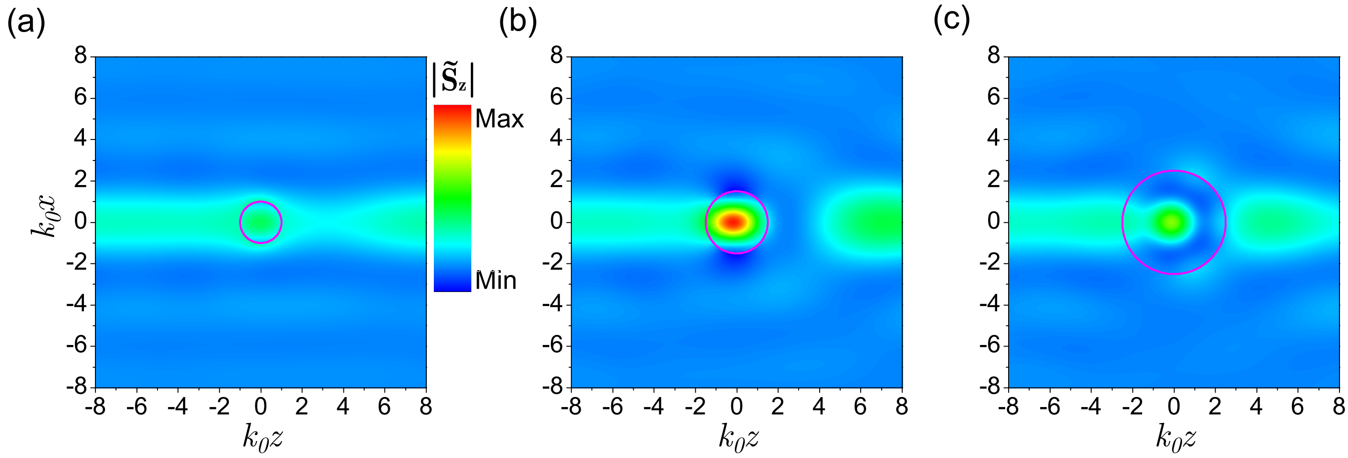

FIG. 1. (Color online) Distribution of the longitudinal Poynting component for particles at the beam axis $d = 0$ for various radiuses, (a) $k_0R = 1.0$, (b) $k_0R = 1.5$, and (c) $k_0R = 2.5$. Parameters: $\varepsilon = 3.9$, $\mu = 1$, $m = 1$, $\alpha = 70°$, $c_1 = 1$, $c_2 = i$. The longitudinal Poynting component is greatly enhanced in the center of the particle, providing large and positive axial Poynting component to give rise to pulling force.

where the fields $\tilde{\mathbf{E}} = \mathbf{E} + \mathbf{E}_{sc}$ and $\tilde{\mathbf{H}} = \mathbf{H} + \mathbf{H}_{sc}$ are the sum of the incident and scattered fields. Then the optical force acting on a particle is the result of the integration over any surface σ embracing the particle: $\langle \mathbf{F} \rangle = \int_\sigma (\mathbf{n} \cdot \hat{\mathbf{T}}) ds$. We use exactly this accurate calculation technique through the whole paper.

2. Distribution of the longitudinal Poynting component for the situations of pushing force and pulling force.

The interaction between Bessel beam and particles can generate various Poynting vectors, including concentrate the Poynting vectors. For small particles (Fig. 1(a)), the interference is weak and the beam shape is preserved. By using appropriate Bessel beam and proper particle size (Fig. 1(b)), much longitudinal Poynting component can be focused at the center of the particle due to the enhanced interaction. According to the expression of optical force $\langle F_z \rangle$, the large positive longitudinal Poynting component inside the particle is the origin of pulling force. For larger particles, higher-order multipoles are developed, thus the dipole approximation is no longer valid.

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