Gain-assisted transformation optics

Tiancheng Han,[1,2] Cheng-Wei Qiu,[2,*] Jiaming Hao,[3] Xiaohong Tang,[1] and Said Zouhdi[3]

[1]*EHF Key Laboratory of Fundamental Science, School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China.*
[2]*Department of Electrical and Computer Engineering, National University of Singapore, Kent Ridge, Singapore 119620, Republic of Singapore.*
[3]*Laboratoire de Genie Electrique de Paris, SUPELEC, Plateau de Moulon 91192, Gif-sur-Yvette, France*
**eleqc@nus.edu.sg*

Abstract: Loss severely degrades the cloaking effect of the device designed by traditional transformation. In this letter, we propose gain-assisted transformation optics to overcome the loss problem by introducing gain media into a spherical cloak. The gain media, which can amplify the electromagnetic fields, is controlled precisely to compensate the inherent loss in experimental realization of cloaks. We discuss the significance of controlling embedded gain materials in the context of the inverse design mechanism, which allows us to wisely select realizable materials with constant gain and loss along the radius. For practical realizations, isotropic spherical gain-assisted cloak is designed. Full-wave simulations validate the proposed design concept, which can be utilized to alleviate the inevitable loss problem in transformational optical devices.

OCIS codes: (160.1190) Anisotropic optical materials; (160.3918) Metamaterials; (230.0230) Optical devices; (260.2065) Effective medium theory.

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#143723 - $15.00 USD Received 7 Mar 2011; accepted 28 Mar 2011; published 18 Apr 2011

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1. Introduction

Based on conformal mapping method [1] and coordinate transformation method [2], different kinds of cloaks have been investigated [3,4]. The researches on invisibility cloaks are not limited to theoretical analysis [1–4], some experimental demonstrations were conducted at microwave frequencies [5,6] or optical frequencies [7] with the assistance of the metamaterials and nanophotonics.

However, the inevitable loss will appear in the process of constructing artificial materials. Generally the losses are orders of magnitude too large for the proposed applications, and the reduction of losses with optimized designs seems to be out of reach. More recently, it is reported that the existence of loss will significantly frustrate the performance of invisibility cloaks [8,9]. Namely, the loss problem plagues the invisibility cloaks and is one of the major restrictions preventing them from wider industrial applications. One solution is to use low-loss dielectric uniaxial materials as macroscopic cloaks experimentally verified in [10,11]. However, the transformation applies to a particular geometry resulting in homogeneous uniaxial materials and also uses high-permittivity background medium to scale up the cloak's parameters (all bigger than, so as to use available calcites [10,11]). When inhomogeneities or parameters smaller than unity are inevitable, another possible solution to this bottleneck is the use of active materials, which can amplify the waves and compensate the loss [12–14]. Unfortunately, the traditional transformation optics usually involve only ideal lossless passive media and cannot overcome the loss problem effectively.

In this work, gain-assisted transformation optics is proposed to overcome the loss problem of 3D spherical invisibility cloaks. Different from traditional transformation that usually involves only ideal lossless passive media, the proposed transformation involves lossy media with negative imaginary part and gain media with positive imaginary part. The gain-assisted transformation has the advantage of traditional transformation of controlling the wave transmission, but compensates its deficiency in dealing with loss problem. Once the loss in cloak is determined, the needed active media can be obtained according to the theory. By compensating the attenuation caused by the lossy parameters, it is, a priori, possible to realize an overall ideal cloak.

2. Theoretical analysis

Gain-assisted transformation optics is introduced with the assistance of inverse design method [15], in which the constitutive parameters can be independently obtained without any knowledge of the corresponding coordinate transformation. We adopt this inverse design because it allows us to effectively spot a simple gain material that is homogenous. The simulations are carried out in MathematicaTM. We know that a spherical cloak is obtained by compressing a spherical region $(r' \le b)$ in virtual space (r', θ', φ') into a shell region $(a \le r \le b)$ in real space (r, θ, φ). The well-known expression for the constitutive parameters in the real space transformed from vacuum is

$$\bar{\bar{\varepsilon}} / \varepsilon_0 = \bar{\bar{\mu}} / \mu_0 = \bar{\bar{J}} \bullet \bar{\bar{J}}^{\mathrm{T}} / \det(\bar{\bar{J}}), \tag{1}$$

where $\bar{\bar{J}} = \partial(r, \theta, \phi) / \partial(r', \theta', \phi')$ is the Jacobian matrix. The transformation is operated along the radial direction $(\theta' = \theta \text{ and } \varphi' = \phi)$, and thus the Jacobian matrix is diagonal, though we still have no information of what the coordinate transformation is. The constitutive parameters can be expressed as

$$\bar{\bar{\varepsilon}} / \varepsilon_0 = \bar{\bar{\mu}} / \mu_0 = \mathrm{diag}\left[\xi_{\mathrm{r}}(r), \xi_{\mathrm{t}}(r), \xi_{\mathrm{t}}(r)\right], \tag{2}$$

#143723 - $15.00 USD Received 7 Mar 2011; accepted 28 Mar 2011; published 18 Apr 2011

where $\xi_r = \left(\frac{r'}{r}\right)^2 \frac{dr}{dr'}$ and $\xi_t = \frac{dr'}{dr}$. By integrating $\xi_t = \frac{dr'}{dr}$, one can obtain $r' = \int_a^r \xi_t(r_1)dr_1 + C_0$, where C_0 is a constant. Due to the boundary condition ($r'=0$ when $r=a$), we have $C_0=0$. Another condition ($r'=b$ when $r=b$) leads to the normalization condition

$$b = \int_a^b \xi_t(r_1)dr_1. \tag{3}$$

Different from the previous study [15] that regarded constitutive parameters to be real, here the parameters are complex. By setting $\xi_t(r) = \mathrm{Re}\left[\xi_t(r)\right] + i\,\mathrm{Im}\left[\xi_t(r)\right]$ and substituting it into Eq. (3), one can obtain

$$b = \int_a^b \mathrm{Re}[\xi_t(r_1)]dr_1, \tag{4a}$$

$$0 = \int_a^b \mathrm{Im}[\xi_t(r_1)]dr_1. \tag{4b}$$

Then we introduce $g(r)$ to generate the real part of the transverse parameter, which is proportional to $\mathrm{Re}(\xi_t)$, i.e., $\mathrm{Re}(\xi_t) = d_0 g(r)$, where d_0 is an arbitrary constant. According to Eq. (4a), $\mathrm{Re}(\xi_t)$ can be expressed

$$\mathrm{Re}[\xi_t] = bg(r) / \int_a^b g(r_1)dr_1. \tag{5a}$$

Observing Eq. (4b), if we can find a function $F(r) = \int \mathrm{Im}[\xi_t(r)]dr$ (generating the imaginary part of the transverse parameter) that satisfies $F(a) = F(b)$, we can thus obtain

$$\mathrm{Im}[\xi_t] = \frac{dF(r)}{dr} = F'(r). \tag{5b}$$

Then the transverse parameter can be represented

$$\xi_t(r) = bg(r) / \int_a^b g(r_1)dr_1 + iF'(r). \tag{6}$$

From the identity $\xi_r \bullet \xi_t = (r'/r)^2$ and $r' = \int_a^r \xi_t(r_1)dr_1$, the radial parameter can be determined

$$\xi_r(r) = \left(\frac{b\int_a^r g(r_1)dr_1}{r\int_a^b g(r_1)dr_1} + i\frac{F(r)-F(a)}{r}\right)^2 \Bigg/ \left(\frac{bg(r)}{\int_a^b g(r_1)dr_1} + iF'(r)\right). \tag{7}$$

Nevertheless, the unknown coordinate transformation can be found

$$r' = b\int_a^r g(r_1)dr_1 \Big/ \int_a^b g(r_1)dr_1 + i[F(r) - F(a)]. \tag{8}$$

To validate the proposed method, we select a specific generating function $g(r)$ = 1 and $F(r) = C\left|r-(a+b)/2\right|$, where C is a constant. Its corresponding coordinate transformation can be expressed as

$$r' = b(r-a)/(b-a) + iC\left[\left|r-(a+b)/2\right| - (b-a)/2\right]. \tag{9}$$

Then the transverse and radial parameters can be derived

$$\xi_t(r) = \frac{b}{b-a} + iCu(r - \frac{a+b}{2}), \tag{10a}$$

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$$\xi_{\mathrm{r}}(r)=\left(\frac{b(r-a)}{(b-a)}+iC\left|r-\frac{a+b}{2}\right|-iC\frac{b-a}{2}\right)^2\Big/r^2\xi_{\mathrm{t}}(r),\qquad(10\mathrm{b})$$

where $u(r)=\begin{cases}-1 & r<0\\ 1 & r\geq 0\end{cases}$.

Obviously, If C = 0, Eq. (9) degenerates into a traditional transformation, and the complex constitutive parameters in Eq. (10) degenerate into real parameters [1].

3. Simulation results and discussion

When a = 0.1 m and b = 0.2 m in Eq. (10), Fig. 1 shows the complex permittivity composed of real part and imaginary part. The imaginary part can be negative and positive constants corresponding to lossy or active materials respectively. Obviously, attenuation and amplification of the EM field will coexist in the realized cloak. Finally, the gain-assisted cloak would behave effectively like a lossless one, though the constitutive parameters are non-ideal. From Fig. 1, it is found that the selection of the value of C significantly affects the material parameters, especially the imaginary part of the parameters. Increasing C may lead to bigger imaginary parts of the parameters, and if C approaches zero, the constitutive parameters will approach the values obtained by the traditional transformation. More importantly, the imaginary part of the transverse parameter keeps homogeneous in Fig. 1(d), which represents the maximum loss or gain according to Eq. (10)a).

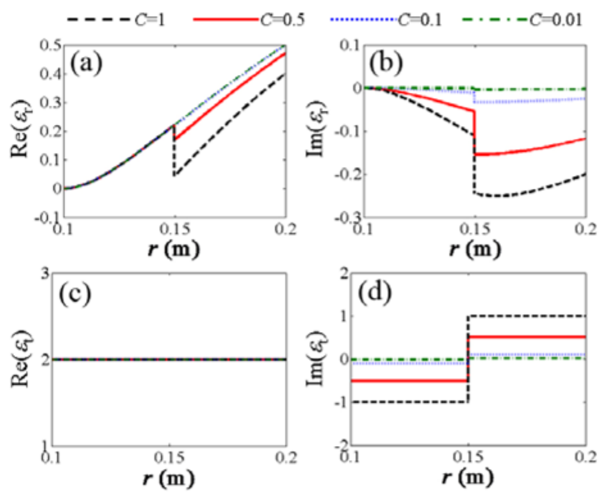

Fig. 1. Permittivity along the radial direction with different C. (a) Real part of radial parameter. (b) Imaginary part of radial parameter. (c) Real part of transverse parameter. (d) Imaginary part of transverse parameter. The permeability curve will be exactly the same since $\mu_{\mathrm{r}}=\varepsilon_{\mathrm{r}}$ and $\mu_{\mathrm{t}}=\varepsilon_{\mathrm{t}}$, resulting in impedance match.

In what follows, we show the full-wave simulations to validate the proposed method. The incident plane wave is along z-axis and the frequency is 2 GHz. A dielectric spherical core is placed in cloaking region with refractive index n = 1.45. Figure 2 shows the total electric field distributions of the gain-assisted cloak with C = 0.5 in x-z plane (Fig. 2(a)) and y-z plane (Fig. 2(b)). As predicted, the EM waves smoothly flow around the cloaked region and then restore their original propagation paths. However, compared to the traditional cloak [16], it is observed that the field distribution is asymmetric in the shell region. This is because the gain-assisted cloak is composed of loss materials and gain materials, hence the phenomena of loss (corresponding to the field attenuation) and compensation (corresponding to the field amplifying) in the shell region can be found.

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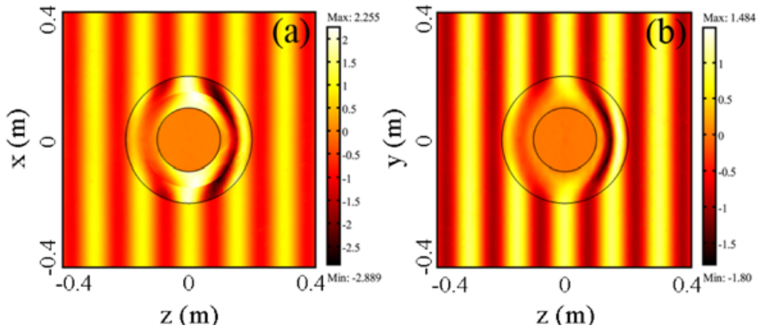

Fig. 2. Snapshots of the total electric field for the spherical gain-assisted cloak with $C = 0.5$. (a) x-z plane, (b) y-z plane.

In order to examine the compensation effect quantitatively, we calculate the cross section of the gain-assisted cloak in Fig. 2 with different amounts of gain embedded, as shown in Fig. 3(a). Obviously, the fully compensated cloak (corresponding to 100% gain) performs much more perfectly than the non-compensated cloak (corresponding to 0% gain), though both of them have very small backscattering which accords very well with the theoretical results. When the gain is introduced insufficiently (corresponding to 50% gain) or overly (corresponding to 150% gain), the backward scattering is very small though evident scattering is induced in the forward direction. To give more insights, the scattering cross sections (σ) normalized by that of the non-compensated cloak (σL) are plotted versus the percentage of the gain (from 0 to 200%) in Fig. 3(b). Diverging from the ideal compensation point (100% gain), σ always increases regardless of that the gain is either decreasing or increasing. Nevertheless, the insufficient compensation makes σ increase much faster than the over compensation. Approximately above 40% gain, the total scattering cross section can decrease more than 10 dB compared to the non-compensated case. It should be noted that if it is difficult to introduce exactly 100% gain, it is better to introduce slightly more gain.

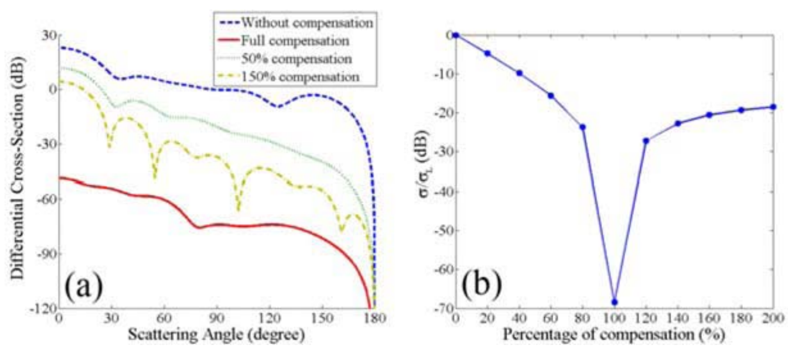

Fig. 3. (a) Differential cross section (exactly the same definition in Ref [15]) for the gain-assisted cloak with different compensation when C = 0.5. (b) The normalized total scattering cross sections σ/σL (normalized by σL of the loss cloak with non-compensation) as a function of the gain introduced. The scattering cross section is defined exactly the same as that in Ref [17].

The spherical gain-assisted cloak could be easily realized through alternating layered isotropic mediums [16]. According to effective medium theory, the shell region is equally discretized into N layers, and each layer consists of medium-A and medium-B with equal thickness. Here we introduce two kinds of parameter profiles including old set and new set. The old set of material parameters for isotropic medium-A and medium-B is $\varepsilon_{\rm A} = \mu_{\rm A} = \xi_{\rm t} + \sqrt{\xi_{\rm t}^2 - \xi_{\rm t}\xi_{\rm r}}$ and $\varepsilon_{\rm B} = \mu_{\rm B} = \xi_{\rm t} - \sqrt{\xi_{\rm t}^2 - \xi_{\rm t}\xi_{\rm r}}$ respectively, and the new set of material parameters of isotropic medium-A and medium-B is $\varepsilon_A = \mu_B = \xi_t + \sqrt{\xi_t^2 - \xi_t\xi_r}$ and $\varepsilon_B = \mu_A = \xi_t - \sqrt{\xi_t^2 - \xi_t\xi_r}$ respectively. It is obvious that the old set corresponds to impedance matching and the new set corresponds to refractive index matching. Figure 4(a) illustrates the real part and imaginary part of medium-A and medium-B. It is found that the parameters of medium-A and media-B keep nearly invariant along the radius, and this property will make the fabrication easier. We also calculate the differential cross section of the isotropic cloak in Fig. 4(b). Clearly, the forward scattering of the old set is large, which can be improved much more by considering the new set. Although there is a small sacrifice in backscattering of the new set compared with the backscattering of the old set, it is evidently worthwhile since the backscattering of the new set is still negligible. It should be noted that the performance is quite pronounced when the discretization increases.

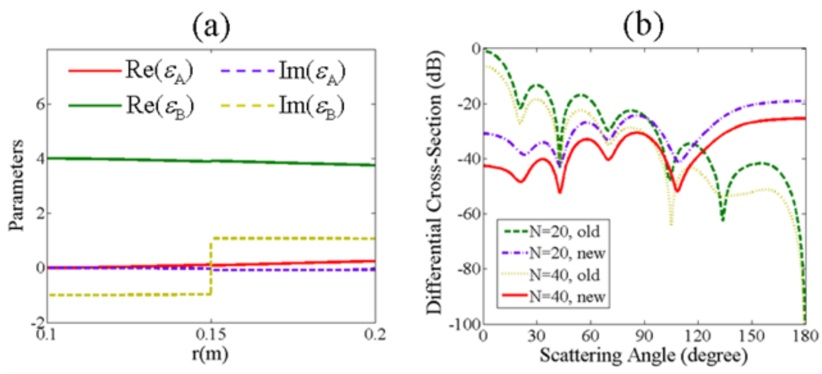

Fig. 4. (a) Constitutive parameters of media-A and media-B along radial direction for isotropic cloak with C = 0.5. (b) Differential cross section for isotropic cloaks designed by the old set and new set parameter profiles with different layer number (N = 20 and N = 40).

4. Conclusion

To conclude, we propose a general transformation that can degenerate into traditional transformation, namely gain-assisted transformation optics, which provides a powerful theoretical tool to overcome the loss problem in invisibility cloaks. Although the current gain materials are not enough to compensate large metallic loss [18], certain heavily doped semiconductors shed the bright light to this problem owing to the low loss. Nevertheless, this paper theoretically considers material's inevitable loss (high or low) into the design process and compensates it by introducing the gain of active materials properly, though the model is still magnetic now. The current method is not limited to the design of invisibility cloak, but can also be widely used in designing other transformational devices, such as perfect lenses, rotators, and illusion devices. It paves an alternative path of alleviating the inevitable loss problem in practical transformational devices.

Acknowledgment

This research was supported by National University of Singapore by grant R-263-000-574-133. It was also supported by France-Singapore collaboration project "CNRS PICS/LIA/GDRI 2009". T.C. Han is working towards his PhD in National University of Singapore.