Supplementary Information

Experimental Demonstration of Bilayer Thermal Cloak

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1. Theoretical analysis of the bilayer thermal cloak

We start from the general three-dimensional (3D) theoretical analysis, directly from thermal conduction equation. Heat flows spontaneously from a high temperature region toward a low temperature region. For a steady state situation without a heat source, the temperature satisfies $\nabla \cdot (\kappa \nabla T) = 0$ in all the regions of space, and can be generally expressed as

$$T_i = \sum_{m=1}^{\infty} \left[A_m^i r^m + B_m^i r^{-m-1} \right] P_m(\cos\theta) \tag{1}$$

where A_m^i and B_m^i (i=1, 2, 3, 4) are constants to be determined by the boundary conditions and T_i denotes the temperature in different regions: i=1 for the cloaking region ($r<a$), i=2 for the inner layer ($a<r<b$), i=3 for the outer layer ($b<r<c$), and i=4 for the exterior region ($r>c$).

A temperature distribution with uniform temperature gradient t_0 is externally applied in the z direction. Taking into account that T_4 should tend to $-t_0 r\cos\theta$ when $r \to \infty$, we only need to consider m=1. Because T_1 is limited when $r \to 0$, we can obtain $B_1^1 = 0$. Owing to the temperature potential and the normal component of heat flux vector being continuous across the interfaces, we have

$$\begin{cases} T_i|_{r=a,\,b,\,c} = T_{i+1}|_{r=a,\,b,\,c} \\ \kappa_i \left.\dfrac{\partial T_i}{\partial r}\right|_{r=a,\,b,\,c} = \kappa_{i+1} \left.\dfrac{\partial T_{i+1}}{\partial r}\right|_{r=a,\,b,\,c} \end{cases} \quad (2)$$

Here, $\kappa_4 = \kappa_b$ and κ_1 is the thermal conductivity of the cloaking object. Considering that the inner layer is perfect insulation material, i.e., $\kappa_2 = 0$, this ensures that an external field does not penetrate inside the cloaking region and the only task is to eliminate the external-field distortion. By substituting Eq. (1) into Eq. (2), we obtain

$$B_1^4 = t_0 c^3 \frac{\kappa_3(2c^3 - 2b^3) - \kappa_b(2c^3 + b^3)}{\kappa_3(2c^3 - 2b^3) + 2\kappa_b(2c^3 + b^3)} \quad (3)$$

By setting $B_1^4 = 0$, we obtain

$$\kappa_3 = \frac{2c^3 + b^3}{2(c^3 - b^3)} \kappa_b \quad (4)$$

Obviously, an ideal bilayer thermal cloak may be achieved as long as Eq. (4) is fulfilled. Eq. (4) shows that the third parameter can be uniquely determined if any two of κ_3, κ_b, c/b are known. If κ_3 and κ_b are given, *c* is proportional to *b*, which means that the geometrical size of the cloak can be arbitrarily tuned without changing the materials of the bilayer cloak.

2. Simulation for 3D thermal cloak

The performance of the proposed bilayer cloak is numerically verified based on a finite element method (FEM), in which the materials of the bilayer cloak are chosen to be the same as those in later experiments: inner layer, outer layer, and background material are expanded polystyrene, an alloy, and a cured sealant with conductivity of 0.03 W/mK, 9.8 W/mK, and 2.3 W/mK, respectively. For a 3D bilayer cloak, we choose *a*=6 mm, *b*=9 mm, and *c*=10.2 mm. An aluminum sphere with conductivity of 205 W/mK is placed in the cloaking region. Fig. S1 shows the simulated temperature distribution, in which (a) and (d) illustrate the perturbation (aluminum sphere) without and with the bilayer cloak, respectively. As expected, the central region without cloak makes the isothermal surfaces significantly distorted, thus rendering the object visible (detected). However, when the central region is wrapped by the bilayer cloak, the isothermal surfaces outside the cloak are restored exactly without distortion as if there was nothing. Figs. S1(b) and S1(c) show the temperature profiles of the perturbation with a single layer of alloy and expanded polystyrene, respectively, in which the single layers have the same thickness as the bilayer cloak of 4.2 mm. It is clear that the isothermal surfaces are significantly distorted in both cases. In Fig. S1(c), though the central region is protected similar to the bilayer cloak (thermal flux goes around the central region), the external field is severely distorted.

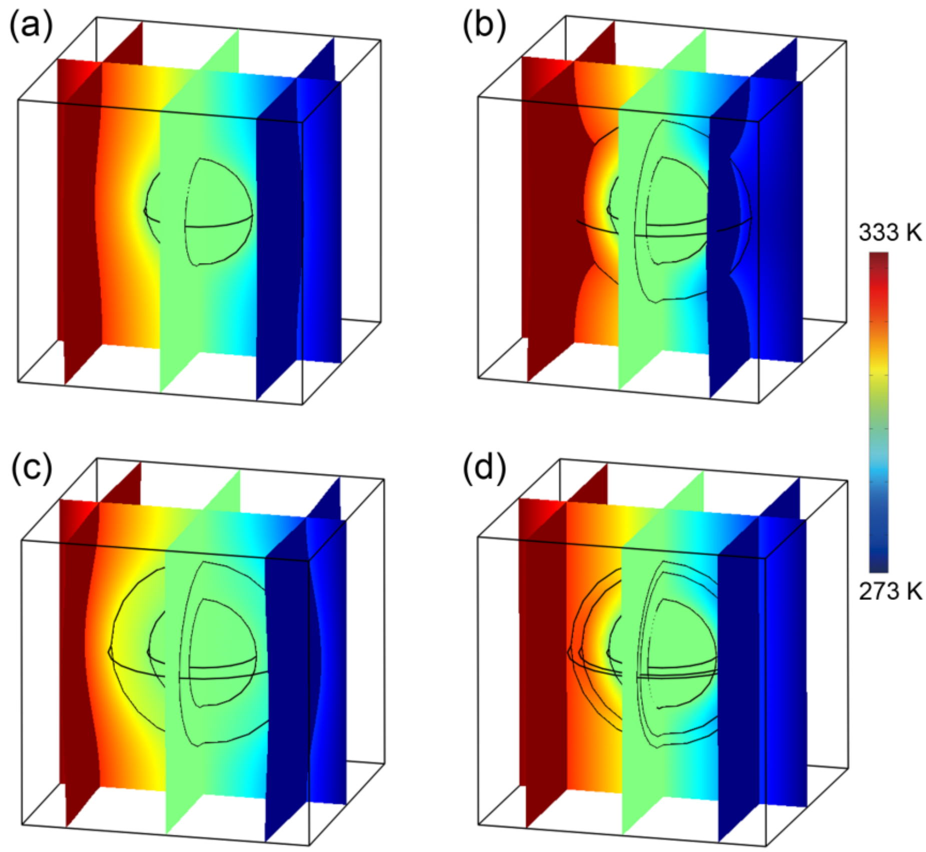

Fig. S1 Simulated temperature distributions for a 3D bilayer cloak with a=6 mm, b=9 mm, c=10.2 mm. (a) A bare perturbation (object) with radius of 6 mm. (b) The object is covered by a single layer of polystyrene with thickness of 4.2 mm. (c) The object is covered by a single layer of alloy with thickness of 4.2 mm. (d) The object is wrapped by the proposed bilayer cloak. Three isothermal surfaces are also represented in panel.

3. Simulation for half 3D thermal cloak

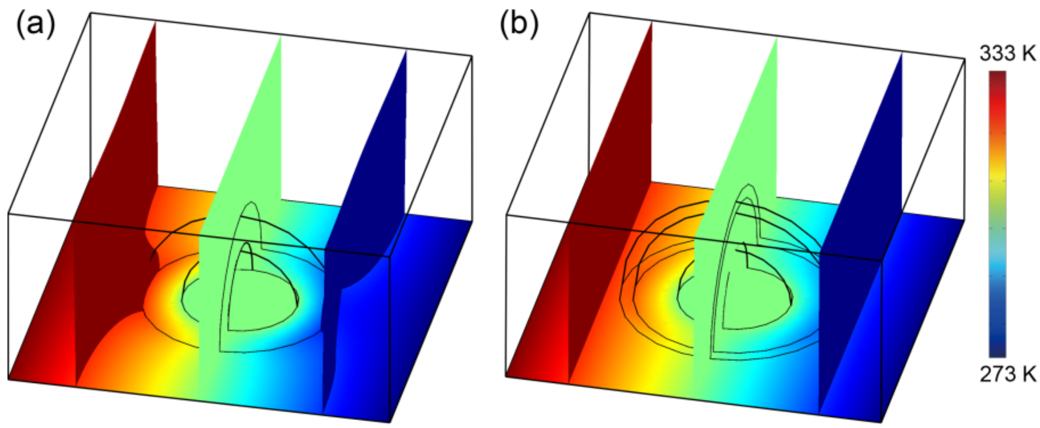

Fig. S2 Simulated temperature distributions for a half 3D bilayer cloak with *a*=6 mm, *b*=9 mm, *c*=10.2 mm. (a) A half aluminum sphere is covered by a single layer of polystyrene with thickness of 4.2 mm. (b) The half aluminum sphere is wrapped by the proposed bilayer cloak. Three isothermal surfaces are also represented in panel.

4. Quantitative comparison

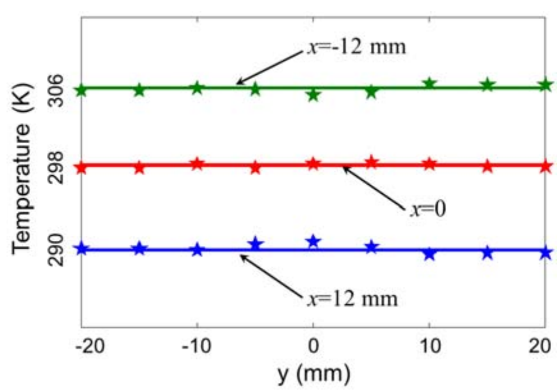

Fig. S3 Simulated (solid lines) and measured (star lines) temperature distributions of thermal cloak in Fig. 2(e) and 2(f).

5. Simulated time-dependent temperature distributions

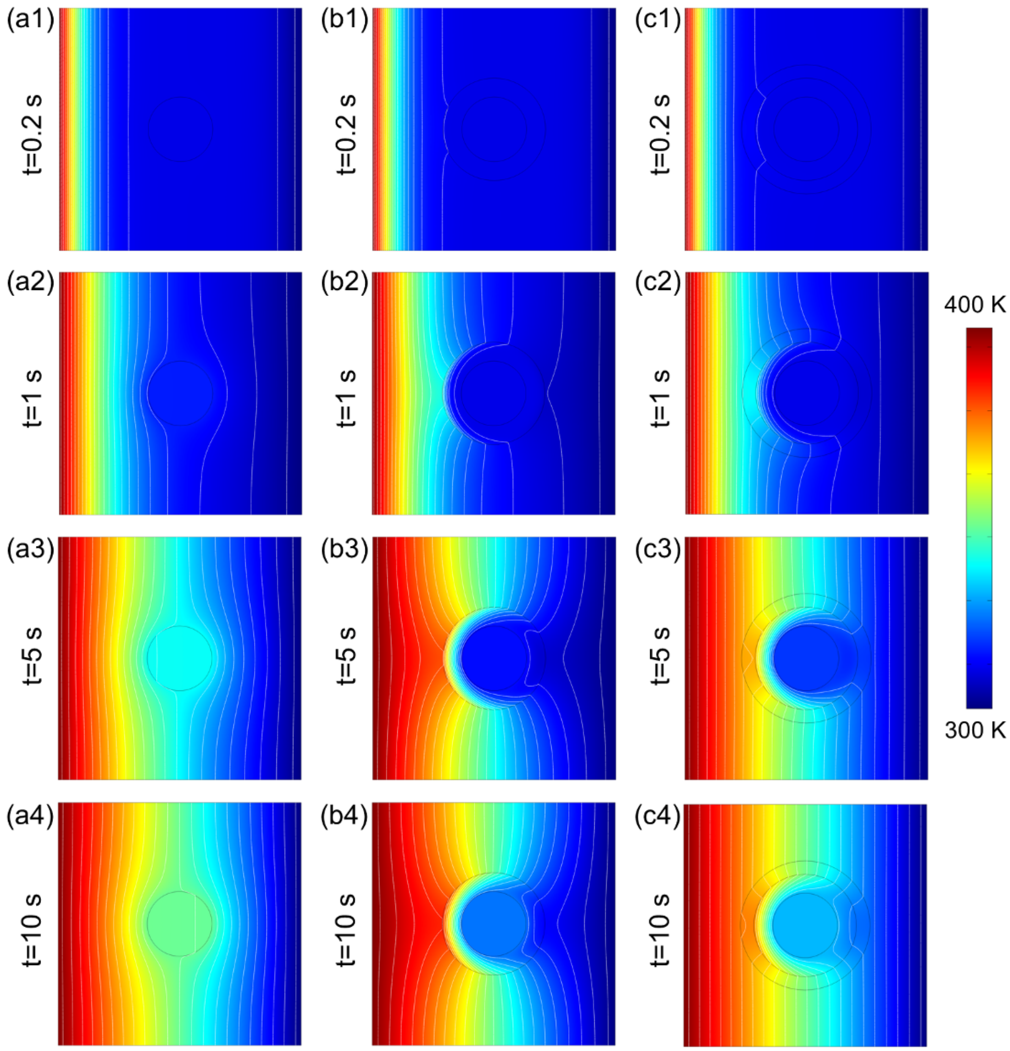

Fig. S4 Simulated temperature distributions at different times *t* as indicated. The first to third columns correspond to a bare perturbation, the perturbation with a single layer of insulation, and bilayer cloak, respectively. The geometric parameters and material parameters in simulations completely accord with those in experiments. Simulated movies will be provided as Supplementary Movies S1, S2, and S3 on the link (http://www.ece.nus.edu.sg/stfpage/eleqc/S1.avi; http://www.ece.nus.edu.sg/stfpage/eleqc/S2.avi; http://www.ece.nus.edu.sg/stfpage/eleqc/S3.avi), corresponding to Figs. S4(a), (b), and (c), respectively.

6. Oblique incidence

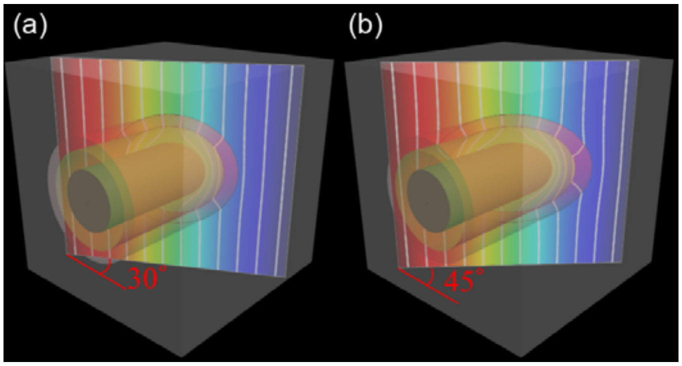

Fig. S5 Simulated temperature distributions for bilayer cloak when the heat conduction is not confined into *x*-*y* plane. (a) Oblique incidence with 30°. (b) Oblique incidence with 45°. Isothermal lines are superimposed as white lines in panel.