

Spherical cloaking with homogeneous isotropic multilayered structures

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We propose a practical realization of electromagnetic spherical cloaking by layered structure of homogeneous isotropic materials. By mimicking the classic anisotropic cloak by many alternating thin layers of isotropic dielectrics, the permittivity and permeability in each isotropic layer can be properly determined by effective medium theory in order to achieve invisibility. The model greatly facilitates modeling by Mie theory and realization by multilayer coating of dielectrics. Eigenmode analysis is also presented to provide insights of the discretization in multilayers.

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Recently, invisibility cloaks [1–4] have received great attention since Pendry *et al.* [5] suggested that an object coated by an inhomogeneous anisotropic shell becomes invisible to electromagnetic waves. The electromagnetic wave interaction with such a cloak was analyzed [6]. In addition to the common method of geometric transformation, various approaches have been proposed to create invisibility cloaks, such as plasmonic resonances [7], scattering cancellation [8], negative index material [9], and stationary Schrödinger equation [10]. Attempts to realize the cloaking idea have been initiated with encouraging results [11].

Conventional cloaks need to be anisotropic and their parameters are functions of position. Although this restriction can be bypassed with the simplified material parameters [11], later it was found that the coating established by simplified parameters cannot perfectly shield the targeted object inside without electromagnetic perturbation to external EM fields [12]. However, the cylindrical invisibility cloak is still difficult to realize due to the limited resource of natural materials exhibiting radial anisotropy [13], not to mention that those tensorial parameters are spatially varying. In this connection, Cai *et al.* [14] investigated a multilayered cylindrical cloak by discretizing the conventional position-dependent cloak into many layered coatings, and the material in each layer is position independent but still anisotropic. Huang *et al.* [15] proposed a cylindrical cloak by replacing one anisotropic coating with a multilayered structure in which each layer is constructed with a certain homogeneous isotropic medium. Some recent works on cylindrical acoustic cloaks were also reported [16,17]. When each layer is thin enough, the effective medium theory can be employed to find out the required permittivity for each layer [18]. More recently, this idea was further applied to three-dimensional acoustic cloak by multilayered isotropic materials [19].

As for electromagnetic spherical invisibility cloaks, material parameters for the perfect cloak are suggested [5]:

$$\epsilon_r = \mu_r = \left(\frac{b}{b-a}\right)\left(\frac{r-a}{r}\right)^2, \quad (1)$$

$$\epsilon_\theta = \mu_\theta = \epsilon_\phi = \mu_\phi = \frac{b}{b-a}, \quad (2)$$

where the cloak occupies the spherical region ($a < r < b$) and its constitutive material parameters are given by

$$\bar{\epsilon} = \epsilon_0 \begin{bmatrix} \epsilon_r & 0 & 0 \\ 0 & \epsilon_t & 0 \\ 0 & 0 & \epsilon_t \end{bmatrix}, \quad \bar{\mu} = \mu_0 \begin{bmatrix} \mu_r & 0 & 0 \\ 0 & \mu_t & 0 \\ 0 & 0 & \mu_t \end{bmatrix}, \quad (3)$$

where the unit dyad is $\bar{I} = \hat{r}\hat{r} + \hat{\theta}\hat{\theta} + \hat{\phi}\hat{\phi}$ and the subscripts r and t denote the parameters along radial (\hat{r}) and tangential direction ($\hat{\theta}$ or $\hat{\phi}$), respectively. The anisotropic cloaking proposed by Pendry *et al.* intrinsically requires the radial parameters (ϵ_r and μ_r) at the innermost boundary $r=a$ to be zero. Also, the position-dependent anisotropic material is a strict restriction for practical realization. In this Brief Report, we will start from the analysis of eigenmodes in an anisotropic medium and present an alternative way to realize spherical cloaks by a series of layered isotropic materials as shown in Fig. 1, while the cloaking effects are still maintained. Thus, Mie theory can be directly applied to study this multilayered structure instead of tailoring Mie scattering model to consider the radial anisotropy.

In Ref. [20], it has been proved that the TE and TM waves are decoupled if off-axis elements are zero, i.e., the uniaxial form as in Eq. (3). The radial components of \mathbf{E} and \mathbf{H} fields have been derived in a more general case (the material studied here is only a subset). Similarly, if we solve the Maxwell equations with two decoupled scalar Debye potentials (i.e., Φ_{TE} and Φ_{TM}) [21] and equate the radial components of $\mathbf{B}_{\text{TE, TM}}$ and $\mathbf{D}_{\text{TE, TM}}$, we have

$$\frac{\epsilon_r}{\epsilon_t} \frac{\partial^2 \Phi_{\text{TM}}}{\partial r^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \Phi_{\text{TM}}}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Phi_{\text{TM}}}{\partial \phi^2} + \omega^2 \mu_0 \epsilon_0 \mu_t \epsilon_r \Phi_{\text{TM}} = 0, \quad (4)$$

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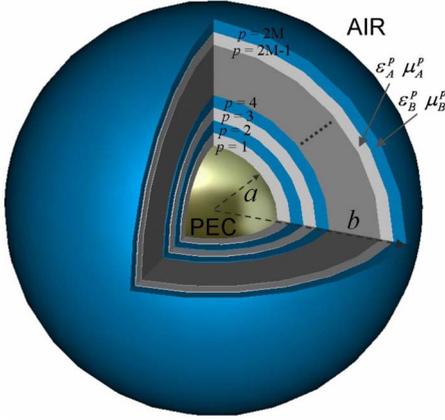


FIG. 1. (Color online) Geometries of the proposed spherical invisibility cloak. Parameters of multilayered isotropic coatings are defined in Cartesian coordinates. The superscript p denotes p th layer ($p=1, 2, \dots, 2M$). The inner core is a perfect electric conductor (PEC) with the radius $a=1\lambda$ and the outermost radius is $b=2\lambda$ which are fixed throughout. The thickness of every coated layer is identical, i.e., $\lambda/2M$.

$$\frac{\mu_r}{\mu_t} \frac{\partial^2 \Phi_{\text{TE}}}{\partial r^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \Phi_{\text{TE}}}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Phi_{\text{TE}}}{\partial \phi^2} + \omega^2 \mu_0 \epsilon_0 \mu_r \epsilon_t \Phi_{\text{TE}} = 0. \quad (5)$$

It can be seen that isotropic case [22] is just a subset. The method of variable separation, where $\Phi=R(r)T(\theta)H(\phi)$ is assumed, further leads us to

$$\left\{ \frac{\partial^2}{\partial r^2} + \left[k_t^2 - A_{e,m} \frac{n(n+1)}{r^2} \right] \right\} R(r) = 0, \quad (6)$$

where $k_t = k_0 \omega \sqrt{\epsilon_t \mu_t}$, $A_e = \epsilon_t / \epsilon_r$ (electric anisotropy ratio), and $A_m = \mu_t / \mu_r$ (magnetic anisotropy ratio). The idea here is to define $v_1(v_1+1) = n(n+1)A_e$ and $v_2(v_2+1) = n(n+1)A_m$, so that Eq. (6) can still fall in the domain of classical Mie theory except for a slight modification in the order of Bessel functions. Thus, the radial function can be expressed in terms of Ricatti-Bessel functions

$$R(r) = \sqrt{\pi k_t r / 2} j_{v_1, v_2}(k_t r), \quad (7)$$

where

$$v_1 = \sqrt{n(n+1)A_e + 1/4} - 1/2 \quad (8)$$

$$v_2 = \sqrt{n(n+1)A_m + 1/4} - 1/2. \quad (9)$$

It is clear that the radial anisotropies can be systematically considered in the order of Bessel functions without changing Mie theory for isotropic cases significantly. Since $T(\theta)$ and $H(\phi)$ are, respectively, associated Legendre polynomials and harmonic functions, we can expand the incident, scattered, and transmitted waves with certain expansion coefficients in terms of those potentials. Those expansion coefficients are to be determined from boundary conditions at each interface, but the derivation is suppressed.

An interesting thing arises when the material parameters in Eqs. (4) and (5) are defined by Eqs. (1) and (2), (i.e.,

$\epsilon_i = \epsilon_\theta = \epsilon_\phi$ and $\mu_i = \mu_\theta = \mu_\phi$). In such cases, the anisotropy ratios become $A_e = A_m = r^2 / (r-a)^2$, and thereby Eqs. (6) and (7) are respectively reduced to

$$\left\{ \frac{\partial^2}{\partial r^2} + \left[k_t^2 - \frac{n(n+1)}{(r-a)^2} \right] \right\} R(r) = 0, \quad (10)$$

$$R(r) = \sqrt{\pi k_t (r-a)} j_n[k_t (r-a)]. \quad (11)$$

One can see that eigenmodes in position-independent radial anisotropic materials consist of Bessel functions with complex order (can be any complex values in general) while eigenmodes in Pendry's cloak [5] become integer-order Bessel functions.

Thus, it comes to our mind that it is, in principle, possible to utilize many thin spherical layers of isotropic materials to mimic a conventional anisotropic cloak since the characteristics of eigenmodes are basically similar in both situations (Bessel functions of integer order). Thus, at a certain distance r , the position-dependent term [i.e., $k_t(r-a)$] in the Bessel function of Eq. (11) for Pendry's cloak can be asymptotically replaced with the term (i.e., $k_p r$) of a Bessel function corresponding to a properly designed isotropic medium. Eventually, the Pendry's cloak can be well mimicked by multilayered isotropic coatings, if M is big enough.

We first discretize the single anisotropic shell into $2M$ layers with identical thickness, and then the radius of each layer in Fig. 1 could be determined as

$$r_p = a + p \frac{b-a}{2M}, \quad p = 1, 2, \dots, 2M. \quad (12)$$

The thickness of each layer should be much less than the wavelength, i.e., the number M needs to be sufficiently large. Then the discrete material parameters for each layer can be obtained by substituting Eq. (12) into Eq. (1). Effective medium theory is applied to design parameters of these two types of alternating isotropic layered materials. From Sten's formula [23],

$$\sigma_\theta = \sigma_\phi = (\sigma_A^p + \sigma_B^p) / 2, \quad (13)$$

$$\frac{1}{\sigma_r} = \frac{1}{2\sigma_A^p} + \frac{1}{2\sigma_B^p} \quad (\sigma = \epsilon \text{ or } \mu), \quad (14)$$

one can obtain the equivalent medium parameters for the isotropic layered structure when the thickness of each layer is identical

$$\epsilon_A^p = \epsilon_\theta - \sqrt{\epsilon_\theta^2 - \epsilon_\theta \epsilon_r}, \quad (15)$$

$$\epsilon_B^p = \epsilon_\theta + \sqrt{\epsilon_\theta^2 - \epsilon_\theta \epsilon_r}, \quad (16)$$

$$\mu_A^p = \mu_\theta - \sqrt{\mu_\theta^2 - \mu_\theta \mu_r}, \quad (17)$$

$$\mu_B^p = \mu_\theta + \sqrt{\mu_\theta^2 - \mu_\theta \mu_r}. \quad (18)$$

Both type- A and type- B materials are magnetic materials with nonunity permeability. In contrast to the cylindrical case, material parameters in Eqs. (1) and (2) cannot be per-

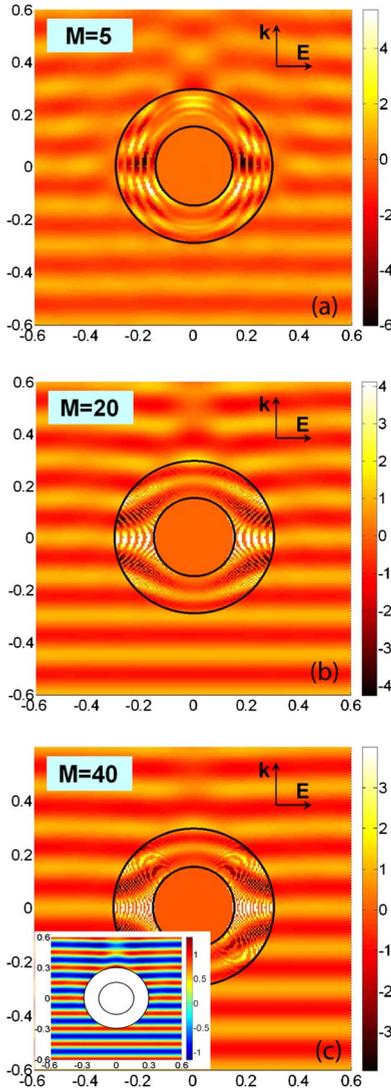


FIG. 2. (Color online) The real part of electric field E_x on z - x plane for a plane wave polarized in x direction and propagating along z direction. The geometry of the proposed cloaking (case I) has been illustrated in Fig. 1, and the operating frequency is $f=2$ GHz. The inset in (c) shows the electric field outside the cloak only.

factly reduced to a nonmagnetic case while the cloaking effects can still be maintained.

The simulation results of the proposed cloaking structure (Case I: PEC-A-B-A-B-....., from inside out; Case II: PEC-B-A-B-A-....., from inside out) illuminated with a plane wave is shown in Fig. 2. Invisibility performance is quite pronounced when the layer number becomes large enough [e.g., the insets in Fig. 2(c) and Fig. 3(c) which only show the fields in the region $r > b$]. It can be speculated that the change in the sequence of type-A and type-B materials will have no remarkable impact upon the cloaking effects if the layer number is sufficiently large, which can be verified from the comparison between Figs. 2(c) and 3(c). However, when we have difficulties to coat so many thin layers which follow the parameter designs in Eqs. (15)–(18), we have to terminate the coating process at a certain medium value of M

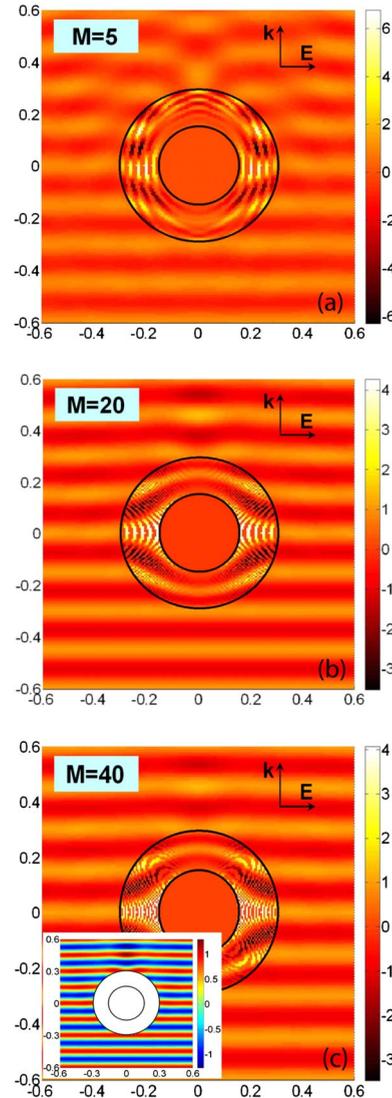


FIG. 3. (Color online) The real part of E_x on z - x plane. The geometry of the proposed cloaking (case II) has been illustrated in Fig. 1 except changing the sequence of A-B into B-A from inside out. The operating frequency is $f=2$ GHz. The inset in (c) shows the electric field outside the cloak only.

considering the fabrication cost and implementation difficulty. Under such circumstances (e.g., $M=20$), it is found that, even though the invisibility performance still holds in the region $r > b$ for both case I [see Fig. 2(b)] and case II [see Fig. 3(b)], the peak value of E_x in Fig. 3(b) (case II) within the cloaking region is smaller than that in Fig. 2(b) (case I). This will result in lower RCS in the far zone because the cloaked PEC is less “visible.” It is worth noting that the impedances of cases I and II are the same, but case II exhibits better cloaking effects because its refractive index of the outermost layer is closer to that of free space than that in case I.

To further demonstrate the capability of the proposed spherical cloakings, we present the bistatic radar cross section (RCS) of the multilayered isotropic structure for cloaking case II in Fig. 4. Mie theory is employed to calculate the bistatic RCS. It can be clearly seen that, when the layer num-

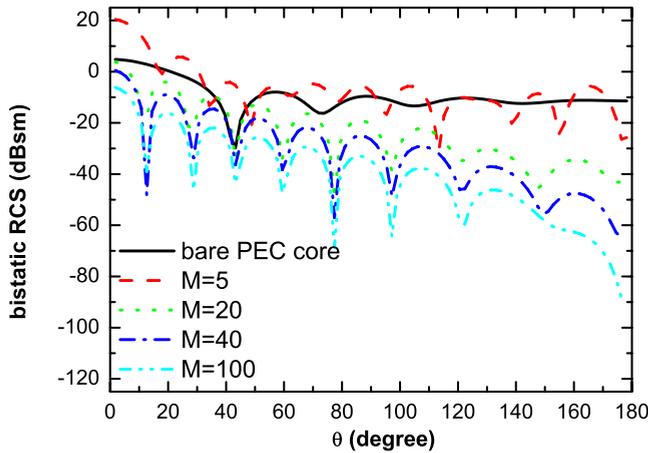


FIG. 4. (Color online) Bistatic RCS of the effective cloak (case II) made of multilayered isotropic structures at different numbers of layers.

ber is increasing (each isotropic layer is thinner) and the parameters are properly selected, the far-field scattering of such multilayered isotropic structures dramatically drops compared with that of a bare PEC sphere. It also verifies the validity of the mechanism of the parameter formulation for the proposed spherical cloak. In Fig. 5, we assume the material in every layer has a loss tangent of 0.01 in both permittivities and permeabilities. It shows that the introduction of the loss in respective layer will degrade cloaking effects in the vicinity of zero degree even though the Pendry's cloak is highly discretized. The far-field quantity will decrease significantly when the angle becomes larger than zero where the cloaking effects are well sustained.

In summary, we have proposed an isotropic multilayered structure as an equivalent spherical cloak. The physics of our

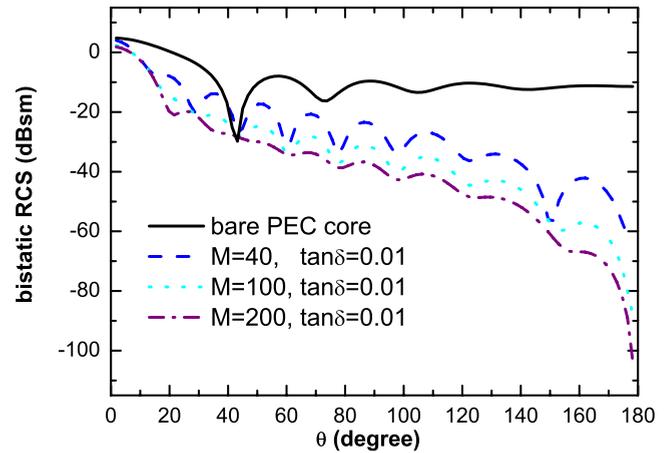


FIG. 5. (Color online) Bistatic RCS of the effective cloak (case II) with the same loss tangent at 0.01 in each layer for different values of M .

spherical invisibility cloak has been interpreted in terms of the property of eigenmodes. The discretization with a medium value of M in our design lifts a lot of strict requirements in realizing conventional anisotropic spherical cloaks in which the material needs to be anisotropic and parameters have to be radius dependent. Also, the multilayered isotropic structure can be solved rigorously by Mie scattering model. The design of the proposed cloak with spherically layered isotropic structures has been analyzed and verified, and the cloaking effects are well demonstrated.

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