Distributed external cloak without embedded antiobjects

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We propose a distributed external cloak without embedded antiobjects, which contains tunable windows and a cloaking area. It consists of closed and standalone systems whose positions and parameters can be separately controlled by the design purpose and folded geometry. Such an external cloak does not rely on either cloaked material or the concept of the antiobject [Phys. Rev. Lett. **102**, 093901 (2009)] in the complementary material, and can thus be kept unchanged when different objects are considered. The object lies outside the cloak and is not blinded in terms of multiple open windows desired by specific designs. Such a distributed cloak enables us to conceal arbitrary objects of varying shape in center areas, which may exceed the inner boundary of a perfect closed cloak. © 2010 Optical Society of America

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Based on coordinate transformation, the electromagnetic (EM) cloak was first proposed by Pendry *et al.* [1]. In the past few years, different kinds of cloaks have been studied, including elliptical cloaks [2], “slab cloaks” [3], toroid cloaks [4], cloaks with twin cavities [5], cloaks of twisted domains [6], and arbitrarily shaped cloaks [7]. Analytical methods based on Maxwell’s equations have been used to reveal the nature of invisibility cloaks [8–10]. Different from these aforementioned approaches, in which the object to be concealed is usually within a closed cloaking shell, the open cloak has been investigated, and the object is still within the cloak [11]. This cloak itself has an open window based on the approximation that the cloak is infinitesimally thin in certain areas, and such discontinuity is thus ignored, resulting in an effective open window. EM scattering still exists outside the open cloak, since it is an approximate approach, which implies the window cannot be large. On the other hand, the concept of complementary media was adopted to design external cloaks that can cloak objects at a distance outside the cloaking shell [12,13]. More recently, another cloaking method to design an active exterior cloak was proposed [14], which needs to know in advance the incoming probing wave, including phase information. However, the “antiobject” [12] inside the shell gives rise to certain restrictions. If the material of the cloaked object turns out to be spherically gyrotropic, angle-dependent, or a perfect electrical conductor (PEC)/perfect magnetic conductor, it is difficult, if not impossible, to apply the concept of antiobject. Moreover, once an illusion device has been designed, it does not work well if the object is changed, because the configuration of the embedded antiobject makes the system sensitive to the size, shape, and constituents of the cloaked object. As a result, the whole cloaking system needs to be rebuilt each time for a new object to be cloaked, making such a design not applicable generally.

In this Letter, we propose a recipe for creating the distributed cloak based on the combination of transformation optics and folded geometry. Such a distributed cloak consists of isolated constitutions with controllable multiple open windows and is independent of the cloaked object.

More importantly, each constitution of the proposed distributed cloak is separated from the other and individually self-contained, which does not require antiobjects embedded in the shell [12,13]. Hence, given an arbitrary target to hide, one can independently design the position, size, and number of the *systems* needed by the distributed cloak, regardless of the target’s shape, varying size, or material property. Then one can place them accordingly around the target. The cloaking area in the center is not blinded so that the communication is feasible through windows, and the area can be adjusted compared to the fixed cloaking area of a classic closed cloak.

A schematic diagram illustrating the concept of designing a distributed cloak is shown in Fig. 1. A closed cloak [1] in virtual space is divided into N segments in Fig. 1(a). Applying the compressing coordinate transformation (CCT) at $\vec{\rho}_i (i = 1, 2, \cdots, N)$, the CCT circular region with the radius c is transformed into another circular region with the radius a, as shown in Fig. 1(b). The folded geometry is employed to compensate the spatial discontinuity from applying CCT, as shown in Fig. 1(c). Finally, a distributed external cloak is composed of N systems. The ith system in Fig. 1(c) is clearly illustrated in Fig. 1(d). Note that all transformations for each system are performed in local cylindrical coordinates at origins $\vec{\rho}_i$.

The relation between the physical space (ρ, ϕ, z) [Fig. 1(b)] and the virtual space (ρ', ϕ', z') [Fig. 1(a)] can be expressed as

$$|\vec{\rho}' - \vec{\rho}_i| = c|\vec{\rho} - \vec{\rho}_i|/a, \qquad \phi' = \phi, \qquad z' = z, \qquad (1)$$

where $0 \le |\vec{\rho} - \vec{\rho}_i| \le a$, $0 \le |\vec{\rho}' - \vec{\rho}_i| \le c$, $\phi = \tan^{-1}(\frac{y-y_i}{x-x_i})$, $\phi' = \tan^{-1}(\frac{y'-y_i}{x'-x_i})$, $\vec{\rho}_i = (x_i, y_i)$, and $c > a > 0$.

The physical space is, however, discontinuous under CCT, which can be compensated by folded geometry. The discontinuous region $(a < |\vec{\rho} - \vec{\rho}_i| < c)$ in Fig. 1(b) is divided into region I $(b < |\vec{\rho} - \vec{\rho}_i| < c)$ and region II $(a < |\vec{\rho} - \vec{\rho}_i| < b)$, respectively denoted by $(\rho^{(1)}, \phi^{(1)},$

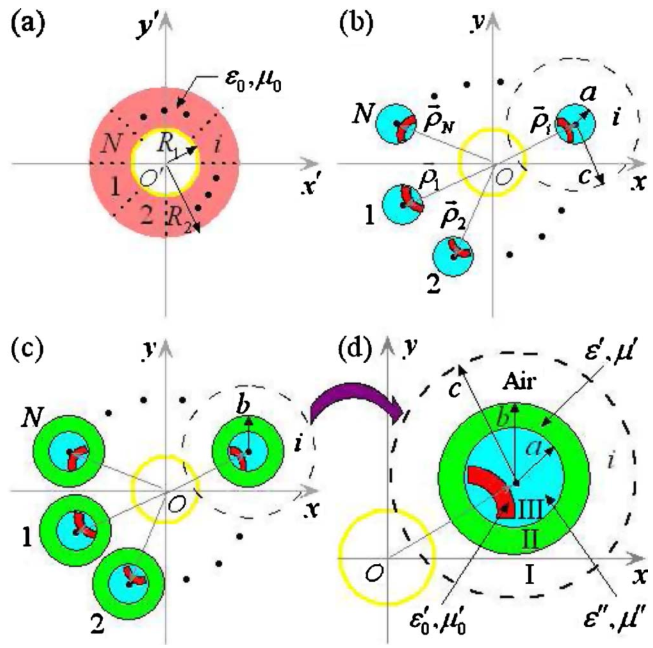

Fig. 1. (Color online) Scheme of designing a reconfigurable distributed cloak. (a) Closed cloak [1] in virtual space is divided into N segments. (b) CCT is operated at $\vec{\rho}_i$, i.e., the distance vector from the origin to the center of the ith system; the core materials (region III, blue areas) are generated; and the ith sector in (a) is compressed into its image sector in region III ($r < a$). (c) Discontinuity induced by CCT is compensated by folded geometry, and the green areas, i.e., region II ($a < r < b$), denote complementary materials. (d) ith system in (c) is zoomed in. Hence, the distributed cloak is composed of N systems.

$z^{(1)}$) and ($\rho^{(2)}$, $\phi^{(2)}$, $z^{(2)}$). The transformations of folding region I into region II are

$$|\vec{\rho}^{(2)} - \vec{\rho}_i| = \frac{a-b}{c-b}(|\vec{\rho}^{(1)} - \vec{\rho}_i| - b) + b,$$
$$\phi^{(2)} = \phi^{(1)}, \qquad z^{(2)} = z^{(1)}, \tag{2}$$

where $\phi^{(1)} = \tan^{-1}(\frac{y^{(1)}-y_i}{x^{(1)}-x_i})$ and $\phi^{(2)} = \tan^{-1}(\frac{y^{(2)}-y_i}{x^{(2)}-x_i})$. As shown in Fig. 1(d), the system is composed of the complementary material [ε' and μ' in region II ($a < |\vec{\rho} - \vec{\rho}_i| < b$)], core material [$\varepsilon''$ and μ'' in region III ($|\vec{\rho} - \vec{\rho}_i| < a$)], and the *image* [$\varepsilon_o'$ and μ_o' within region III]. The image in physical space corresponds to a portion of the closed cloak with ε_o and μ_o [1] in virtual space. Note that the permittivity and permeability are relative parameters throughout. If a closed cloak has the inner boundary R_1 and outer boundary R_2 [Fig. 1(a)], its parameters can be expressed as $\varepsilon_{or}(\rho') = \mu_{or}(\rho') = (\rho' - R_1)/\rho'$, $\varepsilon_{o\phi}(\rho') = \mu_{o\phi}(\rho') = 1/\varepsilon_{or}(\rho')$, and $\varepsilon_{oz}(\rho') = \mu_{oz}(\rho') = \varepsilon_{or}(\rho') \cdot [R_2/(R_2 - R_1)]^2$.

Then the parameters of its corresponding image in the physical space become

$$\varepsilon_{\text{image}} = \mu_{\text{image}} = \mathbf{A}^c \cdot \text{diag}[\varepsilon_{or}(\rho), \varepsilon_{o\phi}(\rho), \varepsilon_{oz}(\rho)] \cdot \mathbf{A}^{cT}/\det(\mathbf{A}^c), \tag{3}$$

where $\mathbf{A}_{ij}^c = \partial x_i/\partial x_j'$ is the Jacobian matrix from Eq. (1), $\varepsilon_{or}(\rho) = (|\vec{\rho} - a\vec{\rho}_i/c| - aR_1/c)/|\vec{\rho} - a\vec{\rho}_i/c|$, $\varepsilon_{o\phi}(\rho) = 1/\varepsilon_{or}(\rho)$, and $\varepsilon_{oz}(\rho) = \varepsilon_{or}(\rho) \cdot [R_2/(R_2 - R_1)]^2$.

Based on the same CCT, the parameters of the core material are

$$\varepsilon_{\text{core}} = \mu_{\text{core}} = \mathbf{A}^c\mathbf{A}^{cT}/\det(\mathbf{A}^c) = \text{diag}[1, 1, (c/a)^2]. \tag{4}$$

Assuming region I to be free space, the parameters of the complementary material can be expressed as $\varepsilon' = \mu' = \mathbf{A}^f\mathbf{A}^{fT}/\det(\mathbf{A}^f)$, where $\mathbf{A}_{ij}^f = \partial x_i^{(2)}/\partial x_j^{(1)}$ is the Jacobian matrix from Eq. (2). Thus, the parameters of the complementary material are

$$\varepsilon_r' = \mu_r' = \frac{k_1|\vec{\rho}^{(2)} - \vec{\rho}_i| - k_2 b}{k_1|\vec{\rho}^{(2)} - \vec{\rho}_i|},$$
$$\varepsilon_\phi' = \mu_\phi' = \frac{k_1|\vec{\rho}^{(2)} - \vec{\rho}_i|}{k_1|\vec{\rho}^{(2)} - \vec{\rho}_i| - k_2 b},$$
$$\varepsilon_z' = \mu_z' = \frac{k_1(k_1|\vec{\rho}^{(2)} - \vec{\rho}_i| - k_2 b)}{|\vec{\rho}^{(2)} - \vec{\rho}_i|}, \tag{5}$$

where $k_1 = (c-b)/(a-b)$, and $k_2 = (c-a)/(a-b)$. The working frequency is 2 GHz under TE polarization, and $R_2 = 2R_1 = 0.2$ m.

In Fig. 2, we consider half of a closed cloak segmented into two parts. By applying Eqs. (3)–(5), two parts of the half-cloak are shifted and folded at controllable positions, where two distributed systems are constructed. From Figs. 2(a) and 2(b), the equivalence in scattering performance is observed, which validates our method. Note that, in our design, after applying CCT from $r = c$ to $r = a$, as Fig. 1(b) shows, each circle with the radius $r = c$ will cut into the center region, as Fig. 1(d) shows. Nevertheless, the overlapping areas in Fig. 1(d) are within the cloaked area, where no field exists. Thus, those overlapping areas between systems and center region will not affect the invisibility performance during the CCT and folded process for an individual system, regardless of what material is actually located in those overlapping areas. As long as individual folded regions ($a < r < b$, i.e., the green regions denoting complementary materials) of any two adjacent systems have no overlapping area, our method is valid, as shown in Fig. 2. The

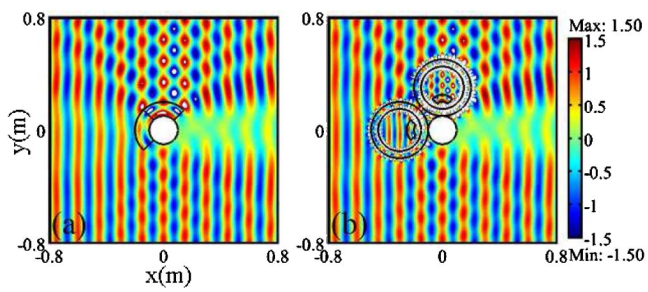

Fig. 2. (Color online) Verification of the design mechanism of distributed cloaks. (a) Bare PEC cylinder with half of a closed cloak [1], which is further divided into two subparts as a demonstration. (b) Distributed cloak composed of two systems designed by the scheme in Fig. 1 with the design parameters of $a = 0.15$ m, $b = 0.2$ m, and $c = 0.3$ m, and the axes of these two systems are located at $(-0.3, 0)$ and $(0, 0.3)$, respectively.

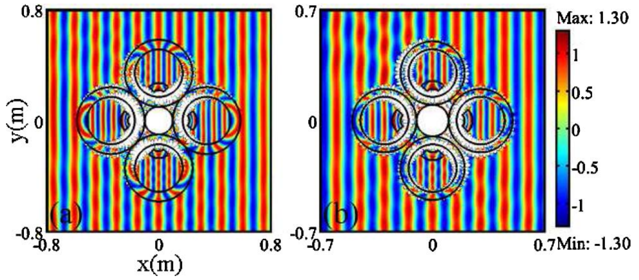

Fig. 3. (Color online) Snapshots of total electric fields for the distributed cloak composed of four systems. (a) Distributed cloak without windows follows the design of $a = 0.1 + 0.05\sqrt{2}$ m, $b = 0.1 + 0.1\sqrt{2}$ m, and $c = 0.2 + 0.1\sqrt{2}$ m. The centers of four systems are respectively located at $(\pm 0.2 \pm 0.1\sqrt{2}, 0)$ and $(0, \pm 0.2 \pm 0.1\sqrt{2})$. (b) Distributed cloak with four windows is designed by $a = 0.15$ m, $b = 0.2$ m, and $c = 0.3$ m. The centers of four systems are respectively located at $(\pm 0.3, 0)$ and $(0, \pm 0.3)$.

material singularity inherited from the classic two-dimensional (2D) closed cloak can be avoided by adopting a nonsingular 2D cloak [15] as the starting point.

If we segment a classic closed cloak [1] into four sectors, a distributed external cloak composed of four standalone systems is presented with open windows [Fig. 3(b)] and without gaps [Fig. 3(a)]. In both cases, the cloaking effect for the distributed cloak is observed. It is evident that the location and number of each system, as well as the radii of a, b, and c, can be controlled to design the size and number of open windows. Note that, in virtue of the proposed cloaking concept, the cloaking area in the center can be significantly enlarged, since one can move the systems farther away from the cloaked region. Also, the complementary materials will be independent of the cloaked object and have no antiobjects embedded. As an example, we choose a design of four systems with identical values of a, b, and c for all systems. One can easily design a distributed cloak with systems of different sizes and at different positions. Thus, the number of open windows, the positions of individual systems, the area of the cloaked region in the center, and the shape of the cloaked area can be tuned.

In this Letter, a distributed external cloak is proposed based on folded geometry. The distributed external cloak can be designed easily with or without windows. Compared to the antiobject cloak [12] and resonant cloak [16], which can also cloak external objects, the proposed distributed cloak is not only able to externally hide an object of arbitrary material property and arbitrary shape, but also the cloaking area/shape of the current design can be adjusted (which can be much larger than the original area of πR_1^2 in closed cloaks) by moving and relocating those systems. Thus, the distributed cloak can be repeatedly used for various objects without removing, rebuilding, or refilling different antiobjects in different shapes into the complementary layer each time.

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