nature materials

SUPPLEMENTARY INFORMATION
https://doi.org/10.1038/s41563-018-0239-6

In the format provided by the authors and unedited.

# Thermal meta-device in analogue of zero-index photonics

**Ying Li[1,8], Ke-Jia Zhu[1,2,8], Yu-Gui Peng[1,3,8], Wei Li[4], Tianzhi Yang[5,6], He-Xiu Xu[1,7], Hong Chen[2], Xue-Feng Zhu[3], Shanhui Fan[4]* and C.-W. Qiu[1]***

[1]Department of Electrical and Computer Engineering, National University of Singapore, Singapore, Singapore. [2]Key Laboratory of Advanced Microstructure Materials (MOE) and School of Physics Sciences and Engineering, Tongji University, Shanghai, China. [3]School of Physics and Innovation Institute, Huazhong University of Science and Technology, Wuhan, China. [4]Department of Electrical Engineering, Ginzton Laboratory, Stanford University, Stanford, CA, USA. [5]Department of Mechanics, Tianjin University, Tianjin, China. [6]Tianjin Key Laboratory of Nonlinear Dynamics and Control, Tianjin, China. [7]Air and Missile Defense College, Air Force Engineering University, Xi'an, China. [8]These authors contributed equally: Ying Li, Ke-Jia Zhu, Yu-Gui Peng.
*e-mail: shanhui@stanford.edu; chengwei.qiu@nus.edu.sg

## Supplementary Note 1

### Effective thermal conductivity of the rotating fluid

We assume a simple velocity field $v(r) = \Omega r$ and consider the following diffusion advection equation:

$$D\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right) = \Omega\frac{\partial T}{\partial \theta}, \tag{1}$$

which can be attempted with a variable separation $T_2(r,\theta) = F(r)G(\theta)$. The subscript 2 means the temperature distribution in the region $R_1 \leq r \leq R_2$. Inserting it to equation (1) gives:

$$(r^2F'' + rF')/F = \left(\frac{\Omega}{D}r^2G' - G''\right)/G. \tag{2}$$

Since $G$ is periodic in $\theta$ and varis slowly in our condition, we assume $G = \exp(i\theta)$ where $i$ is the imaginary unit. This provides a valid solution to equation (2) where $F$ satisfies:

$$r^2F'' + rF' - \left(\frac{i\Omega}{D}r^2 + 1\right)F = 0. \tag{3}$$

By performing a change of variable $r = \sqrt{D/\Omega}x$ equation (3) is modified to the standard form:

$$x^2f'' + xf' - (ix^2 + 1)f = 0, \tag{4}$$

whose solution is the first order Kelvin's function[1]: $f(x) = \mathrm{ber}_1(x) + i\mathrm{bei}_1(x)$. In the limit of large velocity, $x$ is also large. We assume an argument $\phi(x)$ in the solution, which gives $T_2(r,\theta) = M_1[x(r)]\cos(\theta - \phi[x(r)])$, where $M_1(x)$ is the magnitude of $f(x)$. According to the asymptotic expansions of $f(x)$ at large $x$, the temperature gradient $\partial T_2/\partial r = \sqrt{\Omega/\mathrm{D}}M_1\cos(\theta - \phi + \pi/4)$. At the boundary $r = R_2$, the matching condition is:

$$\begin{aligned} T_2|_{r=R_2} &= T_3|_{r=R_2} \\ \kappa_2\frac{\partial T_2}{\partial r}\bigg|_{r=R_2} &= \kappa_3\frac{\partial T_3}{\partial r}\bigg|_{r=R_2} \end{aligned}. \tag{5}$$

At the boundary $r = R_3$, because of the cloaking effect, clearly we have $T_3|_{r=R_3} = E\cos(\theta)$, $\partial T_3/\partial r|_{r=R_3} = F\cos(\theta)$, where $E$ and $F$ are constants. Considering equation (5), there should be deviation of the azimuthal distributions of temperature (containing $\cos(\theta-\phi)$ at $r$ = $R_2$) and temperature gradient (containing $\cos(\theta-\phi+\pi/4)$ at $r$ = $R_2$) along the two boundaries of region 3. At very large rotation speed, the temperature gradient is the dominant term. According to the principle of minimum entropy production, the deviation of temperature gradient should be minimized. Therefore it turns out that $\theta-\phi[x(R_2)]+\pi/4=\theta$, or $\phi[x(R_2)]=\pi/4$, which is confirmed by simulation results. We thus obtain the form of $T_2$ at $r$ = $R_2$ as

$$T_2|_{r=R_2}=M_1[x(R_2)]\cos(\theta+\pi/4)\ . \tag{6}$$

$$\left.\frac{\partial T_2}{\partial r}\right|_{r=R_2}=\sqrt{\Omega/\mathrm{D}}M_1[x(R_2)]\cos(\theta). \tag{7}$$

On the other hand, if the fluid is replaced with solid material of thermal conductivity $\kappa_2^{\mathrm{eff}}$, the temperature distribution has the following form in the first order of $\theta$:

$$T_2^{\mathrm{eff}}(r,\theta)=(A_2r+B_2/r)\cos(\theta), \tag{8}$$

where $A_2$ and $B_2$ are constants. We also have a temperature distribution in the inner most region $r \le R_1$. In the first order $\theta$, $T_1^{\mathrm{eff}}(r,\theta) = A_1 r\cos(\theta)$ if we approximate the interior with an uniform homogeneous material of thermal conductivity $\kappa_1$. At the boundary $r$ = $R_1$, the two distributions should match:

$$\begin{aligned} \left.T_2^{\mathrm{eff}}\right|_{r=R_1} &= \left.T_1^{\mathrm{eff}}\right|_{r=R_1} \\ \kappa_2^{\mathrm{eff}}\left.\frac{\partial T_2^{\mathrm{eff}}}{\partial r}\right|_{r=R_1} &= \kappa_1\left.\frac{\partial T_1^{\mathrm{eff}}}{\partial r}\right|_{r=R_1} \end{aligned}\ . \tag{9}$$

Therefore $T_2^{\mathrm{eff}}(r,\theta)=(A_2r+B_2/r)\cos(\theta)$, which gives:

$$\begin{aligned} A_2 &= \left(1+\frac{\kappa_1}{\kappa_2^{\mathrm{eff}}}\right)\frac{A_1}{2} \\ B_2 &= \left(1-\frac{\kappa_1}{\kappa_2^{\mathrm{eff}}}\right)\frac{A_1R_1^2}{2} \end{aligned}\ . \tag{10}$$

Therefore:

$$\left.T_2^{\mathrm{eff}}\right|_{r=R_2}=\frac{A_1}{2}\left[\left(1+\frac{\kappa_1}{\kappa_2^{\mathrm{eff}}}\right)R_2+\left(1-\frac{\kappa_1}{\kappa_2^{\mathrm{eff}}}\right)\frac{R_1^2}{R_2}\right]\cos(\theta)=C\cos(\theta). \tag{11}$$

$$\left.\frac{\partial T_2^{\text{eff}}}{\partial r}\right|_{r=R_2} = \frac{A_1}{2}\left[\left(1+\frac{\kappa_1}{\kappa_2^{\text{eff}}}\right)-\left(1-\frac{\kappa_1}{\kappa_2^{\text{eff}}}\right)\frac{R_1^2}{R_2^2}\right]\cos(\theta) = D\cos(\theta). \tag{12}$$

In order to generate a similar temperature field as the thermal zero-index cloak outside the device, the temperature and heat flux distribution at the outer boundary $r = R_2$ had better meet:

$$\begin{aligned} T_2|_{r=R_2} &= T_2^{\text{eff}}\big|_{r=R_2} \\ \kappa_2 \left.\frac{\partial T_2}{\partial r}\right|_{r=R_2} &= \kappa_2^{\text{eff}} \left.\frac{\partial T_2^{\text{eff}}}{\partial r}\right|_{r=R_2} \end{aligned}. \tag{13}$$

Looking back at equation (6) and (11), we see that the first condition is not achievable. We should modify it to a weaker version of minimized differenc:

$$\min\left[\int_0^{2\pi}\left(T_2|_{r=R_2} - T_2^{\text{eff}}\big|_{r=R_2}\right)^2 d\theta\right] = \min\left[\pi\left(C^2-\sqrt{2}CM_1+M_1^2\right)\right]. \tag{14}$$

The minimum is reached when $C = \sqrt{2}M_1/2$. The constant $D$ in equation (12) can than be found through its relation with $C$ through $A_1$. The second condition of equation (13) can thereby be rewritten as an equation of $\kappa_2^{\text{eff}}$ (written as $\kappa$ for simplicity):

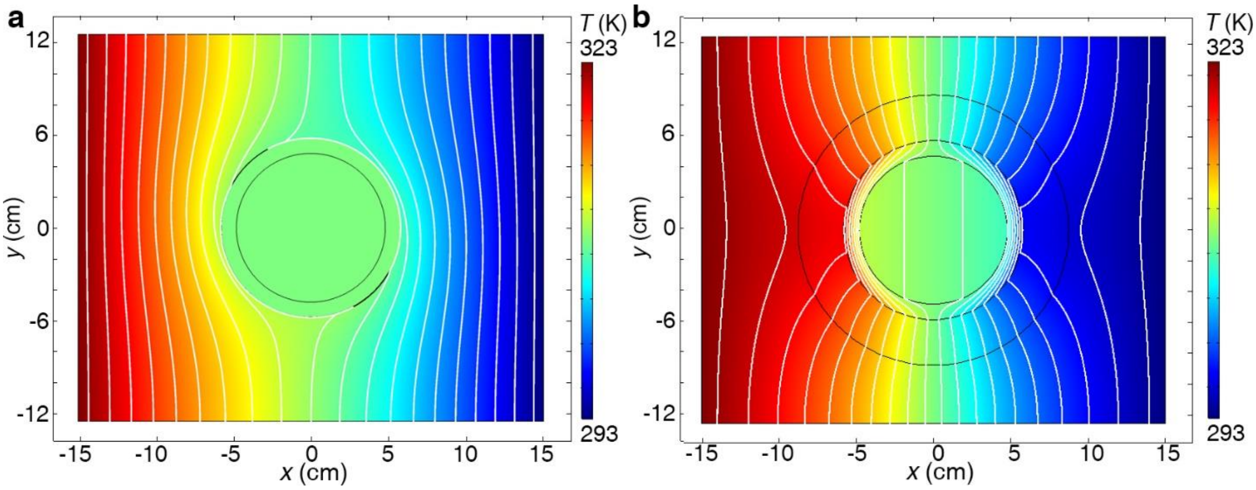
$$(1-\alpha)\kappa^2 + (1+\alpha)\left(\kappa_1 - \kappa_2\sqrt{2Pe}\right)\kappa - (1-\alpha)\kappa_1\kappa_2\sqrt{2Pe} = 0, \tag{15}$$

where $\alpha = R_1^2/R_2^2$, and $Pe = \Omega R_2^2/D$ is the Péclet number. In the limit of large velocity, $\kappa_1 \ll \kappa_2\sqrt{2Pe}$ and can be ignored. The solution is then independent of the interior material as expected:

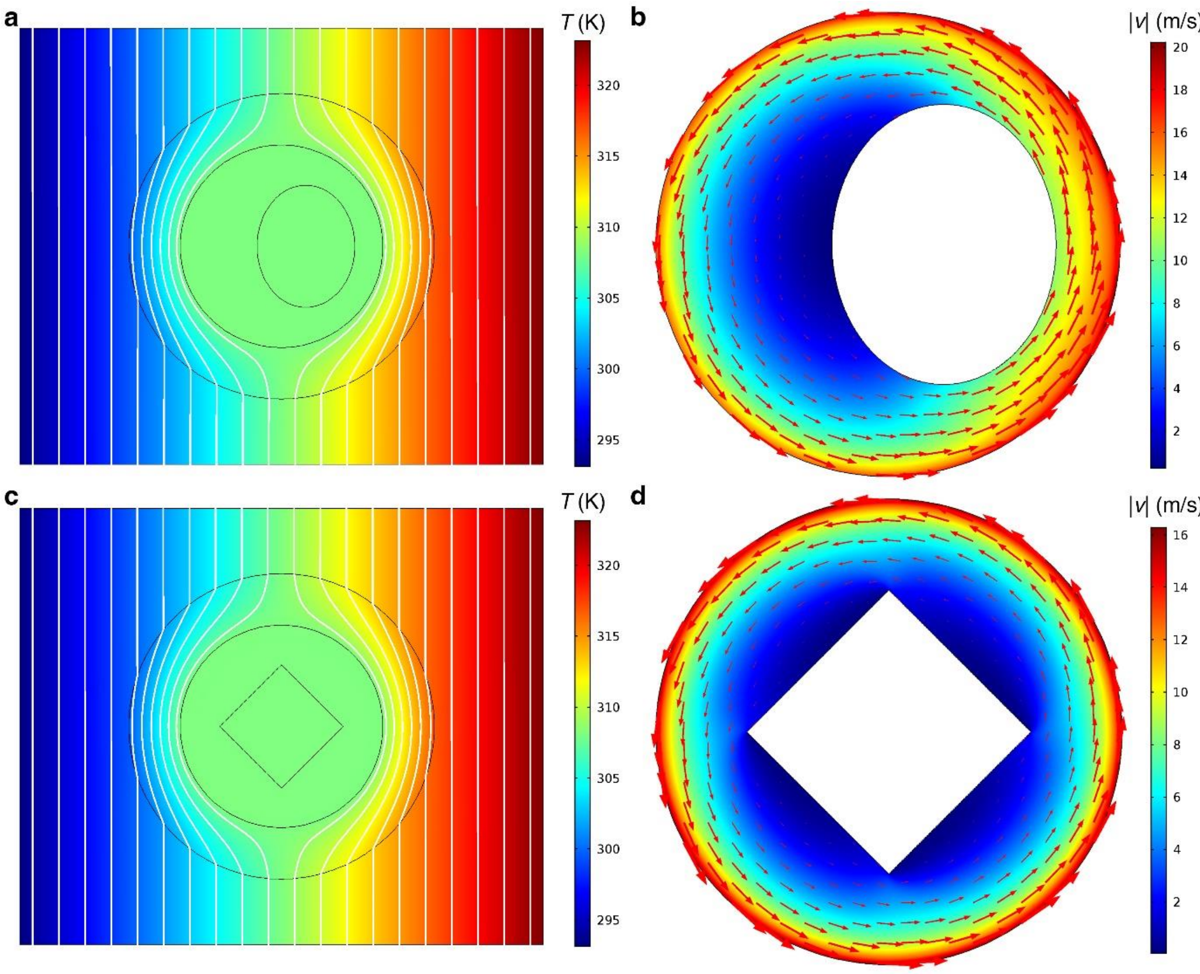
$$\kappa_2^{\text{eff}} = \frac{1+\alpha}{1-\alpha}\kappa_2\sqrt{2Pe} = \frac{R_2^2+R_1^2}{R_2^2-R_1^2}\kappa_2 R_2\sqrt{2\Omega/\text{D}}. \tag{16}$$

**Reference:**

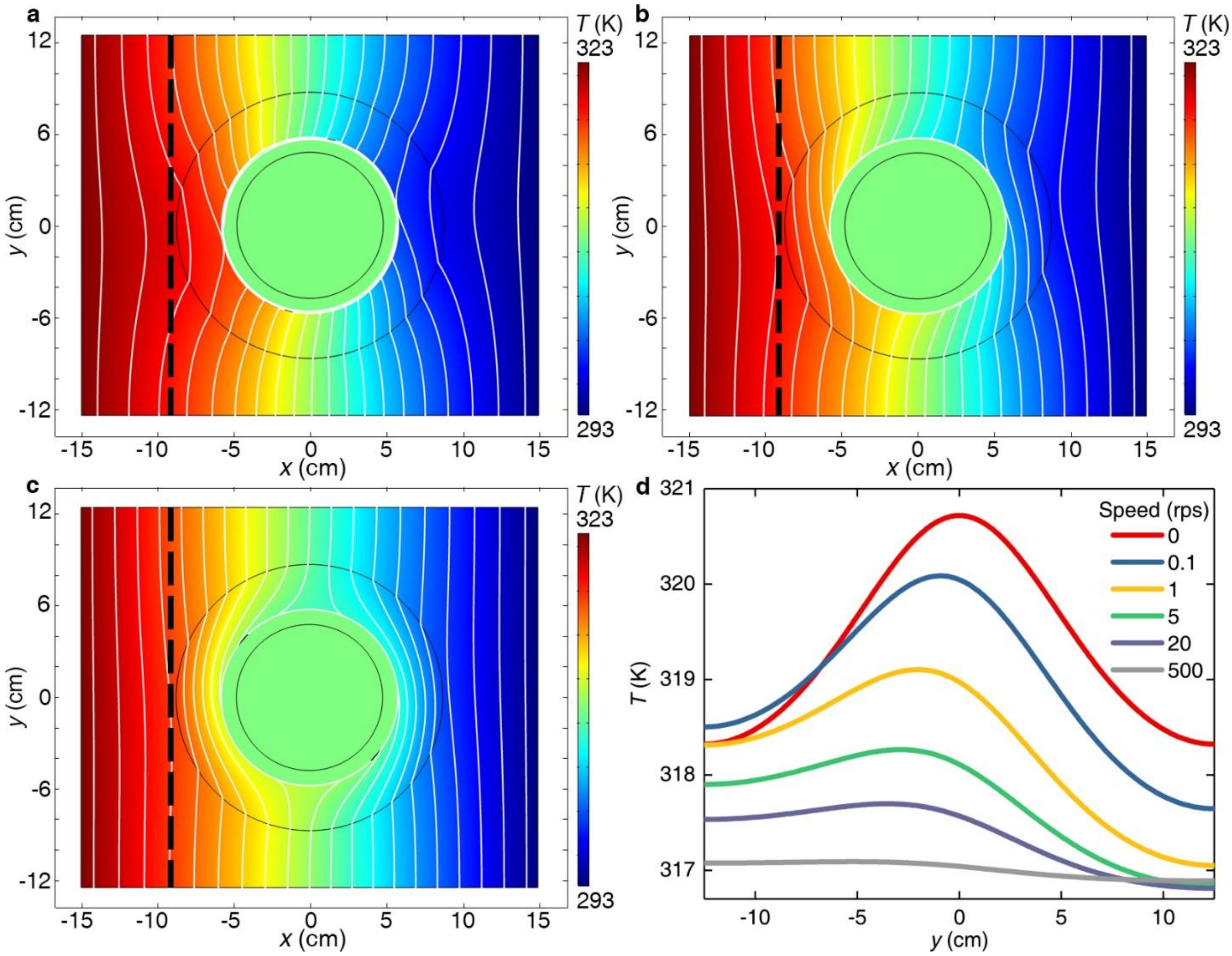
1. Olver, F. W. J. & Maximon, L. C. Bessel Functions, in NIST Digital Library of Mathematical Functions. http://dlmf.nist.gov/, release 1.0.18 of 2018-03-27.



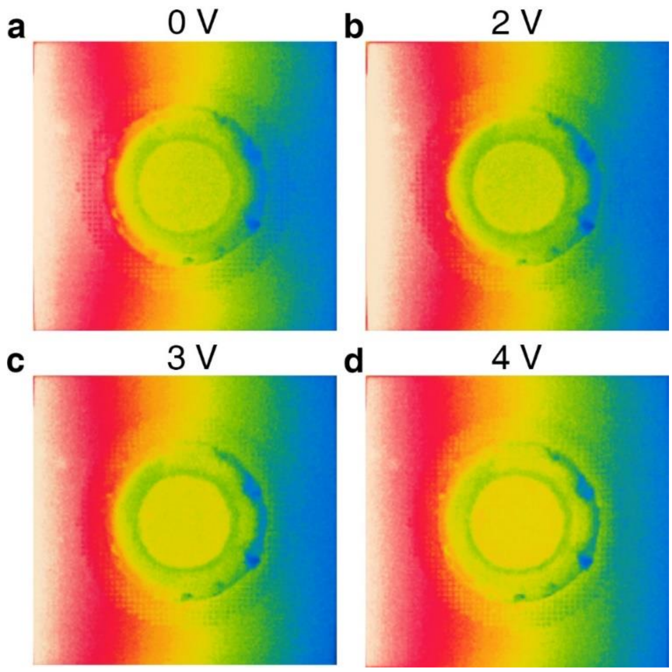

**Supplementary Figure 1. Cases when the cloaking effect is absent. a**, Without the outer complementary layer, the object is just surrounded by a channel filled with rapidly circulating water, which deforms the profile as a scatterer with ultra-high conductivity, equivalent to a near-zero-index material (NZIM). **b**, With the outer layer, but when the water is at rest, the NZIM effects are not triggered.



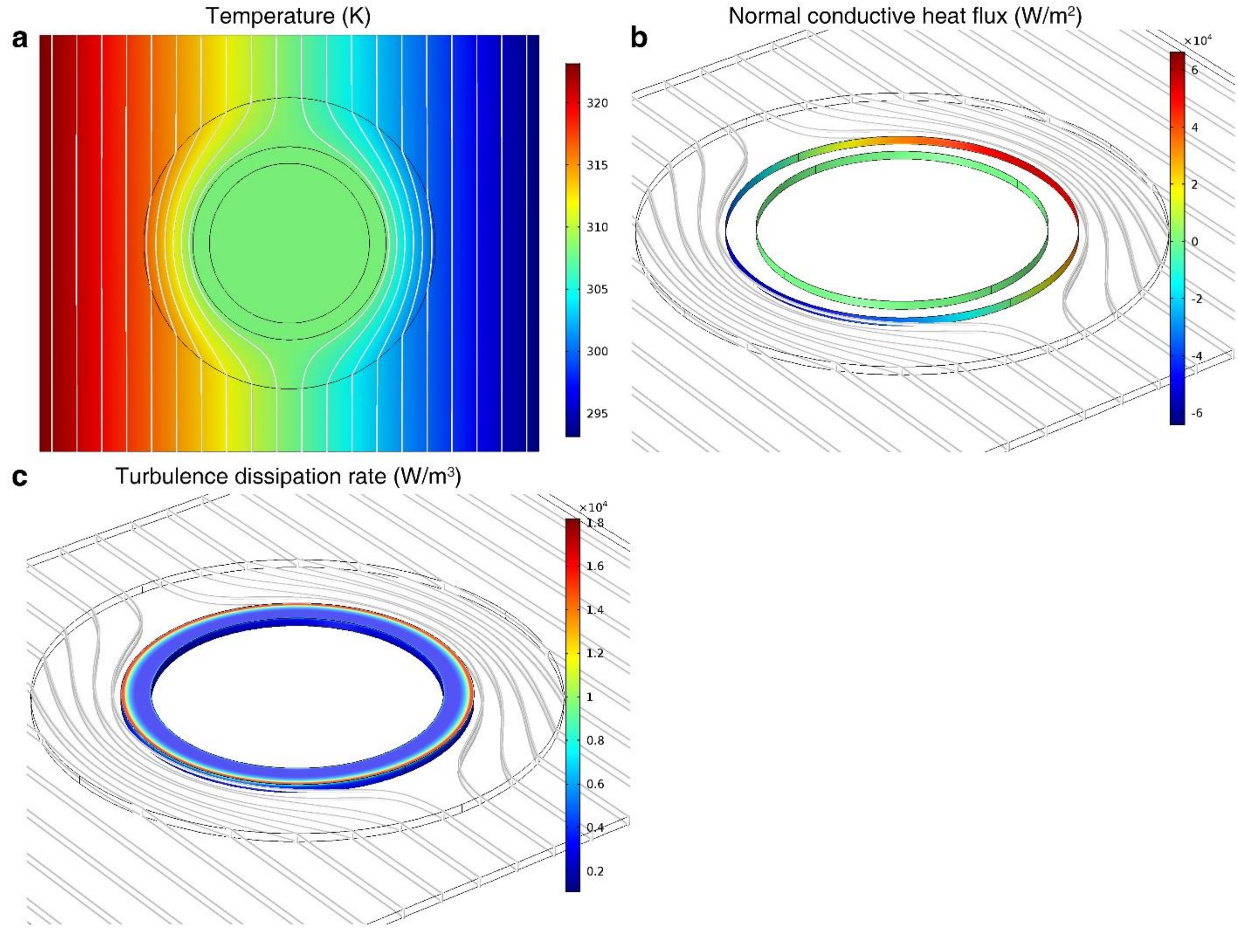

**Supplementary Figure 2. Numerical results with an elliptical (a,b) and a square (c,d) object.** **a**,**c**, Temperature ($T$) profiles, where the white lines are isothermal lines. **b,d**, Velocity magnitude ($|v|$) profiles, where the red arrows represent the directions of the velocity. The velocity fields are driven by a moving outer boundary. The results for objects of other shapes are expected to be similar. The shape of the outer layer is kept circular to satisfy the scattering cancellation condition, which is enough for all conceivable situations. Other shapes of the outer layer are also possible but requires re-calibration of its thermal conductivity.




**Supplementary Figure 3. Simulated temperature profiles at different rotation speeds of the fluid.** **a**, $\Omega = 2\pi \times 0.1$ rad/s. **b**, $\Omega = 2\pi \times 1$ rad/s. **c**, $\Omega = 2\pi \times 5$ rad/s. White lines are isothermal lines. **d**, Temperature distribution along the $x = -8.8$ cm line (indicated as dashed black lines in **a-c**).




**Supplementary Figure 4. Experimentally measured infrared images at different input voltages.** The temperature profile outside the device is gradually restored as the input voltage increases.




**Supplementary Figure 5. Simulation of the more realistic experimental setup with the nonisothermal turbulent flow model. a**, Temperature distribution on the upper surface. **b**, Normal conductive heat flux on the side boundaries of the fluid field. **c**, Turbulence heat dissipation inside the fluid field. The white colour tubes are isothermal lines.