applications (VBA) language. PSO was therefore implemented by VBA macros and launched from the CST environment. For calculations, time domain solver was used.

During the optimization, the layout dimensions of the triple U-slot antenna, the spacing of the holes and their diameter in the EBG substrate were modified to improve the impedance matching in the operation bands. The position of the coaxial feeding probe was changed in very limited bounds. The layout modification consisted in changing the selected state parameters. The number of the selected state parameters was set to 16 (see Fig. 1), including 2, for the EBG substrate setup.

Applying PSO, the swarm consisted of eight agents, and the solution space was bordered by absorbing walls. When a particle hits the boundary of the solution space in one of the dimensions, the velocity in that dimension is zeroed. According to Clerc and Kennedy's original work [4], velocity with constant inertial weights was applied, and the algorithm was performed for 30 iterations. The fitness function was set to be the sum of squares of the s_{11} module for all four central frequencies (925, 1725, 1925, and 2440 MHz).

To avoid a possible overlapping of two objects (the holes in the substrate, the slots in the patch), optimization constraints have to be introduced resulting in different ranges of state parameters: the smallest parameter range (for the feeder position) to the largest one (the largest U-slot branch) is nearly 0.1.

A single realization of the PSO consumed $\sim$30 h of the CPU time (AMD Athlon 2500+, 2 GB RAM, Windows XP, CST Microwave Studio 2006) for the antenna on a conventional substrate, and $\sim$40 h when the EBG substrate was included. To reduce the unacceptably high CPU-time demands, the attention has to be turned to more efficient numerical modeling of the EBG antenna.

5. RESULTS

The proposed EBG substrate was modeled by the MIT Photonic-Bands package so that the upper three bands can be covered by the TM bandgap, which is approximately 47% bandgap to central frequency ratio. The results were verified by two-dimensional full-wave FEMLAB models (see Fig. 3). The triple U-slot antenna design was tuned to make the s_{11} roughly fitting in four operation bands (see Fig. 2). That way, the initial antenna design was completed.

The global optimization was asked then to improve s_{11} at all four central frequencies of the operation bands and to tune s_{11} in the selected boundaries. To verify the principle, just the antenna layout was optimized first. The best global solution lowered the objective function for 33% with respect to the initial state. This relatively small improvement can be caused by the limited non-adaptive boundary settings for the state variables. The inconvenient initial boundary values setup can be the other explanation.

The simulation including the EBG perturbations are despite the relatively modern hardware time consuming and make the core of our current research. For the verification of results, the antenna was optimized with built-in CST optimizer in frequency domain based on the quasi-Newton method. In this initial setup, we were not able to improve simulation results in any way yet.

6. CONCLUSIONS

The way of improving the impedance matching of the four band triple U-slot antenna with the EBG substrate using the particle swarm global optimization method was investigated and presented. Simulation results were discussed for the case of the antenna without the EBG modification. The simulation limitations were presented and discussed.

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ON THE CONSTITUTIVE RELATIONS OF G-CHIRAL MEDIA AND THE POSSIBILITY TO REALIZE NEGATIVE-INDEX MEDIA

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ABSTRACT: *Gyro-chiral (G-chiral) media of different constitutive relations are studied as potential materials to realize the negative-index material (NIM). G-chiral media are believed to have advantages over chiral media: negative refractive index and backward waves can be achieved without requiring the permittivity and permeability to be quite small at working frequency. The gyrotropic parameters favor the realization of NIM far off the resonances of the permittivity and permeability. The possibility of obtaining backward waves and negative refractive index under Post's and Tellegen's relations are discussed. Moreover, the mapping and comparison of G-chiral media of these two relations have been studied. It is found that the Tellegen's relations are more suitable to describing the G-chiral media so as to realize NIM by this kind of medium. Therefore, this paper not only provides an alternative way to achieve NIM from the G-chiral medium but also discusses the constitutive relation's effects on backward wave propagation and refractive indices.* © 2006 Wiley Periodicals, Inc. Microwave Opt Technol Lett 48: 2534–2538, 2006; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.21981

Key words: *metamaterial; chiral materials; G-chiral media; negative-index media; backward waves; constitutive relations*

1. INTRODUCTION

Metamaterials, with simultaneously negative permittivity and permeability in a frequency band, have received intense interests as they exhibit a lot of exotic properties (e.g., reversal Doppler shift and negative refraction [1], reversed circular Bragg phenomenon [2], and perfect lens [3]). Since this artificial material was experimentally verified in Ref. 4, a lot of works have been carried out to retrieve the parameters by using quasi-static Lorentz theory [5] and S-parameter [6], transmission line method [7], and new structures [8]. Instead of designing metal-dielectric structures, naturally occurring materials (e.g., chiral media) provide another possible way to realize the negative-index media (NIM) especially at optical frequencies, which may give benefits in developing high capacity optical data storage and imaging of biomedical samples with unprecedented resolution. Chiral media, which exhibit EM handedness and magnetoelectric coupling, are more naturally realizable at visible light ranges. In the past several decades, chiral media have been widely studied in the scope of polarization, linear and nonlinear propagation, and microscopic physics [9–11]. More recently, chiral media have recaptured interests within research community due to the potentials in negative refraction [12], strong optical activity [13], and perfect lens [14].

Considering the dispersion effects, chiral parameter can be larger than the square root of the product of permittivity and permeability, and then the negative refraction will arise since the phase velocity and energy velocity are in opposite direction in one of the eigenwave. Meanwhile, chirality appears to be small for all the existing chiral materials. Hence, for a chiral medium, a frequency band in which permittivity and permeability are quite small [15] turns out to be a good way to achieve NIM. However, if the medium is working at a frequency far off the resonance, chirality decreases drastically and negative refraction will not appear. This motivates the present work that considers G-chiral media with gyrotropic parameters in different constitutive relations to realize NIM. Different from the previous work on normal chiral media that focus on decreasing permittivity/permeability and increasing chirality [16–18], this paper proposed a new way to achieve backward wave and negative refraction by considering G-chiral media with the help of gyrotropic parameters. As we know, in the case of a chiral medium, chirality, natural, or artificial, cannot be very big; so a working frequency at which permittivity/permeability is quite small becomes necessary to realize the negative refraction. However, the G-chiral media can yield negative refraction far off the resonance without requiring permittivity/permeability to be quite small because the gyrotropic parameters can reduce the refractive indices greatly.

In our paper, not only two different constitutive relations of G-chiral media are studied, but also the possibility for each case to realize negative-index material is discussed. Besides, different approaches are proposed in studying the eigenmodes in different constitutive relations, and we further compare those two relations to show which one is more applicable. It is found that Tellegen's definition is more suitable to describing G-chiral media in the scope of NIM. We also find that backward waves propagation and negative refraction indices arise in G-chiral media far from the resonance because the gyrotropic parameters can decrease the refractive index of the eigenmodes.

2. FORMULATION

In this section, we first discuss the difference between Post's relation given by

$$\boldsymbol{D} = \bar{\varepsilon}_P \cdot \boldsymbol{E} + i\xi \boldsymbol{B} \tag{1a}$$

$$\boldsymbol{H} = i\xi \boldsymbol{E} + \bar{\mu}_P^{-1} \cdot \boldsymbol{B} \tag{1b}$$

and the other two sets of constitutive relations for G-chiral medium, that is, Tellegen's relations given by

$$\boldsymbol{D} = \bar{\varepsilon}_T \cdot \boldsymbol{E} + (\chi - i\kappa)\boldsymbol{H} \tag{2a}$$

$$\boldsymbol{B} = (\chi + i\kappa)\boldsymbol{E} + \bar{\mu}_T \cdot \boldsymbol{H}. \tag{2b}$$

In these two relations, $\bar{\varepsilon}$ and $\bar{\mu}$ (with/without subscripts) are tensorial permittivity and permeability, respectively; κ and ξ denote chirality in corresponding relations; χ is defined as nonreciprocity parameter. Throughout the paper, time dependence of $e^{i\omega t}$ is suppressed. The permittivity and permeability tensors are

$$\bar{\varepsilon} = \begin{bmatrix} \varepsilon & -ig & 0 \\ ig & \varepsilon & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} \tag{3a}$$

$$\bar{\mu} = \begin{bmatrix} \mu & -il & 0 \\ il & \mu & 0 \\ 0 & 0 & \mu_z \end{bmatrix} \tag{3b}$$

where g and l are the electric and magnetic gyrotropic parameters, respectively. This kind of material includes a chiroplasma consisting of chiral objects embedded in a magnetically biased plasma, or a chiroferrite made from chiral objects immersed into a magnetically biased ferrite. As shown, the subject studied in Ref. 17 is only a special case of the present paper in Tellegen's relations.
Note that the elements in Eq. (3) may not be necessarily identical in Post's and Tellegen's relations. The relations of the elements in two descriptions can be obtained by mapping. In the following subsections, the material parameters refer to the value under respective relations. The subscripts of P or T are suppressed for simplicity.

2.1. Backward-Wave Propagation Under Post's Relations

Substituting Eq. (1) into Maxwell equations, we finally have

$$\nabla \times [\bar{\alpha}_P \cdot \nabla \times \boldsymbol{E}] - 2\omega\xi\nabla \times \boldsymbol{E} - \omega^2 \bar{\varepsilon}_P \cdot \boldsymbol{E} = i\omega \boldsymbol{J} \tag{4}$$

where $\boldsymbol{J}$ is the current excitation,

$$\bar{\alpha}_P = \bar{\mu}_P^{-1} = \begin{bmatrix} \alpha_t & -i\alpha_a & 0 \\ i\alpha_a & \alpha_t & 0 \\ 0 & 0 & \alpha_z \end{bmatrix} \tag{5}$$

and

$$\alpha_t = \frac{\mu}{\mu^2 - l^2} \tag{6a}$$

$$\alpha_a = -\frac{l}{\mu^2 - l^2} \tag{6b}$$

$$\alpha_z = \frac{1}{\mu_z}. \tag{6c}$$

Assuming waves of the form $\boldsymbol{E}_{0e}{}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}$ (where $\boldsymbol{k}$ is the wave vector), plane wave propagation in G-chiral media can be examined by setting $\boldsymbol{J}$ zero. Under these conditions, the electric field satisfies

$$\bar{\Phi}\cdot \boldsymbol{E} = 0 \tag{7}$$

with $[\Phi]$ defined as

$$[\bar{\Phi}] = \begin{bmatrix} \omega^2\varepsilon - \alpha_z k_y^2 - \alpha_t k_z^2 & -i\omega^2 g + \alpha_z k_x k_y + i\alpha_a k_z^2 - 2i\xi\omega k_z & \alpha_t k_x k_z - i\alpha_a k_y k_z + 2i\xi\omega k_y \\ i\omega^2 g + \alpha_z k_x k_y - i\alpha_a k_z^2 + 2i\xi\omega k_z & \omega^2\varepsilon - \alpha_z k_x^2 - \alpha_t k_z^2 & \alpha_t k_y k_z + i\alpha_a k_x k_z - 2i\xi\omega k_x \\ \alpha_t k_x k_z - 2i\xi\omega k_y + i\alpha_z k_y k_z & \alpha_t k_y k_z + 2i\xi\omega k_x - i\alpha_z k_x k_z & \omega^2\varepsilon_z - \alpha_t k_x^2 - \alpha_t k_y^2 \end{bmatrix}. \tag{8}$$

Equation (7) only has nontrivial solutions if the determinant of $\bar{\Phi}$ is zero. Note that the obtained polynomial expression for k is tedious to solve. However, a certain case can still be solved, which gives much insight into the physical properties of the G-chiral media. Considering the waves are propagating along z-direction, we can solve $det\bar{\Phi} = 0$ and obtain the wavenumbers supported by the medium. Thus the wavenumbers are found when k_x and k_y are equal to zero. By reducing Eq. (8), we finally obtain:

$$k_{p\pm} = \omega\frac{\pm\xi_c + \sqrt{\xi_c^2 + (\alpha_t \pm \alpha_a)(\varepsilon \pm g)}}{\alpha_t \pm \alpha_a} \tag{9a}$$

$$k_{a\pm} = \omega\frac{\mp\xi_c - \sqrt{\xi_c^2 + (\alpha_t \mp \alpha_a)(\varepsilon \mp g)}}{\alpha_t \mp \alpha_a} \tag{9b}$$

where p and a represents the parallel and antiparallel direction of energy flow (i.e., real part of the Poynting's vector) and the "$\pm$" signs refer to the right-circular polarization (RCP) and left-circular polarization (LCP), respectively. Note that the k_{p-} and k_{a-} could represent the wavenumbers for backward eigenwaves under some situations. The helicity and polarized state of each wavenumber can be obtained by inserting Eq. (9) into Eq. (7). It can be found that the helicity of k_{p+} and k_{a-} is positive and the helicity of k_{p-} and k_{a+} is negative, provided that negative helicity is defined as left-handedness to positive z-direction and right-handedness to negative z-direction.

Two interesting things thus happen: (1) if $\varepsilon - g < 0$, the sign of k_{p-} will change from positive to negative and the polarization state will change from LCP to RCP; and (2) if $\varepsilon + g < 0$, the sign of k_{a-} will change from negative to positive and the polarization state will change from LCP to RCP. Those two situations are the regimes where backward wave propagation arises. It means that, instead of chirality control, the gyrotropic parameters can act as a new effective choice to achieve backward-wave and negative-index materials. Thus the refraction indices of k_{p-} and k_{a-} of particular interest:

$$n_{R1} = \frac{c_0}{(\alpha_t - \alpha_a)}\left[\sqrt{\xi^2 + (\alpha_t - \alpha_a)(\varepsilon - g)} - \xi\right] \tag{10a}$$

$$n_{R2} = \frac{c_0}{(\alpha_t + \alpha_a)}\left[\sqrt{\xi^2 + (\alpha_t + \alpha_a)(\varepsilon + g)} - \xi\right] \tag{10b}$$

where c_0 is light's velocity in vacuum; subscript R denotes RCP; and the subscripts of 1 and 2 correspond to k_{p-} and k_{a-}, respectively. The chirality under Post's relations will appear twice in the final expressions of refractive indices. By amplifying the gyrotropic parameter or increasing the chirality, negative refraction can be achieved.

2.2. Negative Refraction in Tellegen's Relations

In this section, Tellegen's relations for G-chiral media will be discussed so as to realize NIM. The relations can be referred to Eq. (2). The same assumption as in Post's relations is made, that is, the wave is confined to propagate along z-direction. Hence D_z and B_z vanish due to the fact that $\boldsymbol{E}$ and $\boldsymbol{H}$ only have transverse component and the form of the tensorial $\bar{\varepsilon}_T$ and $\bar{\mu}_T$ is gyrotropic. Thus one can have the following relations:

$$\begin{bmatrix} D_x \\ D_y \end{bmatrix} = \begin{bmatrix} \varepsilon E_x - igE_y + (\chi - i\kappa)H_x \\ \varepsilon E_y + igE_x + (\chi - i\kappa)H_y \end{bmatrix} \tag{11a}$$

$$\begin{bmatrix} B_x \\ B_y \end{bmatrix} = \begin{bmatrix} \mu H_x - ilH_y + (\chi + i\kappa)E_x \\ \mu H_y + ilH_x + (\chi + i\kappa)E_y \end{bmatrix}. \tag{11b}$$

Considering ∇ operator can be replaced by $-i\boldsymbol{k}$ for plane waves, Maxwell equations can be rewritten

$$\boldsymbol{k} \times \boldsymbol{E} = \omega\boldsymbol{B} \tag{12a}$$

$$\boldsymbol{k} \times \boldsymbol{H} = -\omega\boldsymbol{D} \tag{12b}$$

where $\boldsymbol{k} = \{0,0,k\}$ is assumed as aforementioned.

By substituting Eq. (11) into Eq. (12), we can express the electric fields in terms of magnetic fields:

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} A & -B \\ B & A \end{bmatrix}\begin{bmatrix} H_x \\ H_y \end{bmatrix} \tag{13}$$

where

$$A = \frac{1}{k^2 + \omega^2(\chi + i\kappa)^2}[i\omega lk - \omega^2\mu(\chi + i\kappa)] \tag{14a}$$

$$B = \frac{-1}{k^2 + \omega^2(\chi + i\kappa)^2}[\omega\mu k + i\omega^2 l(\chi + i\kappa)]. \tag{14b}$$

After careful algebraic formulation of Eqs. (11), (12), and (13), we finally obtain

$$\left(\frac{k}{\omega} + \varepsilon B + igA\right)^2 + [(\varepsilon A - igB) + (\chi - i\kappa)]^2 = 0. \tag{15}$$

Thus two sets of expressions can be obtained as follows

$$\frac{k}{\omega} + (\varepsilon \pm g)(B \pm iA) = \mp i(\chi - i\kappa). \tag{16}$$

From Eq. (14), the following relations that are used in Eq. (16) can be yielded:

$$B \pm iA = -\frac{\omega(\mu \pm l)}{k \mp i\omega(\chi + i\kappa)}. \tag{17}$$

Substituting Eq. (17) in turn into Eq. (16), we can obtain for roots of k:

$$k_{p\pm} = \omega\left[\mp\sqrt{(\varepsilon + g)(\mu + l) - \chi^2} - \kappa\right] \tag{18a}$$

$$k_{a\pm} = \omega\left[\pm\sqrt{(\varepsilon - g)(\mu - l) - \chi^2} + \kappa\right]. \tag{18b}$$

By taking into account Eq. (18) and corresponding polarization state, the refractive index for the backward waves (i.e., RCP k_{p-} and RCP k_{a-}) inside the G-chiral medium can be determined:

$$n_{\mathrm{R1}} = c_0\left[\sqrt{(\varepsilon + g)(\mu + l) - \chi^2} - \kappa\right] \tag{19a}$$

$$n_{\mathrm{R2}} = c_0\left[\sqrt{(\varepsilon - g)(\mu - l) - \chi^2} - \kappa\right] \tag{19b}$$

where subscripts are similarly defined as in Eq. (10). As a special case, chiral nihility requires $\varepsilon\mu = 0$. We found that such media in Tellegen's relations are physically useless. If we map the conditions of chiral nihility in Tellegen's relations to the corresponding cases in Born's relations (which have more physical insight and meaning), it is found that, in Born's relations, the permittivity, permeability, and chirality are all zero, then no waves can be supported to propagate. However, a medium with $\varepsilon\mu \rightarrow 0$ in Tellegen's relations can still have applications. Since the chirality is generally small in natural and composite chiral media, we can reduce the product of $\varepsilon\mu$ in order to achieve NIM. But the difficulty of doing so also increases. Thus the theorem proposed in this paper will be a good alternative way to achieve NIM by increasing the gyrotropic parameters by modern techniques [19] for gyrotropic media. In a confined geometry, with proper arrangement of photon number and angular momentum with crystal lattice and electronic spins, the gyrotropic parameters can be lifted to the same order as those diagonal elements in tensors. Hence the conditions of $\varepsilon - g \approx 0$ and $\varepsilon - g < 0$ are believed to be realizable by the optical parameter generation and amplification technique.

3. COMPARISON

Two different constitutive relations of G-chiral media are discussed. In each category, the wave characteristics, refractive indices, and backward wave phenomenon are studied. Now some comparisons will be made in two aspects: (1) G-chiral media versus chiral media, and (2) G-chiral media described by Post's relations versus by Tellegen's relations.

3.1. G-Chiral Media versus Chiral Media

In the past two decades, normal chiral and bi-isotropic media have received extensive attention. From the viewpoint of material engineering, chiral nihility [17] is shown to be a new choice to realize negative refraction and other related effects in the optical region. However the physical restriction of exact chiral nihility has not been taken into account, besides the phenomena of backward waves and their polarizabilities in normal chiral media have not been well documented. For a long time, it was believed that both of the two eigenwaves are forward due to the restriction of material's parameters [9]. However, the restriction is not essential.

As a step of material research in the upper level, we proposed G-chiral media, which exhibit more applicabilities and advantages. First, in G-chiral media, negative refraction and backward wave propagation can arise without forcing the permittivity and permeability to be extremely small at working frequencies, which means those NIM properties can be achieved off the resonances of ε and μ. Second, those gyrotropic parameters play an important role in making refraction index negative and achieving backward waves. Instead of controlling chirality in normal chiral medium, those gyrotropic parameters show more flexibilities to be controlled by amplification techniques [19]. Third, we have found that chiral nihility will prohibit the propagation of EM waves in the chiral medium because Maxwell's equations cannot be applied. If If $\varepsilon = \mu = 0$ in Tellegen's relation, two parameters (permittivity and permeability) in Born [20] obtained from measurements must be zero by transformation, which will further lead to a "zero" chirality in Tellegen's relation. Hence, in Tellegen's relation, $\varepsilon = \mu = 0$ actually indicate $\varepsilon = \mu = \kappa = 0$ [21], which is problematic. This problem can be removed by using G-chiral media since the gyrotropy parameters can make refractive index negative without requiring chiral nihility. Hence G-chiral media provide an exciting opportunity to realize NIM.

3.2. G-Chiral Media Described by Post's Relations Versus by Tellegen's Relations

Two different constitutive relations of G-chiral media are considered (i.e., Post's and Tellegen's relations). From Eqs. (10) and (19), it can be found that Tellegen's relations are more advisable and suitable to describing G-chiral medium than Post's relations especially in the consideration of the negative-index medium, though it is known that physical properties do not depend on the description formulism.

As we can see in Eq. (10), under Post's relations, $\alpha_t - \alpha_a < 0$ will not change the sign of n_R. In order to make n_R negative, one possible way of obtaining negative refractive indices is to amplify gyrotropic parameters g and l simultaneously to achieve $(\alpha_t - \alpha_a)(\varepsilon - g) > 0$ while $\alpha_t - \alpha_a < 0$. The other way is to increase the chirality ξ. However, for Tellegen's relations, the representations of the refractive indices are explicit. By amplifying any of the parameters g, l, χ, and κ, n_R can be negative. Mathematically, it can be seen that in the description of Tellegen's relations, for G-chiral media, it is not necessary to require gyrotropic parameters to be big enough in order to get negative refractive indices. Even if $\varepsilon - g$ and $\mu - l$ are positive, the amplification of g and l can still lead to negative refraction indices. In summary, Tellegen's constitutive relations are shown to be a better and more explicit description for G-chiral media in the scope of realizing negative refractive media.

4. CONCLUSIONS

In this paper, G-chiral media are discussed in different constitutive relations. In each case, the eigenmodes, backward waves propagation, and the negative refraction have been studied. The present work shows that G-chiral media possess more advantages, physical insights, and feasibilities over normal chiral media. No one has paid attention to phenomena of backward waves and negative refraction in G-chiral media before. Moreover, further investigation on the effects of various constitutive relations of G-chiral media is carried out. It is proved that G-chiral medium in Tellegen's relations is a better choice to realize NIM characteristics in the views of manufacture, parameter amplification, and controlling. G-chiral media can thus be widely used in subwavelength cavity resonator, optics, and photonics. Those areas will be further studied and explored in future.

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WAVES IN 2D ANISOTROPIC L-C LATTICE METAMATERIALS: PHENOMENOLOGY AND PROPERTIES

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ABSTRACT: *Anisotropic metamaterials composed of 2D periodic infinite and finite periodic lattices of lumped inductor (L) and capacitor (C) circuits have been explored. The unique features of wave channeling on such anisotropic lattices and scattering at their interfaces and edges are reviewed and illustrated by the examples of the specific arrangements. The lattice unit cells composed of inductors and capacitors (basic mesh) as well as of assemblies comprised of double series, double parallel, and mixed parallel-series L-C circuits are discussed.* © 2006 Wiley Periodicals, Inc. Microwave Opt Technol Lett 48: 2538–2542, 2006; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/mop.22005

Key words: *metamaterial; L-C mesh; anisotropic lattice; tunable materials; collimated beam*

1. INTRODUCTION

Metamaterials, electromagnetic media with the physical properties unavailable in natural substances, have recently become particular interest for advanced microwave and optical applications. A class of metamaterials based on the 2D periodic lattices of lumped L-C elements [1–3] and transmission line segments [4, 5] has attracted the special attention. In particular, it has been demonstrated that the basic *L-C* mesh (BM) exhibits the properties of wave channeling [1, 3], and the channel direction can be controlled by frequency, and *C* and *L* value variation [2, 6]. The latter feature is particularly attractive for realizing artificial medium with tunable parameters. Also, it was shown that 2D periodic *L-C* BM exhibits the properties of either right- or left-handed media, and interchange of the *L* and *C* positions in the lattice toggles the medium type. On the basis of this property of *L-C* BM, Balmain et al. [1] have demonstrated that the channeled beams impinging onto the interface between the meshes with the orthogonal orientations of the *L-C* axes follow the paths of negatively refracted rays. This phenomenon was further explored in Ref. 3 for *L-C* BM and in Ref. 7 for the transmission line grids.

In this article, the generic mechanisms of wave channeling on 2D anisotropic lattice of lumped reactances are examined and illustrated with the examples of the specific unit cell configurations. The unique features of lattices with the unit cells composed of double series (SSM), double parallel (PPM), and mixed parallel-series (PSM) *L-C* circuits are discussed in comparison with the basic *L-C* mesh (BM). Finally, the channeled waves at the interfaces between dissimilar meshes are considered in the context of the common properties of anisotropic 2D lattices.

2. INFINITE 2D PERIODIC ANISOTROPIC MESH

An infinite 2D periodic lattice of lumped elements can be characterized by a unit cell, which may be described by the equivalent circuit shown in Figure 1(a). The nodal type cell was chosen for our analysis because it permits an explicit definition for power flow on the anisotropic 2D mesh. This feature is of particular significance for identification of the forward and backward types