An arbitrarily shaped cloak with nonsingular and homogeneous parameters designed using a twofold transformation

2010 J. Opt. 12 095103

(http://iopscience.iop.org/2040-8986/12/9/095103)

IOP Publishing Journal of Optics
J. Opt. 12 (2010) 095103 (6pp) doi:10.1088/2040-8978/12/9/095103

An arbitrarily shaped cloak with nonsingular and homogeneous parameters designed using a twofold transformation

Tiancheng Han[1,2], Chengwei Qiu[2] and Xiaohong Tang[1]

[1] EHF Key Laboratory of Fundamental Science, School of Electronic Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, People's Republic of China
[2] Department of Electrical and Computer Engineering, National University of Singapore, Kent Ridge, Singapore 119620, Republic of Singapore

E-mail: tchan@uestc.edu.cn

Received 27 April 2010, accepted for publication 13 August 2010
Published 6 September 2010
Online at stacks.iop.org/JOpt/12/095103

Abstract
An arbitrarily asymmetrical cloak with nonsingular and homogeneous parameters is obtained by dividing an arbitrary N-sided polygonal region into N triangular regions, applying an independent twofold spatial compression, and smoothing the inner boundary of the polygonal cloak. In each triangular region, a small line segment is first stretched along one side of the triangle; this is followed by a second transformation along the other side which is carried out to create a pairing triangle. The inner boundary will thus be continuous and take a shape similar to the outer boundary. The proposed cloak is nonsingular and moreover each region of the cloak is composed of homogeneous materials.

Keywords: transformation optics, arbitrary polygonal cloak, homogeneous material, nonsingularity

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In the past few years, research on electromagnetic (EM) cloaks has become very popular due to their exciting property of completely hiding objects from EM detection. Scientists have developed a conformal mapping method [1] and a coordinate transformation method [2], which can protect the cloaked object of arbitrary shape from EM radiation. On the basis of the coordinate transformation, different kinds of cloaks have been studied, including elliptical cloaks [3, 4], cloaks with twin cavities [5], petal-shaped cloaks [6], isotropic nonmagnetic flat cloaks [7] and arbitrarily shaped cloaks [8]. On the other hand, an inverse mechanism has been proposed for designing and optimizing non-ideal cloaks, which does not need one to know the coordinate transformation before obtaining material parameters [9, 10]. Analytical methods based on Maxwell's equations have been used to reveal the physical mechanism underlying invisibility cloaks [11–13]. The fundamental idea is the invariance of Maxwell's equations under a space-deforming transformation if the material properties are altered accordingly. During this period, experiments on simplified cylindrical cloaks [14] and ground-plane cloaks [15, 16] were also presented.

In contrast to these aforementioned approaches in which the coordinate transformation is operated along the radial direction, an embedded coordinate transformation was proposed—by operating a coordinate transformation along one axis of the Cartesian coordinate system [17]. According to this concept, one directional diamond cloak is achieved with homogeneous materials, in which invisibility can only be achieved for TM waves along a certain direction [18]. On the basis of the combination of complementary media and the embedded optical transformation, a homogeneous external diamond cloak has been proposed [19–22], which can hide an object within a certain distance outside the cloak. More recently, a diamond cylindrical cloak made with homogeneous materials was investigated [23].

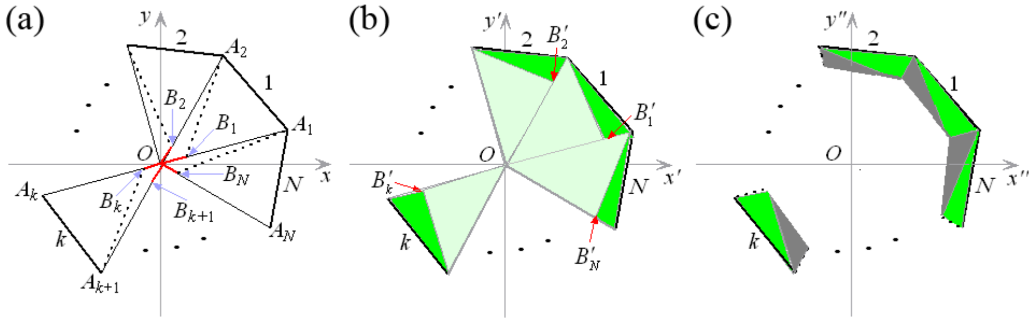

Figure 1. The twofold scheme for designing an arbitrary N-sided polygonal cloak. (a) The original arbitrary N-sided polygonal region composed of N triangles $A_k O A_{k+1}$ with line segments OB_k, where $k = 1, 2, \ldots, N-1$. (b) The line segment OB_k is stretched to OB'_k and then the triangle $A_k B_k A_{k+1}$ is compressed into the triangle $A_k B'_k A_{k+1}$. (c) The stretched line segments are transformed into N triangular spaces and the triangle $B'_k O A_{k+1}$ in (b) is compressed into the triangle $B'_k B'_{k+1} A_{k+1}$. Thus an N-sided polygonal cloak is obtained.

A polygonal cloak may be a promising solution for cloaking an object of irregular shape. N-sided polygonal cloaks of several symmetrical shapes [24] and star-shaped cloaks [25] have been studied. The problems are that (i) those shapes are symmetrical and (ii) such polygonal cloaks inherently require inhomogeneous materials and have singularities at the inner boundaries [24, 25]. This motivates our work, i.e., to remove the two bottlenecks limiting practical applicability of such cloaks. In this paper, a twofold transformation method is proposed for designing arbitrary and asymmetrical polygonal cloaks with homogeneous materials with finite values. We first divide an N-sided polygonal region into N triangular regions and then apply a coordinate transformation to each region, independently. In each triangular region, the coordinate transformations are operated along two sides of the triangle, and a small line segment is stretched into a triangular space. Unlike the diamond cloak approach [23], the design method proposed in this paper can be utilized for designing arbitrary/asymmetrical N-sided polygonal cloaks, including regular and irregular shapes. In this sense, the design method proposed in this paper opens up an avenue to arbitrarily shaped cloaks made with nonsingular and homogeneous materials.

2. Theoretical analysis

A schematic diagram illustrating the design concept is shown in figure 1. Figures 1(a)–(c) show the original Cartesian space (x, y, z), the transitional Cartesian space (x', y', z'), and the transformed Cartesian space (x'', y'', z''), respectively. In figure 1(a), an N-sided polygonal region is divided into N triangles $(A_1OA_2, \ldots, A_kOA_{k+1}, \ldots, A_NOA_1)$ with small line segments $(OB_1, \ldots, OB_k, \ldots, OB_N)$. To design the cloak, we take two steps. First, we stretch the small line segment OB_k to OB'_k, and the triangle $A_kB_kA_{k+1}$ is compressed into the triangle $A_kB'_kA_{k+1}$ with ε' and μ', as shown in figure 1(b). Second, the stretched line segment OB'_k is further expanded into a triangle $OB'_kB'_{k+1}$ to cloak the object, and the triangle B'_kOA_{k+1} is compressed into the triangle $B'_kB'_{k+1}A_{k+1}$ with ε'' and μ'', as shown in figure 1(c). Now, an arbitrary N-sided polygonal cloak is obtained. It should be noted that the coordinates of A_k, B_k, B'_k are constants, and can be expressed as (x_k, y_k), (x_{ka}, y_{ka}), (x_{kb}, y_{kb}), respectively.

Since the coordinate transformation is operated independently in each triangular region, we take the kth triangle A_kOA_{k+1} as an example for calculating the EM parameters. It should be noted that the permittivity and permeability are relative parameters throughout. For simplicity, we consider conformal polygonal cloaks, and we set $x_{ka} = n \cdot x_k$, $y_{ka} = n \cdot y_k$, $x_{kb} = m \cdot x_k$, $y_{kb} = m \cdot y_k$, where $0 < n < m < 1$.

In the first step, the transformation equations for compressing the triangle $A_kB_kA_{k+1}$ into $A_kB'_kA_{k+1}$ can be expressed as

$$x' = A_{11}x + B_{11}y + C_{11}$$
$$y' = A_{12}x + B_{12}y + C_{12} \qquad z' = z \tag{1}$$

where

$$A_{11} = \frac{1-m}{1-n} - \frac{y_k}{x_k}B_{11},$$

$$B_{11} = \frac{(m-n)(x_k - x_{k+1})x_k}{(1-n)(x_{k+1}y_k - x_k y_{k+1})},$$

$$C_{11} = (1-A_{11})x_k - B_{11}y_k, \qquad A_{12} = \frac{y_k}{x_k}\left(\frac{1-m}{1-n} - B_{12}\right),$$

$$B_{12} = \frac{(1-m)(x_{k+1} - nx_k)y_k - (1-n)(y_{k+1} - my_k)x_k}{(1-n)(x_{k+1}y_k - x_k y_{k+1})},$$

$$C_{12} = (1 - B_{12})y_k - A_{12}x_k.$$

According to the principles of transformation optics [1], the permittivity and permeability of the triangle $A_kB'_kA_{k+1}$ become

$$\varepsilon' = \mu' = \frac{1}{Q_1}\begin{bmatrix} \varepsilon'_{xx} & \varepsilon'_{xy} & 0 \\ \varepsilon'_{xy} & \varepsilon'_{yy} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

where

$$\varepsilon'_{xx} = A_{11}^2 + B_{11}^2, \qquad \varepsilon'_{xy} = A_{11}A_{12} + B_{11}B_{12},$$

$$\varepsilon'_{yy} = A_{12}^2 + B_{12}^2, \qquad \text{and} \qquad Q_1 = A_{11}B_{12} + A_{12}B_{11}.$$

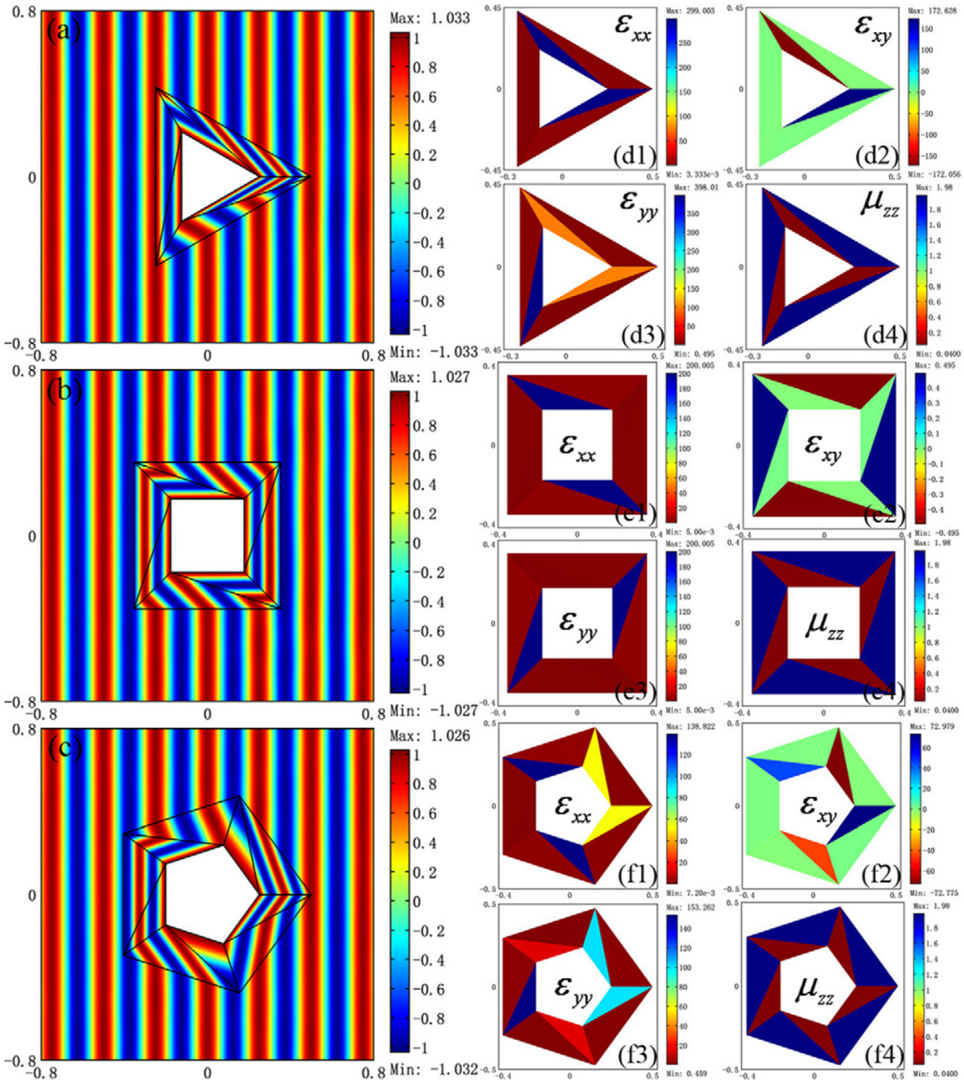

Figure 2. The snapshots of the total magnetic fields ((a)–(c)) and parameter distributions ((d)–(f)) for the symmetrical polygonal cloaks. ((a), (d)) The three-sided polygonal cloak. ((b), (e)) The four-sided polygonal cloak. ((c), (f)) The five-sided polygonal cloak.

The transformation equations that stretch the line segment OB_k to OB'_k are

$$x' = A_{21}x + B_{21}y$$
$$y' = A_{22}x + B_{22}y \qquad z' = z \tag{3}$$

where

$$A_{21} = 1 - \frac{y_{k+1}}{x_{k+1}}B_{21}, \qquad B_{21} = \frac{(m-n)x_k x_{k+1}}{n(x_{k+1}y_k - x_k y_{k+1})},$$
$$A_{22} = \frac{y_{k+1}}{x_{k+1}}(1 - B_{22}), \qquad B_{22} = \frac{m x_{k+1} y_k - n x_k y_{k+1}}{n(x_{k+1}y_k - x_k y_{k+1})}.$$

Then, the transformation equations in the second step that compress the triangle $B'_k O A_{k+1}$ into $B'_k B'_{k+1} A_{k+1}$ can be written as

$$x'' = A_{31}x' + B_{31}y' + C_{31}$$
$$y'' = A_{32}x' + B_{32}y' + C_{32} \qquad z'' = z' \tag{4}$$

where

$$A_{31} = 1 - m - \frac{y_{k+1}}{x_{k+1}}B_{31}, \qquad B_{31} = \frac{(x_{k+1} - m x_k)x_{k+1}}{x_k y_{k+1} - x_{k+1} y_k},$$
$$C_{31} = m x_{k+1}, \qquad A_{32} = \frac{y_{k+1}}{x_{k+1}}(1 - m - B_{32}),$$
$$B_{32} = \frac{(1-m)x_k y_{k+1} - (y_k - y_{k+1})x_{k+1}}{x_k y_{k+1} - x_{k+1} y_k}, \qquad C_{32} = m y_{k+1}.$$

Substituting (3) into (4), we obtain

$$x'' = M_1 x + N_1 y + C_{31}$$
$$y'' = M_2 x + N_2 y + C_{32} \qquad z'' = z \tag{5}$$

where

$$M_1 = A_{21}A_{31} + A_{22}B_{31}, \qquad N_1 = B_{21}A_{31} + B_{22}B_{31},$$
$$M_2 = A_{21}A_{32} + A_{22}B_{32}, \qquad N_2 = B_{21}A_{32} + B_{22}B_{32}.$$

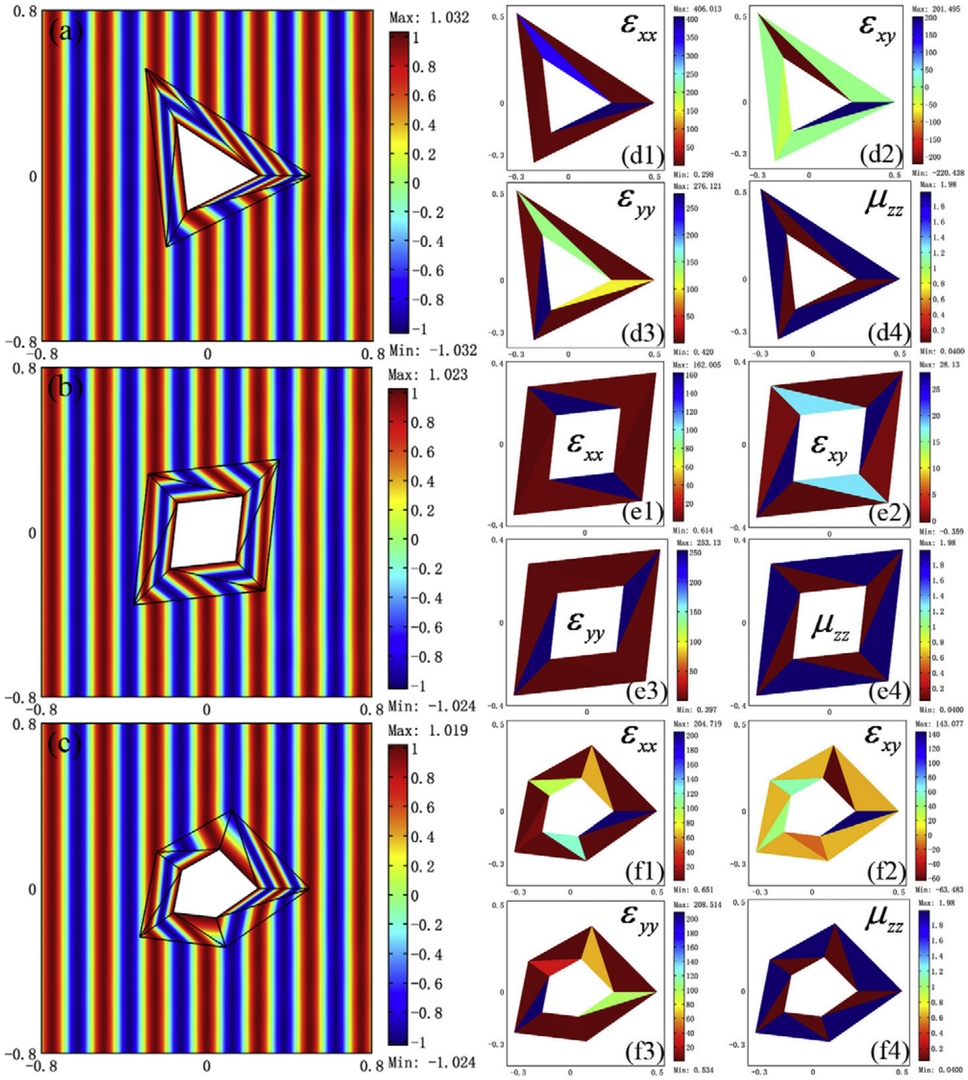

Figure 3. The snapshots of the total magnetic fields ((a)–(c)) and parameter distributions ((d)–(f)) for the asymmetrical polygonal cloaks. ((a), (d)) The three-sided polygonal cloak. ((b), (e)) The four-sided polygonal cloak. ((c), (f)) The five-sided polygonal cloak.

Then the parameters for the triangle $B'_k B'_{k+1} A_{k+1}$ become

$$\varepsilon'' = \mu'' = \frac{1}{Q_2}\begin{bmatrix} \varepsilon''_{xx} & \varepsilon''_{xy} & 0 \\ \varepsilon''_{xy} & \varepsilon''_{yy} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (6)$$

where

$$\varepsilon''_{xx} = M_1^2 + N_1^2, \qquad \varepsilon''_{xy} = M_1 M_2 + N_1 N_2,$$

$$\varepsilon''_{yy} = M_2^2 + N_2^2, \qquad Q_2 = M_1 N_2 + M_2 N_1.$$

The parameters for the kth region are shown in equations (2) and (6), and the parameters for the $(k+1)$th region could be obtained by replacing k with $k+1$. It is interesting to note that the constitutive parameters are spatially invariant once m, n and the vertices of the polygon have been fixed. This is because no other variables are involved in the material parameter tensors. Clearly, there is no singularity in the material parameters. These two features are distinguished from the current polygonal cloak case [24, 25], and thus greatly enhance the realizability of arbitrarily shaped cloaks in practice.

3. Simulations and discussion

In order to verify the design formulas, i.e., equations (2) and (6), a full-wave simulation based on the finite element method (FEM) is carried out to show the properties of the arbitrary N-sided polygonal cloak. The inner boundary of the cloak is PEC (perfectly conducting). The working frequency is 1.2 GHz under TM polarization, and we set $m = 0.5$. As for the case of the TE polarization, the simulation can be done in the same way.

For an N-sided symmetrical polygon, the boundary can be expressed as the combination of N sides, and each side can be expressed through the general expression for the vertices. The general expression for the kth vertex is $x_k = R_0 \cos[2(k-1)\pi/N + \theta_0]$ and $y_k = R_0 \sin[2(k-1)\pi/N + \theta_0]$, where

$R_0 = \sqrt{x_1^2 + y_1^2}$ and $\theta_0 = \arctan(y_1/x_1)$. Snapshots of the total magnetic fields of symmetrical polygonal cloaks with $R_0 = 0.5$ m and $n = 0.01$ are shown in figures 2(a)–(c). Figure 2(a) corresponds to the three-sided polygonal cloak with $\theta_0 = 0$; figure 2(b) corresponds to the four-sided polygonal cloak with $\theta_0 = 45°$; figure 2(c) corresponds to the five-sided polygonal cloak with $\theta_0 = 0$. In figures 3(a)–(c), the fields remain unperturbed with the cloak, as if there were no scattering. The cloaking effect can be clearly found. Under illumination with TM polarized waves, only ε_{xx}, ε_{xy}, ε_{yy} and μ_{zz} are of interest. The distributions of material parameters corresponding to figures 2(a)–(c) are illustrated in figures 2(d)–(f). It is clear that all values are finite and homogeneous in each region.

The design method proposed in this paper is successful not only for arbitrary symmetrical polygonal cloaks but also for asymmetrical polygonal cloaks. For simplicity, we choose the expression for the kth vertex of the N-sided asymmetrical polygon as $x_k = R_k \cos[2(k-1)\pi/N + \theta_0]$ and $y_k = R_k \sin[2(k-1)\pi/N + \theta_0]$, where $R_k = \sqrt{x_k^2 + y_k^2}$ and $\theta_0 = \arctan(y_1/x_1)$.

Figures 3(a)–(c) presents the snapshots of the total magnetic fields for the asymmetrical polygonal cloaks with $n = 0.01$. Figure 3(a) corresponds to the three-sided polygonal cloak with $R_1 = 0.5$ m, $R_2 = 0.6$ m, $R_3 = 0.4$ m, and $\theta_0 = 0$. Figure 3(b) corresponds to the four-sided polygonal cloak with $R_1 = R_3 = 0.5$ m, $R_2 = R_4 = 0.4$ m, and $\theta_0 = 45°$. Figure 3(c) corresponds to the five-sided polygonal cloak with $R_1 = 0.5$ m, $R_2 = R_4 = 0.4$ m, $R_3 = R_5 = 0.3$ m, and $\theta_0 = 0$. Clearly, the invisibility performance of the asymmetrical polygonal cloaks is as good as that of the symmetrical polygonal cloaks. We also calculate the distributions of material parameters corresponding to figures 3(a)–(c), as shown in figures 3(d)–(f). The finite and homogeneous values can be seen, obviously. It can be concluded that the method proposed in this paper can be utilized for designing arbitrarily shaped cloaks with nonsingular and homogeneous parameters.

Unlike the cloaks in which the cloaked region can be crushed to a point [2, 3, 6–9], the arbitrary N-sided polygonal cloak proposed in this paper is designed by expanding N line segments. Hence, we should investigate the influence of the cloaking performance by varying the lengths of the line segments. Taking the three-sided symmetrical polygonal cloak (shown in figure 2(a)) as an example, we have computed the scattered fields for the three-sided symmetrical polygonal PEC case with or without the cloak for different values of n. Figure 4(a) shows the forward scattering at the observation line $x = 0.6$ m, and figure 4(b) shows the backward scattering at the observation line $x = -0.4$ m. Clearly, the smaller the value of n, the more perfect the cloaking performance of such an N-sided polygonal cloak. However, an extremely small value of n may also lead to extreme material parameters according to equations (2) and (6), which are harder to realize. Hence, we should choose the value of n appropriately by considering the practical engineering requirements.

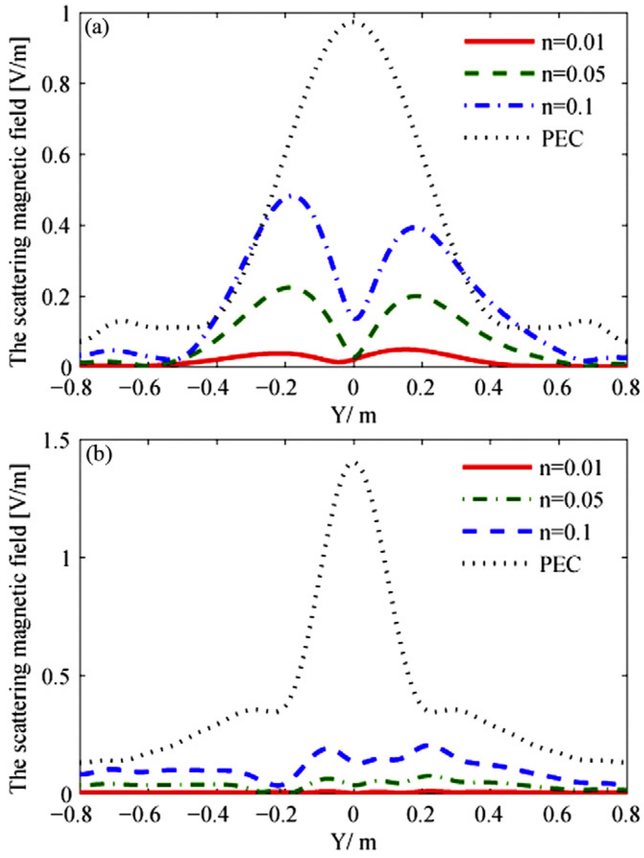

Figure 4. The comparison of the scattered magnetic fields for a three-sided symmetrical polygonal PEC case with or without the cloak for different values of n. (a) The forward scattering at the observation line $x = 0.6$ m. (b) The backward scattering at the observation line $x = -0.4$ m.

4. Conclusion

In this paper, a twofold transformation method for designing an arbitrarily shaped cloak with nonsingular and homogeneous parameters is proposed. The formulas for computing the permittivity and permeability for the kth region are obtained, and these can be expressed using a general equation. Thus we can derive the parameters of the next region conveniently by replacing k with $k + 1$. Numerical simulation shows the good performance of such cloaks and validates the design method. The cloak proposed in this paper is nonsingular and could be constructed with blocks of homogeneous materials. Although the parameters are still anisotropic, they could be easily realized through an alternating layered system made up of two isotropic materials, on the basis of the effective medium theory (EMT) [26, 27] and the improved EMT of [28].

References

[1] Leonhardt U 2006 *Science* **312** 1777
[2] Pendry J B, Schurig D and Smith D R 2006 *Science* **312** 1780
[3] Jiang W X, Cui T J, Yu G X, Lin X Q, Cheng Q and Chin J Y 2008 *J. Phys. D: Appl. Phys.* **41** 085504
[4] Jiang W X, Cui T J, Yang X M, Cheng Q, Liu R P and Smith D R 2008 *Appl. Phys. Lett.* **93** 194102

[5] Chen T Y and Weng C N 2009 *Opt. Express* **17** 8614
[6] Han T C, Tang X H and Xiao F 2009 *J. Electromag. Waves Appl.* **23** 2055
[7] Han T C and Qiu C-W 2010 *Opt. Express* **18** 13038
[8] Hu J, Zhou X and Hu G 2009 *Opt. Express* **17** 1308
[9] Novitsky A, Qiu C-W and Zouhdi S 2009 *New J. Phys.* **11** 113001
[10] Qiu C-W, Novitsky A and Gao L 2010 *J. Opt. Soc. Am.* A **27** 1079
[11] Zhang B L *et al* 2007 *Phys. Rev.* B **76** 121101
[12] Chen H S, Wu B-I, Zhang B L and Kong J A 2007 *Phys. Rev. Lett.* **99** 063903
[13] Qiu C-W, Novitsky A, Ma H and Qu S B 2009 *Phys. Rev.* E **80** 016604
[14] Schurig D, Mock J J, Justice B J, Cummer S A, Pendry J B, Starr A F and Smith D R 2006 *Science* **314** 977
[15] Liu R, Ji C, Mock J J, Chin J Y, Cui T J and Smith D R 2006 *Science* **323** 366
[16] Ergin T, Stenger N, Brenner P, Pendry J B and Wegener M 2010 *Science* **328** 337
[17] Rahm M, Cummer S A, Schurig D, Pendry J B and Smith D R 2008 *Phys. Rev. Lett.* **100** 063903
[18] Xi S, Chen H, Wu B-I and Kong J A 2009 *IEEE Microw. Wireless Compon. Lett.* **19** 131
[19] Lai Y, Chen H Y, Zhang Z Q and Chan C T 2009 *Phys. Rev. Lett.* **102** 093901
[20] Han T C, Tang X H and Xiao F 2009 *J. Phys. D: Appl. Phys.* **42** 235403
[21] Chen H Y and Chan C T 2009 *Opt. Lett.* **34** 2649
[22] Han T C, Tang X H and Xiao F 2010 *IEEE Microw. Wireless Compon. Lett.* **20** 64
[23] Li W, Guan J G, Sun Z G, Wang W and Zhang Q J 2009 *Opt. Express* **17** 23410
[24] Wu Q, Zhang K, Meng F Y and Li L W 2009 *J. Phys. D: Appl. Phys.* **42** 035408
[25] Diatta A, Nicolet A, Guenneau S and Zolla F 2009 *Opt. Express* **17** 13389
[26] Chen H and Chan C T 2008 *Phys. Rev.* B **78** 054204
[27] Xu X F, Feng Y J, Hao Y, Zhao J M and Jiang T 2009 *Appl. Phys. Lett.* **95** 184102
[28] Qiu C-W, Hu L and Zouhdi S 2010 *Opt. Express* **18** 14950