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Review

Transformation Laplacian metamaterials: recent advances in manipulating thermal and dc fields

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Abstract
The full control of single or even multiple physical fields has attracted intensive research attention in the past decade, thanks to the development of metamaterials and transformation optics. Significant progress has been made in vector fields (e.g., optics, electromagnetics, and acoustics), leading to a host of strikingly functional metamaterials, such as invisibility cloaks, illusion devices, concentrators, and rotators. However, metamaterials in vector fields, designed through coordinate transformation of Maxwell’s equations, usually require extreme parameters and impose challenges on the actual realization. In this context, metamaterials in scalar fields (e.g., thermal and dc fields), which are mostly governed by the Laplace equation, lead to more plausible and facile implementations, since there are native insulators and excellent conductors (serving as two extreme cases). This paper therefore is particularly dedicated to reviewing the most recent advances in Laplacian metamaterials in manipulating thermal (both transient and steady states) and dc fields, separately and (or) simultaneously. We focus on the theory, design, and realization of thermal/dc functional metamaterials that can be used to control heat flux and electric current at will. We also provide an outlook toward the challenges and future directions in this fascinating area.

Keywords: transformation optics, thermal/dc metamaterials, thermodynamics, invisibility cloak, scattering cancellation

(Some figures may appear in colour only in the online journal)

1. Introduction
Manipulation and application of light or electromagnetic waves at will has been a long-standing dream for many researchers over the decades. As early as in 1968, Veselago theoretically investigated a medium with simultaneously negative values of $\varepsilon$ and $\mu$ [1]. In 1996 and 1999, Pendry et al realized artificial electric plasma [2] and magnetic plasma [3] using wire medium and split-ring resonators (SRRs), respectively. In 2001, the first artificial left-handed materials (LHMs) were experimentally realized by Shelby et al using a combination of wire and SRRs [4]. With the great development of metamaterials, it is time to explore novel functional devices, such as cloaks [5], which were only possible in science fiction. Transformation optics (TO) builds a bridge between the device functions and the material properties, which tells us the required metamaterial parameters for a specific functional device [6, 7]. After the experimental
realization of invisibility cloaks in the microwave regime [8], much more attention has been paid to exploration and realization of various cloaks, such as optical cloaks [9], non-superluminal cloaks [10], and unidirectional cloaks [11]. Another strategy to further reduce a full cloak into something more practical is the carpet cloak [12], which attempts to hide an object on the ground and mimic a half-infinite vacuum space. Two dimensional (2D) carpet cloaks have been experimentally demonstrated in the microwave range [13] and optical range [14], while three dimensional (3D) carpet cloaks were realized in the microwave range [15] and optical range [16] soon after. More recently, calcite crystals have been employed to fabricate carpet cloaks based on linear transformation [17, 18], and a ray-optics cloak has been experimentally demonstrated [19]. Readers may find more information in a recent review paper [20]. In addition to invisibility cloaking [8–20], TO theory has been applied to design a wide variety of optical devices, such as superlenses [21–23], hyperlenses [24, 25], concentrators [26, 27], rotators [28–30], artificial black holes [31, 32], and bending waveguides [33, 34]. Beyond the optics/ electromagnetics realm [8–34], this theoretical tool has been extended to matter waves [35], elastic waves [36, 37], acoustic waves [38, 39], magnetostatic fields [40–43], thermal conduction [44–58], and dc fields [59–64].

Inspired by TO theory [7, 44], manipulation of heat flux was first investigated by Fan et al [45] and Chen et al [46] in 2008. Later, in 2010, a bifunctional cloak working for both static electric field and steady-state heat conduction was numerically studied by Li et al [47], which is because the two physical processes obey the same conduction equation, i.e. the Laplace equation. In 2012, Guenneau et al extended TO theory to the thermodynamics field to control thermal conduction [48]. Through tailoring the inhomogeneity and anisotropy of conductivities (as well as the specific heat and material density), transient thermal cloaks were realized by Schittny et al [49] and Ma et al [50] in the following year. Meanwhile, steady-state thermal cloaking can be designed with only anisotropic conductivities and its construction can be further simplified by utilizing the multilayered composite approach, as demonstrated both experimentally [51, 52] and theoretically [53]. Recently, inspired by magnetostatic cloaks realized using a layer of ferromagnetic material and a layer of superconductor [40], bilayer thermal cloaks have been experimentally demonstrated in two dimensions [54] and three dimensions [55], respectively. In addition to thermal cloaks, other functional metamaterials have also been theoretically investigated and experimentally demonstrated, such as thermal concentrators [48, 51, 52, 56, 58], rotators [51, 52], thermal camouflage [57], and sensu-shaped thermal metamaterials [58].

On the basis of the form invariance of the Laplace equation [44], Yang et al experimentally demonstrated the first dc electric cloak using an anisotropic and spatially-varying network of resistors in 2012 [59]. Later, in 2013, an ultrathin dc electric cloak with anisotropic but homogeneous conductivities was realized by Jiang et al using similar resistor networks [60]. Meanwhile, with the aid of active sources, an exterior dc cloak [61] and active dc cloak [62] were reported by the same research group. Recently, inspired by magnetostatic cloaks [40], an alternative way to realize a dc electric cloak with two layers of natural bulk materials that is similar to scattering-cancellation technology was experimentally demonstrated [63]. In addition to dc cloaks [59–63], other dc metamaterials with interesting functionalities have also been demonstrated, such as dc concentrators [63, 64], and dc illusion devices [62, 65].

In this review, we first summarize the design approaches for various kinds of thermal/dc metamaterials, including transformation-based cloaks (with inhomogeneous and anisotropic parameters), multilayered cloaks (with homogeneous and anisotropic parameters), bilayer cloaks (with homogeneous and isotropic parameters), concentrators, and rotators. Then we will focus on the most recent advances, both thermal metamaterials and dc metamaterials. Finally, we provide an outlook on the challenges and future directions in this fascinating area.

2. Design approaches

Heat flows spontaneously from a high temperature region toward a low temperature region. Without a heat source, the thermal conduction equation can be written as

$$\nabla \cdot (\kappa \nabla T) - \rho \frac{\partial T}{\partial t} = 0$$

(1)

where $T$ represents the temperature. Moreover, $\kappa$ is the thermal conductivity, $\rho$ is the density and $c$ is the specific heat capacity.

For a steady state, equation (1) becomes

$$\nabla \cdot (\kappa \nabla T) = 0$$

(2)

Similarly, without the source and the external current, the electrical conduction equation can be written as

$$\nabla \cdot (\sigma \nabla V) = 0$$

(3)

where $V$ is the electrical potential and $\sigma$ is the electrical conductivity.

Apparently, the governing equations of steady-state heat conduction (equation (2)) and electrical conduction (equation (3)) have the same form, i.e. the Laplace equation. Therefore, they obey the same design approach and we focus on thermal conduction in the following analysis.

2.1. TO-based cloak

On the basis of the invariance of the heat conduction equation under coordinate transformations, transformation thermodynamics has provided a new method to manipulate heat flux at will [48]. Specifically, in the transformed space, the thermal conduction equation can be written as

$$\nabla' \cdot \left( \kappa' \nabla' T' \right) - \rho' c' \frac{\partial T'}{\partial t'} = 0$$

(4)
where
\[ \kappa'_r = \frac{A_k A^T}{\det(A)}, \quad \rho'c' = \frac{\rho c}{\det(A)} \] (5)
and \( A = \frac{\partial(x', y', z')}{\partial(x, y, z)} \) is the Jacobian of the coordinate transformation.

Considering the 2D case, i.e. a cylindrical cloak, where a circular region \((r \leq b)\) in original space \((r, \phi, z)\) is compressed into an annular region \((a \leq r' \leq b)\) in physical space \((r', \phi', z')\), the transformation equation can be expressed as
\[ r' = \frac{b - a}{b} r + a, \quad \phi' = \phi, \quad z' = z \] (6)

Substituting (6) into (5), we can obtain the transformed conductivity and the transformed product of density and heat capacity:
\[ \kappa'_r = \kappa_0 \begin{pmatrix} \frac{r' - a}{r} & 0 & 0 \\ 0 & \frac{r'}{r' - a} & 0 \\ 0 & 0 & \left(\frac{b}{b - a}\right)^2 \frac{r' - a}{r'} \end{pmatrix} \] (7a)
\[ (\rho'c') = \left(\frac{b}{b - a}\right)^2 \frac{r' - a}{r'} (\rho_0 c_0) \] (7b)
where \( \kappa_0, \rho_0, \) and \( c_0 \) are the thermal conductivity, density, and specific heat capacity of the background medium, respectively.

For the 3D case, i.e. a spherical cloak, where a sphere \((r \leq b)\) in original space \((r, \theta, \phi)\) is compressed into a shell \((a \leq r' \leq b)\) in physical space \((r', \theta', \phi')\), the transformation equation can be expressed as
\[ r' = \frac{b - a}{b} r + a, \quad \theta' = \theta, \quad \phi' = \phi \] (8)

Substituting (8) into (5), we can obtain the transformed conductivity and the transformed product of density and heat capacity:
\[ \kappa'_r = \kappa_0 \begin{pmatrix} \frac{b}{b - a} & \left(\frac{r' - a}{r'}\right)^2 & 0 \\ 0 & \frac{b}{b - a} & 0 \\ 0 & 0 & \frac{b}{b - a} \end{pmatrix} \] (9a)
\[ (\rho'c') = \left(\frac{b}{b - a}\right)^3 \left(\frac{r' - a}{r'}\right)^2 (\rho_0 c_0) \] (9b)

From equations (7) and (9) we find that the transformed conductivities are inhomogeneous and anisotropic. It is also found that the transformed product of density and heat capacity is inhomogeneous. Furthermore, the ideal thermal cloaks have singularities \( \left(\kappa'_r \right)_{\theta = 0} \rightarrow \infty; \ (\rho'c')_{2D} \rightarrow 0 \) at the inner boundary \( (r' = a) \). These exotic parameters are difficult to achieve in practice. Therefore, simplification has to be made in the practical realization of a thermal cloak.

2.2. Multilayered cloak
If we consider a steady-state case, the governing equation is shown in equation (2), where only thermal conductivity is involved. Therefore, the conductivity of a steady-state thermal cloak is \( \kappa'_r = \frac{1}{\kappa_0} = \frac{r'}{r} \). Clearly, the transformation-based steady-state thermal cloak is still inhomogeneous and anisotropic. Surprisingly, Narayana et al. experimentally demonstrated a multi-layered cylindrical cloak with only two kinds of isotropic materials [51], which means that the inhomogeneity of the transformation-based cloak has been removed. A detailed theoretical analysis was later provided to give a physical interpretation [53]. The conductivities in the cloaking shell follow the condition
\[ \kappa'_r = \frac{1}{\kappa'_0} = C \kappa_0 \] (10)
where \( C \) is a constant with \( 0 < C < \frac{b - a}{b} \) and \( \kappa_0 \) is the thermal conductivity of the background medium. Thus, a homogeneous/anisotropic cloak without a singularity is achieved. On the basis of effective medium theory (EMT), such a cloak could be easily realized using alternating layered isotropic media, and only two types of isotropic material (medium A and medium B) are needed throughout.

Following the design strategy for a 2D cylindrical cloak [53], Chen et al. proved that a 3D spherical cloak can also be constructed using homogeneous/anisotropic materials [66]. The conductivities in the cloaking shell follow the condition
\[ \kappa'_r = \frac{1}{2 \kappa'_0} = \frac{1}{C} \kappa_0 \] (11)
where \( \kappa'_r = \kappa'_0 = \kappa'_0 \), \( C \) is a constant with \( 0 < C < \frac{b - a}{b} \) and \( \kappa_0 \) is the thermal conductivity of the background medium.

Theoretical analysis indicated that higher anisotropy corresponds to better performance, with the price of more difficult fabrication though. We want to have perfect performance and small anisotropy simultaneously. However, this is not possible. In fact, each solution is some compromise between these two quantities, and each quantity can be improved by trading off the other one [53].

2.3. Bilayer cloak
With the aid of the TO method, the derived cloaks are always inhomogeneous and (or) anisotropic. Different from the TO method, another approach is to use the scattering cancellation method to achieve invisibility [5]. Inspired by this method, a magnetostatic cloak has been realized using a layer of ferromagnetic material and a layer of superconductor [40]. For steady-state thermal conduction, its governing equation is also the Laplace equation, as shown in equation (2), which can lead to a bilayer thermal cloak based on the scattering cancellation method.
We design a bilayer thermal cloak that is composed of an inner layer \((a < r < b)\) and an outer layer \((b < r < c)\) with conductivities of \(\kappa_2\) and \(\kappa_3\), respectively. The conductivity of the background is \(\kappa_0\). Considering the inner layer to be a perfect insulation material, i.e., \(\kappa_2 = 0\), when the external field distortion is completely eliminated, we obtain [54]

For the 2D case: \(\kappa_3 = \frac{c^2 + b^2}{c^2 - b^2} \kappa_0\) \hfill (12)

For the 3D case: \(\kappa_3 = \frac{2c^3 + b^3}{2(c^3 - b^3)} \kappa_0\) \hfill (13)

Equations (12) and (13) show that the third parameter can be uniquely determined if any two of \(\kappa_3, \kappa_0, c/b\) are known. It should be noted that equations (12) and (13) are derived from the steady-state conduction equation, leading to perfect performance only in the steady-state situation, in principle.

### 2.4. Concentrator

A thermal concentrator is able to focus the heat flux in a certain region. TO theory can still be used to design a thermal concentrator, leading to inhomogeneous and anisotropic parameters [48]. Again, Narayana et al experimentally demonstrated a thermal concentrator with only two kinds of isotropic material [51], which meant that the inhomogeneity of the transformation-based concentrator was removed. A detailed theoretical analysis was later provided to give a physical interpretation [56]. Assuming the inner and outer radii of the concentrator are \(a\) and \(b\), respectively, we obtain

\[
\kappa'_r = \kappa'_\theta = \kappa_0 \quad r \leq a \\
\kappa'_r = \frac{1}{\kappa_0} = 2^n \quad a < r \leq b
\]

For the 2D case: \(n = b\) \hfill (14)

\[
\kappa'_r = \kappa'_\phi = \frac{b}{a} \kappa_0 \quad r \leq a \\
\kappa'_r = \frac{1}{\kappa'_\phi} = 2^n \quad a < r \leq b
\]

For the 3D case: \(n = b\) \hfill (15)

where \(n\) is a constant of \(n > 0\) and \(\kappa_0\) is the thermal conductivity of the background medium. A larger \(n\) corresponds to a higher concentrating efficiency. It is interesting that the concentrator, with an arbitrary value of \(n\), can always focus the heat current into the inner core without any reflection and distortion.

### 2.5. Rotator

A thermal rotator is able to twist the thermal flux in a certain manner. We assume the inner and outer radii of the rotator are \(a\) and \(b\). For the steady state situation, similar to an electromagnetic rotator [28], the transformation equation can be expressed as

\[
r' = r, \quad \phi' = \phi + \phi_0 \frac{\ln b - \ln r}{\ln b - \ln a}, \quad z' = z
\]

We further derive the conductivity of the rotator as

\[
\kappa'_r = \left(\begin{array}{ccc}
1 & \frac{\phi_0}{\ln b - \ln a} & 0 \\
0 & 1 + \left(\frac{\phi_0}{\ln b - \ln a}\right)^2 & 0 \\
0 & 0 & 1
\end{array}\right)
\]

where \(\phi_0\) is the twisting angle. In particular, a thermal inverter can be achieved when \(\phi_0 = \pm \pi\). From equation (17), we know that the parameters of the rotator are homogenous and anisotropic, which can be realized using alternating layers of two isotropic materials with an overall bend in a spiral manner [51].

### 3. Progress in manipulating thermal fields

Let us start with a thermal invisibility cloak that should fulfil its task: (1) keeping the cloaking region colder than its surroundings by reducing the external heat flux entering the cloaking region; (2) keeping the temperature distribution outside the cloak undisturbed as if no perturbation were there. The parameters of an ideal transient thermal cloak are described in equation (7), which are difficult (if not impossible) to be realized in practice. Mathematically [48], Guenneau et al derived the following reduced thermal conductivities and the reduced product of density and heat capacity

\[
\kappa''_{2D} = \frac{\kappa''_{2D}}{\det(A)} = \left\{\begin{array}{ccc}
\left(\frac{b}{b-a}\right)^2 & 0 \\
0 & \left(\frac{b}{b-a}\right)^2 \\
0 & 0
\end{array}\right\}
\]

From equation (18), we notice that the reduced conductivity has no singularity and only one component is spatially varied. Moreover, the reduced product of density and heat capacity is spatially constant. However, the reduced parameters in equation (18) lead to an approximation of equation (1) as

\[
\nabla \cdot \left(\tilde{k}''_{2D} \nabla T\right) = -\frac{1}{\det(A)} \nabla \cdot \left(\tilde{k}'_{2D} \nabla T\right) + \nabla \left(\frac{1}{\det(A)} \cdot \tilde{k}'_{2D} \nabla T\right)
\]

This approximation can be satisfied only when the perturbation \(\nabla \left(\frac{1}{\det(A)} \cdot \tilde{k}'_{2D} \nabla T\right)\) is small enough, which means that
Figure 1. Demonstration of thermal cloaks based on different design approaches. (a) Transient thermal cloak with inhomogeneous and anisotropic conductivities [49]. (b) Measured temperature distribution of (a). (c) Steady state thermal cloak with homogeneous and anisotropic conductivities [51]. (d) Measured temperature distribution of (c). (e) Snapshot of 2D bilayer thermal cloak with homogeneous and isotropic conductivities [54]. (f) Measured temperature distribution of (e). (g) Snapshot of 3D bilayer thermal cloak [55]. (h) Measured temperature distribution of (g). Figure reprinted with permission: (a) and (b) from [49], Copyright 2013 American Physical Society; (c) and (d) from [51], Copyright 2012 American Physical Society; (e) and (f) from [54], Copyright 2014 American Physical Society; (g) and (h) from [55], Copyright 2014 American Physical Society.

\[
\det(A) = \left(\frac{b-a}{b}\right)^2 \frac{r' - r}{r - r'} > 1. 
\]
This condition is no longer satisfied near the outer boundary of the cloak \((r = b)\), which induces an ‘impedance mismatch’ between the cloak and the background. Nevertheless, it is a valid approximation where the thermal protection functionality is still maintained [48].

The reduced thermal cloak described in equation (18) was later experimentally realized by Schittny et al [49]. They first uniformly discretized the cloaking shell into 5 layers, and the conductivity of each layer can be approximated as homogeneous. To remove the anisotropy of each layer, one layer can be replaced by two materials with thermal conductivity \(\kappa_A, \kappa_B\) and thickness \(d_A, d_B\), respectively. The layered structure is stacked along the radial direction, and the effective heat conductivities are [49]

\[
\kappa_r = \frac{\kappa_A \kappa_B (d_A + d_B)}{d_A \kappa_B + d_B \kappa_A}, \quad \kappa_\phi = \frac{d_A \kappa_A + d_B \kappa_B}{d_A + d_B} \quad (20)
\]

By adjusting the thickness of each material, the corresponding parameters described in equation (18) can be realized. For practical realization, copper and PDMS were used to realize the cloak, as shown in figure 1(a). The measured temperature distribution is shown in figure 1(b), which demonstrates that the central region is colder than its surroundings without disturbing the external profile of the isotherms.

For the steady state case, the governing equation is described in equation (2), where only thermal conductivity is involved. Narayana et al first experimentally demonstrated the realization of a steady-state thermal cloak with homogenous anisotropic materials [51], which was further interpreted and validated through theoretical analysis [53]. In Narayana et al’s work [51], 40 altering layers of natural latex rubber and silicone elastomers containing boron nitride particles were used, as shown in figure 1(c), to construct a multilayered thermal cloak working in a background of agar-water. The measured temperature distribution is shown in figure 1(d). A similar design was also experimentally demonstrated using a glass-epoxy FR-4 printed circuit board (PCB) [52].

Different from the designs based on coordinate transformation introduced above, thermal invisibility can also be achieved using the scattering cancellation method [5]. Inspired by the design method for static magnetic fields [40], bilayer thermal cloaks with only homogeneous isotropic materials have been demonstrated in both 2D [54] and 3D [55] cases. For 2D bilayer thermal cloaks, as shown in figure 1(e), the inner and outer layers are expanded polystyrene and Inconel 625 alloy, and a thermally conductive sealant was used as the host background material. The measured temperature distribution is shown in figure 1(f). Meanwhile, a single layer 3D thermal cloak was demonstrated, as shown in figure 1(g), where copper and stainless steel were selected as the shell and background materials, respectively. The measured temperature distribution is shown in figure 1(h). It is noted that the bilayer thermal cloaks have been validated as exact cloaks, thus leading to very good...
performance in the steady state case. More interesting is that experiments illustrate that these cloaks have fairly good performance in the transient state. This can be definitely achieved if the time-varying term is much smaller than the space-varying term in equation (1); thus the time-varying term can be ignored. In these two experiments [54, 55], both slowly varying temperature and the thin thickness of the cloaks made sure that the cloaks still had fairly good performance in the transient state.

In addition to thermal invisibility cloaks, many other interesting thermal metamaterials have been experimentally demonstrated, such as thermal concentrators, rotators, and camouflage. In Narayana et al’s work [51], 240 azimuthally altering layers of latex rubber and silicone elastomers were used, as shown in figure 2(a), to construct a thermal concentrator working in the background of agar–water. The measured temperature distribution is shown in figure 2(b). Narayana et al also demonstrated a thermal rotator composed of 96 alternating layers of copper and polyurethane, as shown in figure 2(c). The measured temperature distribution is shown in figure 2(d). Thermal camouflage can potentially transform an actual perception into a pre-controlled perception, thus empowering unprecedented applications in thermal illusion [57]. The above functionality can be derived based on a two-fold operation. The first is to eliminate the scattering of the original object by using an exact bilayer cloak, and the second is to create the desired scattering signature by placing expected objects. The fabricated illusion device and its equivalent object are schematically illustrated in figures 2(e) and (g), respectively. The camouflage device was fabricated by drilling holes into a copper plate and filling them with PDMS. The measured temperature distributions of figures 2(e) and (g) are shown in figures 2(f) and (h), respectively, demonstrating good agreement. The thermal camouflage effect was experimentally confirmed in both time-dependent and temperature-dependent cases. In addition to experiments, theoretical investigations on thermal cloaking [75, 76] and illusions [77, 78] have been performed based on the TO method, which usually requires extreme conductivities.

Recently, a new class of sensu-shaped thermal metamaterial (SSTM) units was designed to create a uniform heating region, heat flux focusing, and heat flux concentration through the positioning of multiple identical units [58]. The SSTM unit was composed of 18 copper wedges and 18 PDMS wedges. The fabricated structure for thermal focusing is schematically illustrated in figure 3(a). The measured temperature distribution, as shown in figure 3(b), demonstrates that the enhanced thermal field of the heat source is harvested by the second SSTM unit and focused into its center. The fabricated structure for the creation of a uniform heating region is schematically illustrated in figure 3(c). The measured temperature distribution, as shown in figure 3(d), demonstrates that a nearly uniform heating region is formed between the four heat sources, which is different from the inhomogeneous plate heater implemented using appropriate combinations of aluminium and artificially-synthesized graphite sheets [73]. A concentrator is achieved by the combination of two back-to-back SSTM units, as shown in figure 3(e). The measured temperature distribution is shown in figure 3(f).
4. Progress in manipulating dc fields

On the basis of TO theory [7, 44], the transformed conductivity for the 2D dc cloak is expressed as

\[
\tilde{\sigma} = \begin{pmatrix}
\frac{r'}{r} & 0 \\
0 & \frac{r'}{r'}
\end{pmatrix}
\]

\[
= \frac{r' - a}{r'} \begin{pmatrix}
\frac{r' - a}{r} & 0 \\
0 & \frac{r' - a}{r'}
\end{pmatrix}
\]

(21)

where \( \sigma_0 \) is the thermal conductivity of the background medium, and \( a \) and \( b \) represent the inner and outer radii of the cloak.

Equation (21) shows that the realization of a dc cloak requires inhomogeneous and anisotropic conductivities, which is difficult in wave dynamics but easy in steady current fields. These complicated conductivities can be easily emulated using circuit theory. The continuous material is first discretized using polar grids, and then each elementary cell in the grid can be implemented by two resistors [59]

\[
R_r = \frac{\Delta r}{\sigma_r r \Delta \phi h}, \quad R_\phi = \frac{r \Delta \phi}{\sigma_0 \Delta r h}
\]

(22)

where \( h \) represents the thickness of the continuous material, and \( \Delta r \) and \( \Delta \phi \) are step lengths in the radial and tangential directions, respectively. Thus, the inhomogeneous and anisotropic conductivity shown in equation (21) can be realized using different resistors in different locations.

In 2012, Yang et al experimentally demonstrated the first dc cloak for steady current fields using resistor networks [59]. The fabricated resistor network is illustrated in figure 4(a), which is built on a PCB. The network contains 15 concentric layers in the radial direction, and 36 nodes in the tangential direction. The cloak occupies 5 layers, and the remaining 10 layers are used for the background material. Matching resistors are added to the outer layer to mimic an infinite material. The measured potential distribution is shown in figure 4(b), demonstrating excellent cloaking performance. In 2013, Jiang et al experimentally demonstrated an ultrathin dc electric cloak using resistor networks, where only anisotropic materials were required [60]. Theoretical analysis indicated that the inhomogeneity of the transformation-based thermal cloak can be removed [53], which is also appropriate for dc cloaks because both thermal and electric conduction are dominated by the Laplace equation. Therefore, a dc cloak may have excellent performance as long as its conductivities satisfy \( \sigma'_r = \frac{1}{\sigma'_\phi} = C \sigma_0 \), where \( C \) is a constant. An ultrathin dc cloak can be achieved when \( C \to 0 \). The fabricated ultrathin dc cloak is shown in figure 4(c), and its measured potential distribution is shown in figure 4(d).

Beyond using complicated resistor networks to construct dc cloaks, another approach is to use bulk natural materials. Similar to bilayer thermal cloaks [53], bilayer dc cloaks have also been experimentally demonstrated [63]. A schematic and snapshot of a bilayer dc cloak are shown in figures 4(e) and
(g), where stainless steel was selected as background medium. The inner and outer layers are insulating material (air) and copper, respectively. The cloaking performance can be evaluated by measuring the potential distribution along the observation lines near the bilayer cloak, as shown in figures 4(f) and (h). Apparently, without the cloak, the perturbation strongly distorts the original equipotential lines. When the central region is wrapped by the bilayer cloak, the distortions are eliminated and the potential profiles restore exactly to the original equipotential lines.
In addition to dc cloaks, dc concentrators, that can focus electric currents into a central region, thus enhancing the electric field, have been experimentally demonstrated using different implementation schemes. In 2013, Jiang et al experimentally demonstrated a dc concentrator with inhomogeneous and anisotropic conductivity [64]. In analogy to dc cloaks [59], they used resistor networks to mimic the required conductivity. The fabricated resistor network is illustrated in figure 5(a). The measured potential distribution is shown in figure 5(b). The potential distributions for certain observation lines are shown in figure 5(c), demonstrating good concentrating performance. Another approach for the realization of a dc concentrator is to use bulk natural materials [59]. Theoretical analysis indicated that the dc (or thermal) concentrating performance is mainly dominated by anisotropy, which means that ultra-high concentrating efficiency may be achieved by judiciously selecting the natural materials [56]. A fan-shaped concentrator, including 36 copper wedges and 36 air wedges, as shown in figure 5(d), has been experimentally demonstrated [59]. An exact concentrator has to satisfy two conditions: (1) the external field outside the concentrator should be undisturbed, and (2) the electric field or current density should be focused onto a small region. Figures 5(e) and (f) validate the two conditions, respectively.

Furthermore, voltage sources were introduced into the resistor networks to realize exterior [61] and active [62] dc cloaks. According to TO theory [70], an exterior dc cloak, which is placed away from an object and hides the object at a distance, requires negative electric conductivity. In Yang et al’s experiment [61], a circuit model composed of two resistors and a controlled voltage source was used to mimic the negative electric conductivity. The fabricated resistor network is illustrated in figure 6(a). The measured potential distribution is shown in figure 6(b), indicating the good cloaking performance. In addition to the passive cloaking schemes mentioned above, an active invisibility cloak, which makes use of controlled sources around the protected object, has been theoretically demonstrated [71]. By using a resistor network to simulate a conducting medium, Ma et al used 36 power supplies as the controlled sources to realize an active cloak for steady current fields [62]. A photograph of the fabricated active dc cloak is shown in figure 6(c), and the measured potential distribution is shown in figure 6(d), confirming the correctness of the design. Recently, a light-controlled dc illusion device was experimentally demonstrated,
which is made of a resistor network with light-sensitive semiconductor resistors embedded [65]. By tuning the intensity of the illumination light, the functionalities of the device metamorphosed from a dc cloak to varying dc illusion devices. A photograph of the fabricated dc illusion cloak is shown in figure 6(e). Figure 6(f) shows the measured potential distributions along the observation line when the illuminating light turns ‘on’ or ‘off’, which markedly demonstrates the difference.

5. Progress in manipulating thermal-electric fields

Both thermal and electric conduction in steady state satisfy the Laplace equation, which provides the possibility to manipulate thermal-electric fields simultaneously. A bifunctional cloak was first theoretically explored by Li et al in 2010 [47], which possessed both electrical and thermal cloaking functionality. According to TO theory, the electrical and thermal conductivities (for steady state) of a bifunctional cloak should satisfy equation (21) simultaneously (thermal conductivity is obtained by replacing $\sigma$ with $\kappa$ in equation (21)). In Li et al’s design, they theoretically proposed to distribute nonspherical nanoparticles with appropriate electrical and thermal conductivities, shape aspects, and volume fractions to realize the required electrical and thermal conductivities. In 2014, Moccia et al theoretically designed a metamaterial shell that simultaneously behaved as a thermal concentrator and an electric cloak via a mixture of inclusions of different shapes and materials embedded in a host medium [67]. However, the complicated shape control and spatial distribution of the nanoparticles are obviously challenging for experimental realization.

The realization of a bifunctional cloak is difficult using the TO method, but can be easily achieved using the scattering cancellation method. An electric-thermal bifunctional cloak can be achieved as long as the electric and thermal conductivities of the shell satisfy equation (12) simultaneously (electric conductivity is obtained by replacing $\kappa$ with $\sigma$ in equation (12)). To implement this device, it only needs to satisfy the relation $\kappa_3/\kappa_0 = \sigma_3/\sigma_0$. Ma et al used a typical semiconductor material to realize the first bifunctional cloak

![Image](image.png)
tributions are demonstrated in figure 7(a). The measured temperature and potential distributions are demonstrated in figures 7(b) and (c), respectively. Recently, Yang et al experimentally demonstrated the first invisible sensor in electric-thermal fields [68]. An invisible sensor is a sensing system that may receive and transmit information, while its presence is not perceived by the surroundings [72]. In the experimental setup, the sensor, shell, and background were made of copper, stainless steel, and magnesium alloy, respectively. A photograph of the fabricated sample device is shown in figure 7(d). The measured temperature and potential distributions are demonstrated in figures 7(e) and (f), respectively. Apparently, excellent performance can be achieved for both electric and thermal fields. More recently, an electric-thermal bifunctional concentrator has also been experimentally demonstrated [74].

6. Conclusion and outlook

Exploring thermal/dc metamaterials is inspired by the development of electromagnetic metamaterials. Different from the wave dynamics metamaterials governed by Maxwell’s equations, both electric and thermal conduction in the steady state are dominated by the Laplace equation. Therefore, phase preservation, which has to be considered in the design of wave dynamics metamaterials, is not involved in thermal and dc metamaterials. In that sense, many novel functional devices, i.e. thermal/dc cloaks, concentrators, rotators, etc, can be exactly realized using natural bulk materials. As we have seen, this area has been attracting much research effort in recent years.

Although the development of thermal/dc metamaterials has made great progress, many challenges still exist and need to be clarified. For example, the implementation of thermal/dc functional devices, whether using artificial resistor networks or selected natural materials, is mainly limited to the ‘proof-of-concept’ stage, and needs to be further developed for incorporation into industry-level applications. On the other hand, there are three types of heat transfer: heat conduction, heat radiation and heat convection. In heat radiation, heat is transferred by electromagnetic waves. In heat convection, heat is transferred by mass transport. However, the current research on thermal metamaterials mainly focuses on thermal conduction, thus resulting in limitations for practical application.

As a new research branch in the field of metamaterials, thermal/dc metamaterials still need substantial development. It is gratifying to see the multiple physical metamaterials demonstrated recently, which function in both electric and thermal fields. This is indeed a good start. It is expected that more and more multiple physical metamaterials will be experimentally demonstrated in the near future.

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