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# **Electromagnetic Scattering Properties in a Multilayered Metamaterial Cylinder**

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**SUMMARY** Electromagnetic scattering properties of metamaterial cylinders due to a line source are studied by a multilayer algorithm based on eigenfunctional expansion. Closed forms of electric and magnetic fields are formulated. Both the fields inside the cylinder and field in outer space are plotted for different sizes of the cylinder. The focusing phenomena and the wave propagation in the presence of metamaterial cylinders are investigated and shown. Electromagnetic field distributions are presented for subwavelength metamaterial cylinders and cylinders fabricated by magnetoelectric materials, and resonant scattering and focusing properties are reported. Special designs of scatterer cloaking are proposed and calculated by multilayer algorithm which can reduce scattering cross sections.

key words: multilayer algorithm, metamaterial, magnetoelectric coupling, subwavelength cylinder, cloaking

# 1. Introduction

The permeability and permittivity of a material are two of the fundamental parameters that determine how the material interacts with electromagnetic (EM) waves propagating and scattering in the materials. Metamaterials, with simultaneously negative permittivity and permeability in a frequency band, have received intensive interests, which exhibit a lot of exotic properties (e.g., reversal Doppler shift and negative refraction [1], reversed circular Bragg phenomenon [2] and perfect lens [3]). These materials exhibit a left-handed rule which are referred to as left-handed materials (LHM) in literature [4]–[6]. The double negative (DNG) material has shown special optical properties, and could lead to a perfect lens [7], [8].

The plane wave propagating through layered DNG media were formulated by Kong [9] for planar structures. In this paper, existing application is extended to the line-source incidence associated with cylindrical multilayers, so as to gain more insight into the hybrid effects of metamaterials and cylindrical curvature. Eigenfunction expansion technique [10]–[12] is employed to establish a systematic

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scheme, which can incorporate the material functionality, the geometrical parameters and incidence characteristics together. By using the multilayer algorithm, single cylinders are considered so as to investigate the subwavelength imaging and wave interaction. Different kinds of metamaterial single cylinders are studied. Interesting results such as focusing properties and energy localization are shown. Next, we further study the cylinder with magnetoelectric coupling as a special case. In this case, the cylinder is characterized by 3 parameters: permittivity, permeability and chirality. The effect of magnetoelectric coupling is presented and resonant scattering is discussed. Finally, the cloaking of cylinders and its effects on cross section are examined.

#### 2. Preliminaries

#### 2.1 Eigenfunction Expansion

Consider a parallel infinite line source with electric current *I*, which is placed at  $(\rho_0, \phi_0)$  from the cylinder as depicted in Fig. 1. The incident field due to the line source can be expressed

$$E^{i} = -\frac{k^{2}I}{4\omega\varepsilon_{0}} \sum_{n=0}^{\infty} (2 - \delta_{n0}) H_{n}^{(1)}(k\rho_{0}) N_{n}^{(1)}(k) e^{-jn\phi_{0}}$$
(1)



**Fig.1** Cross-section view of a multilayered cylinder with the line source in 1st region.

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$$\boldsymbol{H}^{i} = -\frac{kI}{4j} \sum_{n=0}^{\infty} \left(2 - \delta_{n0}\right) H_{n}^{(1)}\left(k\rho_{0}\right) \boldsymbol{M}_{n}^{(1)}\left(k\right) e^{-jn\phi_{0}}$$
(2)

where the vector wave functions (VWFs) have been given in [13], [14]

$$N_n^{(1)}(k) = \widehat{z} J_n(k\rho) e^{jn\phi}$$
(3)

$$\boldsymbol{M}_{n}^{(1)}(k) = \left[\widehat{\boldsymbol{\rho}}\frac{jn}{k\rho}J_{n}(k\rho) - \widehat{\boldsymbol{\phi}}J_{n}'(k\rho)\right]e^{jn\phi}.$$
(4)

One can however note that the superscript of VWFs would change to 3 if the multiple reflection and transmission are taken into accout due to the interfaces in Fig. 1:

$$N_n^{(3)}(k) = \hat{z} H_n^{(1)}(k\rho) e^{jn\phi}$$
(5)

$$\boldsymbol{M}_{n}^{(3)}(k) = \left[\widehat{\boldsymbol{\rho}}\frac{jn}{k\rho}H_{n}^{(1)}(k\rho) - \widehat{\boldsymbol{\phi}}H_{n}^{\prime(1)}(k\rho)\right]e^{jn\phi}.$$
(6)

Based on the eigenfunction expansion, the electromagnetic fields in intermediate layers (e.g., *f*th layer) are as follows

$$E_{f} = \sum_{n=0}^{\infty} \left\{ a_{nf} N_{n}^{(3)} \left( k_{zf} \right) + b_{nf} M_{n}^{(3)} \left( k_{zf} \right) + a'_{nf} N_{n}^{(1)} \left( k_{zf} \right) + b'_{nf} M_{n}^{(1)} \left( k_{zf} \right) \right\},$$
(7)

$$H_{f} = \frac{1}{j\eta_{f}} \sum_{n=0}^{\infty} \left\{ a_{nf} M_{n}^{(3)} \left( k_{zf} \right) + b_{nf} N_{n}^{(3)} \left( k_{zf} \right) + a_{nf}' M_{n}^{(1)} \left( k_{zf} \right) + b_{nf}' N_{n}^{(1)} \left( k_{zf} \right) \right\}.$$
(8)

In Eq. (7), scattering coefficients  $a'_{nf}$  and  $b'_{nf}$  denote incoming TM and TE modes (Bessel function being convergent approaching the origin), and  $a_{nf}$  and  $b_{nf}$  represent outgoing TM and TE modes (Hankel function being finite in the infinity). Therefore, in the 1st region, the scattered electric field only contains Hankel function. In *N*th region, the scattered electric field only contains Bessel function. This is the principle of eigenfunction expansion. In the intermediate layers, the fields are just the superposition of inward and outward scattered fields due to the multiple interfaces. Note that  $k_{zf}$  and  $\eta_f = k_f/\omega\mu_f$  denote longitudinal wavenumber and wave impedance in *f*th layer, respectively.  $k_f^2 = \omega^2 \mu_f \epsilon_f$ is the wavenumber for incidence at arbitrary angles, and it reads  $k_f^2 = k_\rho^2 + k_z^2$  where  $k_\rho$  is the radial wavenumber in *f*th layer.

Those scattering coefficients can be solved in a multilayer algorithm associated with boundary conditions at each interface, which will be shown later. Thus, we can have the electromagnetic field expansions based on the field coupling together with the incoming/outgoing waves superposition. The electric fields in 1st and *N*th layer are given, respectively

$$\boldsymbol{E}_{1} = \boldsymbol{E}^{i} + \sum_{n=0}^{\infty} \left\{ a_{n1} N_{n}^{(3)}(k_{z1}) + b_{n1} \boldsymbol{M}_{n}^{(3)}(k_{z1}) \right\}$$
(9)

$$\boldsymbol{E}_{N} = \sum_{n=0}^{\infty} \left\{ a_{nN}' \boldsymbol{N}_{n}^{(1)}(k_{zN}) + b_{nN}' \boldsymbol{M}_{n}^{(1)}(k_{zN}) \right\}.$$
 (10)

Note that the vectors M(k) and N(k) have the following relations

$$\nabla \times \boldsymbol{M}(k) = k\boldsymbol{N}(k) \tag{11}$$

$$\nabla \times N(k) = kM(k). \tag{12}$$

Therefore, the magnetic fields in 1st and *N*th can be obtained as follows

$$\boldsymbol{H}_{1} = \boldsymbol{H}^{i} + \frac{1}{j\eta_{1}} \sum_{n=0}^{\infty} \left\{ a_{n1} \boldsymbol{M}_{n}^{(3)}(k_{z1}) + b_{n1} \boldsymbol{N}_{n}^{(3)}(k_{z1}) \right\}$$
(13)

$$\boldsymbol{H}_{N} = \frac{1}{j\eta_{N}} \sum_{n=0}^{\infty} \left\{ a_{nN}' \boldsymbol{M}_{n}^{(1)}(k_{zN}) + b_{nN}' \boldsymbol{N}_{n}^{(1)}(k_{zN}) \right\}.$$
(14)

# 2.2 Multilayer Algorithm

The electromagnetic fields satisfy the boundary conditions at each interface  $\rho = \rho_f$ 

$$\widehat{\boldsymbol{\rho}} \times \begin{bmatrix} \boldsymbol{E}_f \\ \boldsymbol{H}_f \end{bmatrix} = \widehat{\boldsymbol{\rho}} \times \begin{bmatrix} \boldsymbol{E}_{(f+1)} \\ \boldsymbol{H}_{(f+1)} \end{bmatrix}.$$
(15)

Inserting Eqs. (7) and (8) into Eq. (15), we have a linear equation

$$\left[F_f\right]\left[C_f\right] = \left[F_{(f+1)}\right]\left[C_{(f+1)}\right] \tag{16}$$

where  $[F_f]$  is a 4 × 4 matrix

$$\begin{bmatrix} F_f \end{bmatrix} = \begin{bmatrix} F_{f1} \\ F_{f2} \\ F_{f3} \\ F_{f4} \end{bmatrix}$$
(17)

with the definition

$$\begin{bmatrix} F_{f1} \end{bmatrix} = \begin{bmatrix} 0 & -H'_n{}^{(1)}(k_f \rho) & 0 & -J'_n(k_f \rho) \end{bmatrix} \\ \begin{bmatrix} F_{f2} \end{bmatrix} = \begin{bmatrix} -H_n{}^{(1)}(k_f \rho) & 0 & -J_n(k_f \rho) & 0 \end{bmatrix} \\ \begin{bmatrix} F_{f3} \end{bmatrix} = \begin{bmatrix} -\frac{H'_n{}^{(1)}(k_f \rho)}{j\eta_f} & 0 & -\frac{J'_n(k_f \rho)}{j\eta_f} & 0 \end{bmatrix} \\ \begin{bmatrix} F_{f4} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{H_n{}^{(1)}(k_f \rho)}{j\eta_f} & 0 & -\frac{J_n(k_f \rho)}{j\eta_f} \end{bmatrix}.$$

Note that the derivative is with respect to the argument. The  $[C_f]$  is a vector of unknown scattering coefficients

$$\begin{bmatrix} C_f \end{bmatrix} = \begin{bmatrix} a_{nf} & b_{nf} & a'_{nf} & b'_{nf} \end{bmatrix}^T.$$
 (18)

By defining a new matrix

$$\left[T_f\right] = \left[F_{f+1}\right]^{-1} \left[F_f\right],\tag{19}$$

we can rewrite

$$[C_N] = \left[T_N^{(1)}\right][C_1]$$
(20)

where

$$\left[T_{N}^{(f)}\right] = \left[T_{N-f}\right] \left[T_{N-(f+1)}\right] \dots \left[T_{2}\right] \left[T_{1}\right].$$
(21)

Therefore, the coefficient relationship between inner- and outer-most layer can be established

$$\begin{bmatrix} 0\\0\\a'_{nN}\\b'_{nN} \end{bmatrix} = \begin{bmatrix} T_{N}^{(1)} \end{bmatrix} \begin{bmatrix} a_{n1}\\b_{n1}\\a'_{n1}\\b'_{n1} \end{bmatrix}.$$
 (22)

In view of Eq. (1), we have

$$a'_{n1} = -(2 - \delta_{n0}) \frac{k_1^2 I}{4\omega\varepsilon_0} H_n^{(1)}(k_1\rho_0) e^{-jn\phi_0}, \qquad (23)$$

$$b'_{n1} = 0$$
 (24)

where  $k_1$  is just the wave number in free space. Since  $a'_{n1}$  and  $b'_{n1}$  are already known,  $a_{n1}$  and  $b_{n1}$  can be determined via the first two rows of  $[T_N^{(1)}]$  in Eq. (22), regardless of the value of  $a'_{nN}$  and  $b'_{nN}$ . Provided  $[C_1]$  is obtained,  $[C_f]$  is straightforward by using Eq. (21). As a result, electromagnetic fields in any region can be formulated.

# 2.3 Magnetoelectric Coupling

To consider a more general case, the dielectric material is just a subset of the bi-isotropic material, which exhibit magnetoelectric coupling in the form of

$$\boldsymbol{D} = \boldsymbol{\epsilon}_r \boldsymbol{\epsilon}_0 \boldsymbol{E} + j \boldsymbol{\kappa} \sqrt{\boldsymbol{\epsilon}_0 \boldsymbol{\mu}_0} \boldsymbol{H}$$
(25)

$$\boldsymbol{B} = -j\kappa\,\sqrt{\epsilon_0\mu_0}\boldsymbol{E} + \mu_r\mu_0\boldsymbol{H} \tag{26}$$

where the chirality  $\kappa$  denotes the degree of magnetoelectric coupling.

Lindell et al have presented the wavefield decomposition method in [15]. Instead of treating the electromagnetic problems in bi-isotropic media directly, the field components can be split into two partial fields

$$\boldsymbol{E} = \boldsymbol{E}_+ + \boldsymbol{E}_- \tag{27}$$

$$\boldsymbol{H} = \boldsymbol{H}_{+} + \boldsymbol{H}_{-} \tag{28}$$

with a pair of equivalent isotropic mediums:

$$\epsilon_{\pm} = \epsilon (1 \pm \kappa) \tag{29}$$

$$\mu_{\pm} = \mu(1 \pm \kappa). \tag{30}$$

Hence, the previous formulated algorithm can still work even for bi-isotropic media. As a special case, chiral nihility was proposed by Tretyakov et al. in [16] to achieve backward wave and negative refraction. Later on, Qiu et al studied the energy transport in the chiral nihility and revisited the condition of chiral nihility for slabs in [17]. In this paper, we further extend our study on chiral nihility to cylindrical structures with the possibility of imaging and negative rafraction due to cylindrical curvatures. The chirality effect on subwavelength imaging will be discussed in detail in the following section.

#### 3. Results and Discussion

In this section, only one-layer and two-layer cylinders are considered because they possess enough interesting scattering properties. The present work can be extended to study 3-layer or 4-layer cylinders straightforwardly since the multilayer algorithm developed above is suitable for arbitrary layers.

#### 3.1 One-Layer Isotropic Cylinder

First, let us investigate the scattering properties in the presence of one-layer isotropic cylinder due to the line-source radiation. The material inside the cylinder in Fig. 2 is antivacuum (i.e,  $\epsilon = -\epsilon_0$  and  $\mu = -\mu_0$ ). The line source is placed at the position of  $(4.5\lambda, 0^{\circ})$ , which is very close to the surface of cylinder. As is known, a slab filled with anti-vacuum can act as a perfect lens to focus light. However, if we consider perfect cylindrical lens, the parameters of the filling material must be dependent on the position. Nevertheless, partial focusing phenomenon can still be observed in Fig. 2. Also, some reflection can be observed at the positions behind the line source in Fig. 2 because of the cylindrical curvature. It can be seen that ripple occurs at the cylinder's surface close to the line source, which is the incoming window of the incident wave. Certain points of this ripple carry comparably high energy as the line source. In Fig. 3 and Fig. 4, the line source is put further away to the cylinder's surface, while the radius of the cylinder keep unchanged. When the source is far away from the surface, the ripple effect in the incoming window on the cylinder will be reduced as expected. In Fig. 3 and Fig. 4, foculas are formed due to the imperfect focusing condition. By changing the position of line source, the energy distributions outside the cylinder are greatly modified as can be seen from Fig. 2 to Fig. 4. It is due to the fact that when the line source is very close to the cylinder's surface, the curvature is quite flat within the incoming window of incident wave, which is close to slab configuration. If the line source is moved faraway, the incoming window becomes larger and the cylindrical curvature takes effect.



**Fig. 2** Normalized magnitude of Poynting vector of a cylinder of  $a = 4\lambda$  filled with anti-vacuum and the line source at  $(4.5\lambda, 0^{\circ})$ .



**Fig. 3** Normalized magnitude of Poynting vector of the same cylinder as in Fig. 2 except the line source at  $(6\lambda, 0^\circ)$ .



**Fig. 4** Normalized magnitude of Poynting vector of the same cylinder as in Fig. 2 except the line source at  $(12\lambda, 0^\circ)$ .

We further investigate scattering properties of subwavelength cylinders due to the line source radiation. The radii in Fig. 5 and Fig. 6 are both equal  $0.05\lambda$ , while the distance of line source from the surface is  $0.2\lambda$  and  $1\lambda$ , respectively. Note that the energy distribution around the line source is suppressed and only the distribution near the cylinder is plotted. Two foculas with giant energy distribution are found in Fig. 5 when the distance of line source from the surface is four times of the radius. Of particular interest is that the focula is not located inside the subwavelength cylinder any more. Instead, the foculas are formed in two particular areas around the cylindrical surface. If the distance of line source increase to 20 times of radius (see Fig. 6), the magnitude of the two foculas decrease and the positions are more apart from each other.

#### 3.2 One-Layer Bi-isotropic Cylinder

Now the bi-isotropic cylinder with magnetoelectric couplings, as described by Eqs. (25) and (26), is studied. The degree of magnetoelectric couplings is represented by the chirality of  $\kappa$ . Different chiralities are chosen so as to study the effect of magnetoelectric coupling upon the scattering properties of the cylinder.

As a special case of bi-isotropic media, chiral nihility is considered first since it yields two special equivalent mediums: one is vacuum and the other is anti-vacuum, which can be verified in Eqs. (29) and (30). The judicious selection



**Fig. 5** Normalized magnitude of Poynting vector of a cylinder of  $a = 0.05\lambda$  filled with anti-vacuum and the line source at  $(0.25\lambda, 0^{\circ})$ .



**Fig. 6** Normalized magnitude of Poynting vector of a cylinder of  $a = 0.05\lambda$  filled with anti-vacuum and the line source at  $(1.05\lambda, 0^{\circ})$ .

of parameters for chiral nihility is based on the findings in [17]. It is shown in Fig. 7 that there are several foculas inside the cylinder and both energy distributions inside and outside the cylinder are greatly modified by the existence of magnetoelectric couplings. Compared with Fig. 3, the energy in Fig. 7 behind the cylinder is enhanced and those enhanced distributions forms some ribbon-shaped areas.

In what follows, we consider normal bi-isotropic mediums with the same  $\epsilon_r$  and  $\mu_r$ , but different  $\kappa$ . The reason why  $\epsilon_r = \mu_r$  is that the wave impedance of bi-isotropic medium is independent of  $\kappa$  and identical with the impedance of free space. From the comparison between Fig. 8 and Fig. 9, it can be seen that the focusing appears more obvious for bigger  $\kappa$  value. More interestingly, in Fig. 9, it not only presents two focusing points, but also enhance the energy inside the cylinder and high energy distribution is confined along the diameter.

#### 3.3 Cloaking for Subwavelength Cylinders

Here the cloaking for a thin cylinder and its selection of parameters are studied. We consider two combinations of the core and cloaking layers: double positive against double negative pair (DPS-DNG) and epsilon negative against mu negative pair (ENG-MNG). Such combinations may give rise to the interface resonances, provided proper choice of radii ratios. In analogy with what have been noticed for thin planar resonators [18], [19], the condition of having a no-



**Fig.7** Normalized magnitude of Poynting vector of a bi-isotropic cylinder of  $a = 2.5\lambda$  filled with chiral nihility medium of  $\epsilon_r = 1e-5$ ,  $\mu_r = 1e-5$ , and  $\kappa = 1$  and the line source at  $(4.8\lambda, 0^\circ)$ .



**Fig.8** Normalized magnitude of Poynting vector of a cylinder of  $a = 2.5\lambda$  filled with bi-isotropic medium of  $\epsilon_r = 1$ ,  $\mu_r = 1$ , and  $\kappa = 2$  and the line source at  $(4.8\lambda, 0^\circ)$ .



**Fig. 9** Normalized magnitude of Poynting vector of a cylinder of  $a = 2.5\lambda$  filled with bi-isotropic medium of  $\epsilon_r = 1$ ,  $\mu_r = 1$ , and  $\kappa = 4$  and the line source at  $(4.8\lambda, 0^\circ)$ .

cut-off propagation mode in a thin cloaking cylinder filled with a pair of DPS-DNG or ENG-MNG layers, depends on the ratio of the radii of the core cylinder (i.e.,  $\rho_2$ ) and the cloaking layer (i.e.,  $\rho_1$ ) instead of the sum of radii or the outer radius. As shown in Fig. 10, a cloaking cylinder consists of 2 layers (i.e., 3 regions). The radius of the cloaking is  $\rho_1$  and the radius of the core-cylinder is  $\rho_2$ . The line source is placed at ( $\rho_0 = 0.8\lambda$ ,  $\phi_0 = 0^\circ$ ). The outer radius of the cloaking layer is fixed to be  $\rho_1 = 0.01\lambda$  and scattering cross section (SCS) is plotted against the radii ratio in order to find resonances. SCS is defined as the ratio of the power



**Fig. 10** Scattering cross section versus ratio of core layer over cloaking layer in two pairs of combinations: DNG-DPS and DPS-DPS. The outer space is free space.



**Fig. 11** Scattering cross section versus ratio of core layer over cloaking layer in two pairs of combinations: DNG-DPS and DPS-DPS. In the case of DNG-DPS pairing, the cloaking layer is filled with DNG medium of  $(-3\epsilon_0, -2\mu_0)$ , and in the case of DPS-DPS pairing, the cloaking layer is filled with DPS medium of  $(3\epsilon_0, 2\mu_0)$ . The core layer remains the same DPS medium of  $(2\epsilon_0, \mu_0)$  for both pairings.

scattered by the scatterer to the incident power per unit area

$$SCS = \frac{\frac{1}{2}Re[E^{sca} \times H^{sca*}]}{\frac{1}{2}Re[E^{inc} \times H^{inc*}]}$$
(31)

In Fig. 11, the dash line refers to the case that both the core and cloaking layers are DPS mediums. Hence, we can see that the SCS is very small since the geometry of this scatter is in subwavelength size. Note that the SCS of dash line is very close to zero, but not exactly zero because we plot DNG-DPS in the same figure as well. Interestingly, a resonant ratio can be observed for DNG-DPS pairing at  $\rho_2/\rho_1 \approx 0.331$ , where the SCS is significantly enhanced. It is attributed to the polaritons which are supported by this pairing. It also shows that a proper cloaking on a subwavelength conventional cylinder can yield a comparably large scattering width similar to that obtained from very thick cylinder. Therefore, the scattering properties of a geometrically small cylindrical scatterer can be amplified up to a big scatterer, if the cloaking material and the ratio of radii are properly chosen.

Analogous scattering properties are obtained for ENG-MNG cloakings. In the solid line in Fig. 12, core layer is 2428



**Fig. 12** Scattering cross section versus ratio of core layer over cloaking layer in two pairs of combinations: ENG-MNG and DPS-DPS. In the case of ENG-MNG pairing, the cloaking layer is filled with ENG medium of  $(-3\epsilon_0, \mu_0)$ , while core layer is occupied by MNG medium of  $(4\epsilon_0, -2\mu_0)$ . In the case of DPS-DPS pairing, the cloaking and core layers are filled with DPS medium of  $(3\epsilon_0, \mu_0)$  and  $(4\epsilon_0, 2\mu_0)$ , respectively.

a mu-negative cylinder while the cloaking is an epsilonnegative coating. Resonant ratio can be found at  $\rho_2/\rho_1 \approx$ 0.152. In contrast to DNG-DPS pairing, the resonant scattering in Fig. 12 is not as sensitive to ratio as in Fig. 11. By coating a MNG subwavelength cylinder by an ENG cloaking, the scattering is still much amplified compared to DPS-DPS combination even if the radii ratio is smaller than the resonant ratio at about 0.152. However, when the inner radius  $\rho_2$  of MNG core layer keeps increasing, the scattering cross section will reduce to that of DPS-DPS combination as the dash line in Fig. 12.

# 4. Conclusion

In this paper, a multilayer algorithm is proposed to study the scattering properties of multilayered metamaterial cylinders by the radiation of a line source. The algorithm can treat arbitrary layers with arbitrary filling dielectric materials. Both single isotropic/bi-isotropic subwavelength cylinder and subwavelength cloaking cylinders are investigated. Focusing phenomena and enhanced resonant scattering are presented.

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