Fano resonance of three-dimensional spiral photonic crystals: Paradoxical transmission and polarization gap

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(Received 18 January 2011; accepted 5 February 2011; published online 25 February 2011)

Extraordinary Fano resonance with right handedness is present in a right-handed polarization gap of compound spiral photonic crystals. The structure is composed of unit cells with double helices of π phase shift arranged in square lattice. Such a paradoxical phenomenon is derived from mode interferences between left-handed propagating modes along spiral axis and negative group velocity modes in extended Brillouin zone. By analyzing Fourier spectra and chirality of photonic eigenmodes, intrinsic mechanism is well understood by mode coupling theory. © *2011 American Institute of Physics*. [doi:10.1063/1.3560338]

Chiral media[1–11] have been paid tremendous attentions due to the characteristics such as polarization gap (PG) (Ref. 5) and negative refraction,[7,8] as well as the potential applications such as circular polarizer,[5,9] poor-man's optical isolator,[3] polarized thermal radiation,[2,10] and perfect lenses.[11]

Spiral photonic crystal is a type of three-dimensional (3D) chiral structures. The resonances in spiral structures have been interpreted following the lines of cholesteric liquid crystals: a resonance occurs if the spiral pitch equals the effective material wavelength. So the resonances in spiral structure stem from individual spirals. Previously, there are many theoretical[1,4,5] and experimental[3,8,9] works on spiral photonic crystal constructed by periodic spirals with the same alignment (zero phase shift in spiral direction) and the same handedness. However, compound spiral photonic crystal (CSPC), which is composed of spirals with nonzero phase shift in between and spirals with opposite handedness in between, is lack of study, although some types of CSPCs have been fabricated very recently.[6] In fact, the alignment and handedness of the spirals should play an important role in optical characteristics of CSPCs, and will lead to other interesting characteristics and potential applications.

In this Letter, a type of CSPC with double right-handed (RH) spirals is investigated. We find that there are extraordinary RH transmission peaks with Fano line-shape in the RH PG. It seems to violate our common sense that the electromagnetic (EM) wave is impeded in the bandgap of defect-free periodic structure. We will give a clear physical picture to understand the paradox.

Consider a CSPC composed of two RH spirals with π phase shift in between arranged in checkerboard manner. It can be generated by holographic method [Fig. 1(a)] and may be achievable by holographic fabrication.[12,13] The spirals with the refractive index of 3.6 are aligned in air background, which can be achieved by silicon double inversion technique.[14] The square base and spirals' pitch are $(a/\sqrt{2})$ and $(a/\sqrt{6})$, and three basis vectors are $\mathbf{a}_1=[(a/2\sqrt{2}),(a/2\sqrt{2}),(a/2\sqrt{6})]$, $\mathbf{a}_2=[0,(a/\sqrt{2}),0]$ and $\mathbf{a}_3=[-(a/2\sqrt{2}),(a/2\sqrt{2}),(a/2\sqrt{6})]$. Here a is lattice constant. Figure 1(b) shows the Brillouin zone (BZ). Note that T point $([0,0,(2\sqrt{6}\pi/a)])$ is outside the first BZ. Figure 1(c) is photonic band structure along the spiral axis, calculated by plane wave expansion method.[15] We find that a RH PG (light blue or gray region) appears in the RH spiral structure. Moreover, both left-handed (LH) and RH bands have negative group velocity modes in the extended zone [shading area in Fig. 1(c)], which will be discussed below.

We use scattering matrix method[16] to calculate the transmission spectrum of a finite CSPC with the thickness of ten unit cells, an RH-polarized beam is normally incident. The number of plane waves is 300 to guarantee the convergence. Figure 2(a) shows that the RH transmittance is lower than 10% from 0.9518 (c/a) to 1.0174 (c/a), showing the characteristic of the RH PG predicted by the band structure. How-

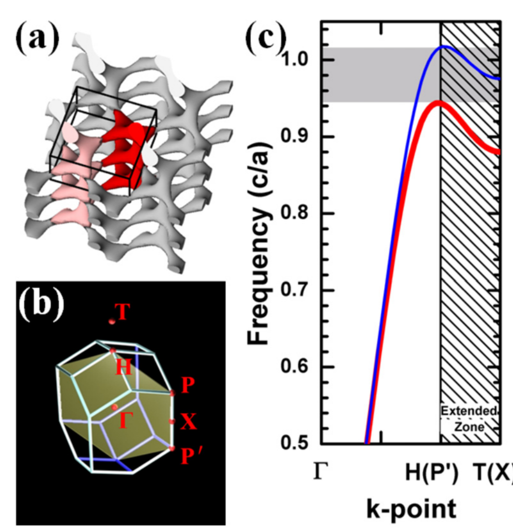

FIG. 1. (Color online) (a) Holographic CSPC composed of double helixes of π phase shift arranged in checkerboard manner. (b) is 3D BZ and (c) is photonic band structure. Thin (thick) solid curve is LH (RH) polarized band. Light gray box highlights the RH polarization gap.

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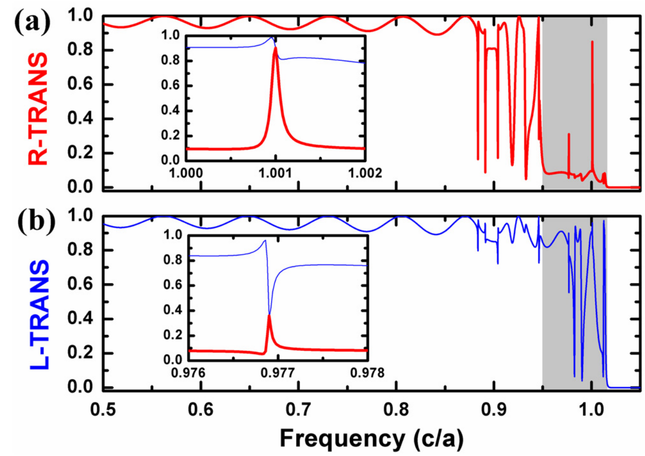

FIG. 2. (Color online) (a) RH and (b) LH-polarized transmission spectra. Insets are the zoom-in graphs near the Fano resonances in the RH PG. Light gray box highlights the RH PG.

ever, it is unexpected that there are two sharp transmitted peaks at 1.0010 (c/a) and 0.9769 (c/a) inside the RH PG. We highlight the spectra near these two frequencies in the insets of Fig. 2. Their asymmetric line shapes are similar to Fano resonance.[17–19] The asymmetric resonances are also found in the LH-polarized spectrum [Fig. 2(b)] when an LH-polarized beam is normally incident upon the structure. As the thickness of CSPC increases, the frequencies of Fano peaks will shift, and the number of Fano peaks will increase. These Fano peaks can be observed as long as the negative group velocity modes appear in the extended BZ.

Fano resonance in the CSPC stems from the coupling between multiple modes. In order to understand the mechanism, we investigate Fourier components and chirality of eigenmodes within the frequency region of Fano resonances. We decompose Fourier components of four eigenmodes at the frequency of 0.98 (c/a) [dashed line in Fig. 3(a)]. Here, four modes are labeled as mode N2, N1, P1, and P2, where P (N) denotes k-points on positive-k (negative-k) axis and number 1 (2) stands for the k-point in the first (second) BZ. Insets of Fig. 3(a) show their Fourier components. The dominant Fourier components (highlighted by colors) of the modes N2 and N1 point to the negative-z direction but their group velocities point to opposite directions. It indicates that the mode N2 (N1) is a mode with negative (positive) group velocity.

Next we study the chirality of the modes. The chiral quality (CQ), defined as $\mathrm{CQ}=\langle E_x\rangle / i\langle E_y\rangle$ where $\langle\cdots\rangle$ is the spatial average over the cubic unit cell, is a good indicator for the chirality. It characterizes the electric field's evolution over time. When $\mathrm{CQ}=1+0i$, the electric field vector rotates clockwise as time goes; while $\mathrm{CQ}=-1+0i$, it rotates anticlockwise. Otherwise, the chirality is weakened and the mode is no longer circular polarization. Figure 3(b) shows the value of CQ in the LH-polarized band. In the first BZ without negative group velocity modes, $\mathrm{CQ}\approx 1+0i$ ($\mathrm{CQ}\approx -1+0i$) for negative (positive) semi-k axis. It indicates that the modes are circular polarization. However, the imaginary part of CQ deviates from zero near the zone boundary, and the value of CQ deviates from $1+0i$ (or $-1+0i$) in the extended zone, showing that the chirality is weakened. The reason is that the off-axis k-components of the modes [see,

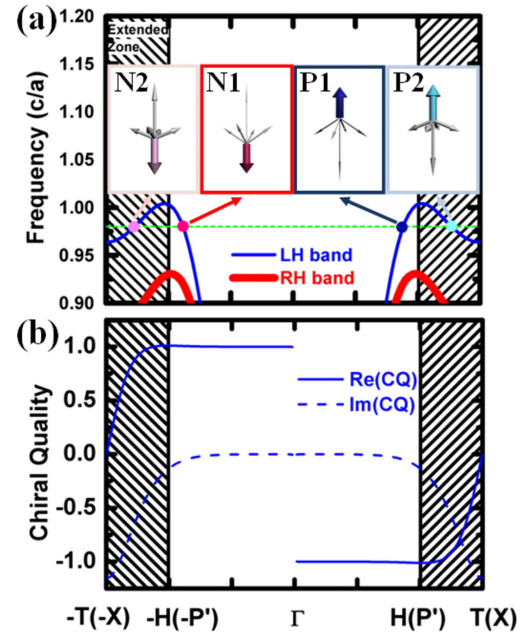

FIG. 3. (Color online) (a) Photonic band structure same as Fig. 1(c). Insets are Fourier transformations of modes at the frequency of 0.98 (c/a). Arrow (radius) denotes k-component's vector (amplitude). The dominant k-component is highlighted by chromatic color. (b) CQ of the LH-polarized band.

e.g., tilted arrows for the mode N2 in Fig. 3(a)] have larger magnitude than those in the first BZ without negative group velocity modes.

We now come to understand the Fano resonance in the CSPC by analyzing the coupling efficiencies between different modes. The sketch map is shown in Fig. 4. When the EM wave is incident upon the CSPC, the mode N2 and P1 are excited due to their positive energy flow propagating directions. The EM wave will propagate forth and back inside the CSPC due to the interface reflections, and it will form different kinds of resonances. There are mainly two pathways. One is an indirect pathway with high quality formed by the C_{N2P2} coupling between the mode N2 and P2 (thin horizontal channel in Fig. 4). As we know, two modes are well coupled when their Fourier components and CQ are consistent with each other. Figure 3(a) shows each k-component in the mode N2 has its opposite in the mode P2, indicating their Fourier components match well. Furthermore, their chiralities are not so good, and there should be amount of wave components with same handedness in both of them. Therefore, when the EM waves reflect at the interface of the CSPC and outside

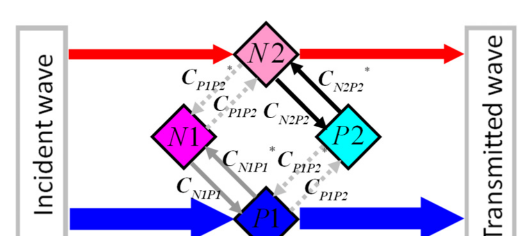

FIG. 4. (Color online) Sketch map of mode couplings. Thin and thick horizontal arrows show two transmission channels. Black (gray) solid arrows show the C_{N2P2} (C_{N1P1}) coupling with high (low) quality. Dashed arrows indicate the weak and negligible couplings.

medium, the mode N2 can easily couple to the mode P2 with large coupling efficiency.

Another channel is a direct pathway (thick horizontal channel in Fig. 4) with low quality formed by the C_{N1P1} coupling. Figure 3(b) shows the magnitude of CQ in the mode N1 and P1 are very close to unity but their signs are opposite. Meanwhile, the mode N1 and P1 are in the first BZ, and easily leaky outside than others. As a result, the C_{N1P1} coupling should be smaller than the C_{N2P2} coupling. However, the C_{N1P1} coupling will not vanish because Im(CQ) deviates from zero a little near the zone boundary. As a whole, these two pathways will interfere with each other, and finally Fano resonance occurs.

We note that the exciting efficiency of the direct pathway by the external wave is stronger than that of the indirect pathway. So, the transmittance through the direct pathway is dominant and determines the background of spectrum except for the frequencies when Fano resonance occurs. There are also other mode couplings but they are much weaker and may be negligible. For example, both the C_{P1P2} and C_{N1N2} couplings are small due to their k-component mismatching. The C_{P1N2} and C_{P2N1} couplings are neglectable because of their same energy flow directions with different propagating constants.

In conclusion, we studied a CSPC arranged in a checkerboard manner, consisting of two sets of RH spirals with π phase shift. The transmission calculations show that there are extraordinary RH Fano resonances in the RH PG in such a chiral structure. The Fano resonances can be well understood by the interference between the LH propagating modes along spiral axis and the hybrid-handed modes partially perpendicular to spiral axis. These Fano peaks in RH PG should have potential applications with circular polarized signatures.[20,21]

We would like to thank Professor C. T. Chan for useful discussions. This work is supported by National Natural Science Foundation of China (Grant Nos. 10804131, 11074311, and 10874250), Fundamental Research Funds for the Central Universities (Grant No. 2009300003161450), and Guangdong Natural Science Foundation (Grant No. 10451027501005073). This work is also supported by HK RGC grant (Grant No. 600308).

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