Integrated Servo-Mechanical Design of a Fine Stage for a Coarse/Fine Dual-Stage Positioning System

Haiyue Zhu, Student Member, IEEE, Chee Khiang Pang, Senior Member, IEEE, and Tat Joo Teo, Member, IEEE

Abstract—Traditionally, mechanical design of coarse/fine stages in dual-stage positioning system is conducted separately without consideration about their control performance in dual-stage integration. In this paper, a flexure-based Lorentz motor fine stage is designed simultaneously with a simple PID controller to perform dual-stage positioning, based on the existing coarse stage. The design of fine stage is carried out using a proposed integrated servo-mechanical design approach, where various specifications are considered and formulated as constraints in an optimization problem. Through the approach, both the plant modal parameters and controller parameters of fine stage are simultaneously solved. It demonstrates that through suitable mechanical plant design, the fine stage can fulfill various control specifications by only using a simple PID controller, e.g. to compensate sensitivity peak, maintain stability and etc. Meanwhile, the proposed design approach also ensures certain open loop positioning performance and the decoupling property of coarse/fine stages. The prototype of designed fine stage is fabricated, and experimental investigation indicates that the sensitivity peak is effectively reduced from 14.5 dB of coarse stage to 7.1 dB in dual-stage system, and the fine stage is able to achieve submicron accuracy. The maximal tracking error is also reduced significantly from about 20 μm in coarse stage to less than 2 μm through dual-stage positioning.

Index Terms—Coarse/fine stages, dual-stage positioning, flexure mechanism, integrated servo-mechanical design, mechatronics design.

I. INTRODUCTION

POSITIONING on micro/nanometer scale precision is a key technology for a variety of scientific research and engineering. To achieve micro/nano-positioning over large traveling stroke, dual-stage is widely adopted in many positioning systems [1], [2]. In dual-stage system, coarse stages deliver large stroke coarse motion, yet due to the nonlinear effects such as friction and backlash [3], the achievable motion quality of coarse stage is normally restricted. Consequently, fine stage is utilized additionally to achieve high accuracy fine positioning.

Although lead zirconium titanate (PZT) [4]–[6] actuator is widely adopted as fine stage to achieve high performance motion, one inherent limitation of PZT actuator is its very limited motion stroke, that make it unsuitable for many applications. Besides, PZT actuation is also reported that has significant non-linear effects, e.g. creep and hysteresis [4]. Instead of PZT actuator, electromagnetic actuation is of much better linearity [3] and its stroke is not restricted by actuation but by the guiding mechanism. Fortunately, large displacement flexure mechanism [7]–[9] are well studied and modeled to provide frictionless guiding in recent years.

Traditionally, dual-stage systems are typically implemented that the fine stage is fully mounted above the coarse translator [10], [11]. As a result, for electromagnetic fine stages containing actuation coils, this design increases the weight and size of coarse translator. Furthermore, the unavoidable energy chain for fine stage degrades the dynamic performance. In this work, a dual-stage positioning system is implemented that only the fine translator is connected with coarse translator using flexure mechanism. Through this arrangement, the dual-stage system becomes more compact, and the coarse stage dynamics is also improved.

Recently, integrated design gains increasing attention that considering control aspects in early mechanical design stage [12]. Simultaneous design and optimization of both the plant and controller is adopt in [13]–[15] to optimize the performance index in structural designs, e.g. $H_2/H_\infty$ performance. Integrated design methods using iterative approach are proposed in [16], [17], where the plant parameters are searched in every iteration, followed by synthesizing the optimal controller. Lately, integrated design are proposed in [18] to redesign the PZT actuator considering finite frequency properties. However, in most existing works of dual-stage system, the mechanical design of coarse/fine stages are conducted separately without considering their control performance in dual-stage integration. Although controller design can be focused to improve the closed-loop performance, non-optimal or even unsuitable mechanical plant design increases the complexity of controller and also leads to sub-optimal performance.

In this paper, the mechanical plant of fine stage is designed simultaneously with a simple PID controller for a dual-stage positioning system, that the fine stage can fulfill various desired mechanical and control specifications. The contributions of this paper are summarized as twofold. First, a novel integrated servo-mechanical design approach that systematically designing the fine stage for dual-stage system is proposed. Second, a flexure-based Lorentz motor fine stage is designed and implemented on an existing coarse stage, by applying the proposed design approach.

The rest of paper is organized as follow. Section II presents the proposed integrated design approach that designing fine stage for dual-stage system. The proposed design approach is implemented in Section III to design a flexure-based Lorentz...
motor fine stage based on existing coarse stage. Section IV introduces the experimental investigation results, and a conclusion is given in Section V.

II. FINE-STAGE DESIGN APPROACH FOR DUAL-STAGE SYSTEMS

In this section, an integrated design approach is proposed to design the fine stage for dual-stage system.

A. Dual Feedback Configuration

The dual-stage systems considered in paper are of dual feedback configuration, as shown in Fig. 1(a), where $G_c$ and $G_f$ denote the coarse and fine plants, respectively, $C_c$ and $C_f$ represent the coarse and fine controllers, respectively. In dual feedback configuration, consider output disturbance $w_c$ and $w_f$ are injected into the system, then

$$y = \frac{C_cG_c + C_cG_f + C_cG_cG_f}{(1 + C_cG_c)(1 + C_cG_f)}r + \frac{1}{1 + C_cG_c}w_c$$

$$+ \frac{1}{1 + C_cG_f}w_f.$$  

For dual-stage system, $w_c$ is typically much significant than $w_f$, since $w_c$ always contains mechanical friction, commutation error and etc. As a result, the sensitivity transfer function for dual-stage system is focused on $w_c$ that

$$S_d = \frac{1}{(1 + C_cG_c)(1 + C_cG_f)} = S_cS_f,$$  

where $S_c$ and $S_f$ are the sensitivity transfer functions of decoupled coarse and fine stages, shown as Fig. 1(b) and (c), respectively, and $S_c = 1/(1 + C_cG_c)$, $S_f = 1/(1 + C_cG_f)$.

In this work, the characteristics and performance of the coarse stage in Fig. 1(b) is assumed to be already known, and the objective of the proposed integrated design approach is to optimally determine the design parameters in both $G_f$ and $C_f$ for fine stage that fulfil various control and mechanical design specifications.

Fig. 2 introduces detail procedures of the proposed fine-stage design approach. To begin with, the design of fine stage is based on the characteristics and performance of existing coarse stage, from where some control and mechanical specifications of fine stage are analyzed. Next, by formulating the analyzed specifications as the constraints in an optimization problem, both the nominal plant model and controller model are solved accordingly. The obtained nominal plant model is realized by a mechanical design that possess the same characteristics. Finally, the prototype of mechanical plant and control system are integrated to evaluate its performance.

B. Integrated Servo-Mechanical Design

The integrated design approach is carried out on the decoupled loop of fine stage, as shown in Fig. 1(c). Without loss of generality, the plant model of fine stage is of assumption

$$G_f(s) = \frac{g_f}{m_fs^2 + d_fs + k_f} = \frac{n_p}{d_p(\lambda_p)},$$  

where $m_f$, $d_f$, $k_f$, and $g_f$ represent the mass, damping, stiffness, and gain of fine plant, respectively, $n_p$ and $d_p$ denote the plant numerator and denominator polynomials, and $\lambda_p$ denotes the unknown design parameters in plant. A standard PID controller is utilized for the fine stage with the structure

$$C_f(s) = K_p + K_i \frac{s}{1 + T_d s} + \frac{K_ds}{1 + T_d s} = \frac{n_k(\lambda_k)}{d_k},$$  

where $K_p$, $K_i$, $K_d$, and $T_d$ are the PID controller parameters, and $T_d$ is a fixed known constant, $n_k$ and $d_k$ are the controller numerator and denominator polynomials, and $\lambda_k$ denotes the controller design parameters vector $[K_p, K_i, K_d]^T$. Under this formulation, the sensitivity transfer function and complementary sensitivity transfer function of the fine stage closed-loop
system in Fig. 1(c) can be expressed by

\[
S_f = \frac{d_p(\lambda_p)d_k}{n_p n_k(\lambda_p) + d_p(\lambda_p)d_k}, \\
T_f = \frac{n_p n_k(\lambda_k) + d_p(\lambda_p)d_k}{n_p n_k(\lambda_k)}
\]

through the design parameter vectors \(\lambda_p\) and \(\lambda_k\).

The fine-stage design approach is conducted on the objective to achieve desired performance specifications, by characterizing the constraints on the sensitivity transfer function \(S_f\) and complementary sensitivity transfer function \(T_f\) through convex optimization. The objective of optimization is to minimize the performance index subject to a list of design specifications, which are represented via linear matrix inequality (LMI) constraints. The details about choosing the design specifications for fine stage will be discussed in the following.

C. Control Design Specifications

The control design specifications are detailed to optimally design the fine stage for dual-stage system.

1) Stability: For control systems, the stability is the most important issue to consider. The stability condition of the closed-loop system can be expressed by the following positive realness property [20]. Consider the closed-loop system with \(S_f\) and \(T_f\) in (5), the stability condition is guaranteed if and only if

\[
\frac{n_p n_k(\lambda_k) + d_p(\lambda_p)d_k}{d_c} \in \mathbb{S},
\]

where \(d_c\) is a stable central polynomial, and \(\mathbb{S}\) denotes the set of all proper positive-real transfer functions.

Numerically, it is noted that the positive realness property (6) can be verified and ensured by the Kalman-Yakubovic-Popov (KYP) Lemma [21] provided in Appendix. By specifying this LMI condition, the stability of the closed-loop system can be guaranteed through the solution of \(\lambda_k\) and \(\lambda_p\).

2) Disturbance Rejection: For standard output feedback in Fig. 1(c), the fine stage tracking error \(e\) is directly related to \(|S_f|\). Therefore, the design specification for rejecting the disturbance in finite frequency is expressed as

\[
|S_f| = \left| \frac{d_p(\lambda_p)d_k}{n_p n_k(\lambda_p) + d_p(\lambda_p)d_k} \right| < \rho_S, \quad \omega \in \Omega_S
\]

where \(\Omega_S\) denotes the target frequency range of disturbance rejection, and \(\rho_S\) serves as the objective of the LMI optimization. It is worth noting that if the frequency of \(w_c\) and \(w_T\) is unknown exactly that concentrate on narrow bandwidth, the fine stage can be designed with strong disturbance rejection ability for such disturbance.

The finite frequency bounded realness condition (7) is able to be handled using general Kalman-Yakubovich-Popov (GKYP) lemma [21] provided in Appendix. However, the unknown design parameters \(\lambda_k\) and \(\lambda_p\) appearing in the denominator of \(S_f\) destroy the linearity of (30). To overcome this limitation, a convex separable parametrization method introduced in [18] is utilized to convert (7) into two conditions,

\[
\left| d_p(\lambda_p)d_k \right| < (1 - \delta)\rho_S, \quad \omega \in \Omega_S
\]

and

\[
\left| \frac{n_p n_k(\lambda_k) + d_p(\lambda_p)d_k}{d_c} \right| < \delta, \quad \omega \in \Omega_S
\]

where \(\delta \in (0, 1)\). As a result, (8) and (9) can be treated directly using GKYP lemma.

In dual-stage positioning systems, to achieve strong low frequency disturbance rejection ability, the coarse stage open loop transfer function are typically designed with large magnitude in low frequency. According to Bode sensitivity integral theorem and waterbed effect [22], sensitivity transfer function often peaks significantly in coarse stage, and the control performance is degraded severely as the disturbance is amplified within sensitivity peak frequency range. As analysed in (2), the sensitivity transfer function of dual-stage \(S\) is the product of two individual sensitivity transfer functions \(S_c\) and \(S_f\). Consequently, if choosing \(\Omega_S\) as the frequency range where \(S_c\) peaks, the specification (7) can also be utilized to compensate the sensitivity peak appearing in \(S_c\) and shape the \(S\) with lower peak magnitude.

3) High Frequency Roll-off: To avoid the excitation of unmodeled high frequency resonant modes in fine stage, the \(T_f\) should be designed to roll-off in high frequency ranges. This high frequency roll-off condition is expressed by

\[
|T_f(j\omega)| = \left| \frac{n_p n_k(\lambda_k)}{d_p(\lambda_p)d_k} \right| < \rho_T, \quad \omega \in (\omega_h, +\infty)
\]

that (10) ensures \(|T_f|\) less than a specified performance index \(\rho_T\) in \(\omega \in (\omega_h, +\infty)\). Similarly, (10) can be handled by the same technique as introduced for (7).

D. Mechanical Design Specifications

The mechanical design specifications for fine stage are also detailed in this part.

1) Open Loop Positioning Performance: The open loop positioning performance considered here includes two aspects, that the open loop positioning resolution \(x_o\) and maximal achievable acceleration \(a_m\). For flexure-based positioning stages, open loop positioning resolution is fundamentally determined by

\[
x_o = \frac{g_i i_t}{k_t},
\]

where \(i_t\) denotes the resolution of current amplifier, and \(g_i\) is actually the force resolution. Obviously, larger \(k_t\) leads to better positioning resolution for certain \(g_i\). Similarly, the maximal achievable acceleration \(a_m\) of fine stage is given by

\[
a_m = \frac{g_i i_m}{m_i},
\]

where \(i_m\) denotes the maximal current the amplifier provides. As \(i_t\) and \(i_m\) are always fixed for certain amplifier, the gain \(g_i\) and the stiffness \(k_t\) should be designed suitably to ensure the certain specifications of open loop positioning resolution and acceleration. Accordingly, to achieve an open loop positioning resolution specification \(x_o\), a lower boundary is established that \(k_t > g_i i_t/ x_o\).
2) Decoupling Issue: In dual feedback configuration Fig. 1(a), coarse/fine stages are assumed to be decoupled with each other. However in reality, the relationship between two stages is shown like in Fig. 3, which is not ideally decoupled. Consequently, to maintain the decoupling assumption, the decoupling issue is an important design consideration. It is reported in [23], the fine stage with relatively smaller mass \( m_f \) will reduce the coupling effect in Fig. 3, which provides a guideline to determine the \( m_f \). Furthermore, the force acting on \( m_f \) results a coupling force \( F_c \) on the coarse stage. The maximal coupling force is predicted by

\[
F_c = k_f x_s, \tag{13}
\]

where \( x_s \) denotes the displacement stroke of fine stage. As a result, to maintain the decoupling assumption and reduce the coupling effect between coarse and fine stages, \( F_c \) is restricted to less than \( \bar{F}_c \). According to the stroke \( x_s \), an upper bound condition is established for the fine stage stiffness \( k_f \), namely, \( k_f < \bar{F}_c/x_s \). As a result, the feasible range for the stiffness of fine stage is given as

\[
g(t_i) \frac{\Delta t_i}{x_i} < k_f < \frac{\bar{F}_c}{x_s}. \tag{14}
\]

For the fine stage damping \( d_f \), to ensure the result solved from the optimization is feasible to be realized mechanically, the parameter \( d_f \) is restricted in a reasonable range

\[
d_f < d_f < \bar{d}_f, \tag{15}
\]

that ensure obtained \( d_f \) can be realized by selection of flexure material or using additional eddy current damper.

Summarized from the control and mechanical design specifications, the optimization problem to determine the design parameters of fine stage is formulated as

\[
\begin{align*}
\min & \quad (\lambda_f, \lambda_k) \quad \rho S \\
\text{subject to:} & \quad (6), (7), (10), (14), \text{and (15)}. \tag{16}
\end{align*}
\]

By solving the optimization problem, the design parameter vectors \( \lambda_f \) and \( \lambda_k \) in (3) and (4) can be determined accordingly. The obtained \( G_f \) and \( C_f \) are denoted as nominal plant model and nominal controller of fine stage, respectively. The nominal plant model \( G_f \) will be realized by customized mechanical design.

### III. Implementation of Fine Stage Design

The proposed integrated design approach is implemented to design a fine stage for a dual-stage positioning system. The work principle of proposed dual-stage system is illustrated in Fig. 4(a). In this work, only the fine translator is connected with the coarse translator using flexure mechanism, while the actuation parts of both the coarse and fine stage lay underneath their translators accordingly. As a result, the size and weight of coarse translator are minimized, which improve the dynamics performance and energy efficiency. In addition, since no energy chain is attached with the translator to provide energy for fine stage, many nonlinear factors are further eliminated.

#### A. Existing Coarse Stage and Its Limitations

In this dual-stage system, the coarse stage employs a moving magnet linear motor (MMLM) [19] to perform large stroke coarse motion. The MMLM is consisted by two parts, a translator and a stator, and the translator is formed by a Halbach permanent magnet array delivering magnetic field. The stator of MMLM lying under the translator contains three-phase coils. By controlling the current energized in the coils underneath the Halbach array, propulsion force is generated on the translator accordingly, which finally leads to the displacement.

Fig. 4(b) shows the implemented MMLM coarse stage, and system identification is conducted on MMLM coarse stage, where an additional weight equivalent to fine stage is attached to the MMLM. Fig. 5 plots the measured frequency response of coarse stage in frequency range of 1–1000 Hz. A second order transfer function is identified to model the measured frequency response,

\[
G_C(s) = \frac{1.08}{s^2 + 54s} \quad \text{N/m.} \tag{17}
\]

Since the derivative control amplifies the high frequency noise introduced from the commutation error of MMLM, a PI
controller is used for controlling the coarse stage to achieve a closed-loop bandwidth about 20 Hz, while provides sufficient disturbance attenuation ability in low frequency, given as

$$C_c(s) = 39000 \left(1 + \frac{1}{0.077s}\right). \quad (18)$$

The closed-loop sensitivity transfer function of the coarse stage MMLM is plotted in Fig. 6 against the simulation, which indicates that an obvious sensitivity peak appearing around 32 Hz. As analyzed, due to the low stiffness characteristics of MMLM and nonlinear effects introduced from the commutation error and mechanical friction, etc., the positioning performance of the coarse stage is limited.

**B. Design of Fine-Stage System**

To improve the positioning performance, a flexure-based Lorentz motor fine stage is utilized to perform fine positioning. The schematics of the flexure-based Lorentz motor is shown in Fig. 7(a), and the real fabricated motor is shown in Fig. 7(b). This Lorentz motor contains two parts, a stator assembled by coil arrays and a moving part made of permanent magnet. As friction caused by mechanical contact severely degrade the positioning performance, a customized compact flexure is designed to provide frictionless motion guiding for the moving part of Lorentz motor. The high stiffness property of flexure also improves the positioning resolution.

![Figure 7](image-url)  
**Fig. 7.** Schematics (a) and photo (b) of the flexure-based Lorentz motor.

As illustrated in Fig. 7(a), the mathematical model of the flexure-based Lorentz motor can be represented by the plant model (3) in Section II, where $m_f$ represents the total mass containing magnet, sensor target and their support holders, $d_f$ denotes the damping of the fine stage, $k_f$ represents the stiffness of the flexure support, and $g_f$ is the amplification gain from current to generated force. Here, the mass $m_f$ can be controlled easily by mechanical design, certain stiffness $k_f$ can be realized by flexure design, the damping parameter $d_f$ can be fulfilled by flexure material selection or using additional eddy current damper. The $g_f$ is actually determined by the electromagnetic configuration, e.g., the magnetization of magnet, coil turn number, and the air gap between the coils and magnets, as indicated in Fig. 7(a).

In this work, the mass $m_f$ is viewed as a known constant, and the remaining plant parameters ($d_f$, $k_f$, and $g_f$) and controller parameters ($K_p$, $K_i$, and $K_d$) are the design parameters which will be determined optimally through the integrated design approach. To avoid the bilinearity caused by the product between $n_p$ and $n_k$, $K_p$, $K_i$, and $K_d$ are used to replace $K_p g_f$, $K_i g_f$, and $K_d g_f$, respectively, that

$$d_p = m_f s^2 + d_1 s + k_f, \quad n_p = 1, \quad d_k = s(1 + T_d s),$$

and

$$n_k = (K_d + K_p T_d) s^2 + (K_p + K_i T_d) s + K_i,$$  \quad (19)

since the value of $n_p$ and $n_k$ can be reallocated easily.

According to the design considerations discussed in Section II, a small $m_f$ is preferred as it helps to reduce decoupling and improve the open loop performance. As a result, the mass $m_f$ is chosen as 0.05 kg including both the mass of magnet and laser mirror, that is only about 1/10 of the coarse translator mass. $T_d$ is set as 1/990 in this work. The central polynomial $d_k$ is selected as

$$d_k = (s^2 + 195 s + 76050)(s^2 + 390 s + 114075),$$  \quad (20)

which is actually the desired closed-loop characteristics polynomial.

To formulate the optimization problem (16), the first specification is chosen to compensate the sensitivity peak of coarse stage, where the frequency range $\Omega_S$ in (7) is selected as $(2\pi \times 31, 2\pi \times 33)$ rad/s, according to the coarse stage sensitivity transfer function plotted in Fig. 6. Two following specifications are chosen to maintain the stability (6) and ensure high frequency roll-off conditions (10), where $\rho_S$ is set as -3 dB, and $\omega_h$ is chosen as 150 Hz. To ensure $x_i < 200$ nm and...
restrict $F_f < 1.5$ N, $k_f$ follows

$$1000 \text{N/m} < k_f < 5000 \text{N/m}. \quad (21)$$

Finally, for feasible $d_f$ in (15), $d_f'$ and $\bar{d_f}$ are chosen as 0.8 Ns/m and 2 Ns/m, respectively.

Through solving the optimization problem (16), the design parameters $K'_p, K'_s, K'_t, d_1, d_2$, and $k_i$ are obtained accordingly. By reallocating $\bar{d_f} = 0.65$, the resulted plant and controller from the design algorithm are given as,

$$G_f(s) = \frac{0.65}{0.05 s^2 + 0.8 s + 2054}, \quad (22)$$

and

$$C_f(s) = 2.14 \times 10^3 + \frac{8.01 \times 10^5}{s} + \frac{18.23 s}{1 + s/990}. \quad (23)$$

C. Mechanical Synthesis of Fine Stage

In this part, a mechanical design of flexure-based Lorentz motor is proposed to realize the nominal fine-stage plant model $G_f$. A compact flexure hinge is designed to guide the fine translator and provide required stiffness $k_f$. Figs. 8(a) and (b) show the flexure hinge in 2D and 3D views, respectively. The flexure contains two Secondary Platforms and one Primary Platform, which are linked and supported by eight beam-based flexures from the Base. The moving part of Lorentz motor is designed to be assembled in the Primary Platform through two threaded holes, as indicated in Fig. 8(b), while the Base of flexure will be mounted on the coarse stage.

To model the stiffness of proposed flexure hinge, a semi-analytic model [7] is used to analyze the flexure in this work. According to the semi-analytic model, the deflection of flexure hinge is given as

$$\delta = \left(1 + \frac{L}{2s}\right) \sin \alpha, \quad (24)$$

where $L$ denotes the length of flexure, as indicated in Fig. 8(a), and $l$ denotes the length of rigid part. In this work, $l = 0$ since no rigid part is in the beam. $\alpha$ represents the deflection angle and $\delta$ is a Sinc function, i.e., $\delta = \sin \alpha / \alpha$. The loading force $F_z$ is expressed as

$$F_z = \frac{E l \alpha}{l + \rho L / (2 \alpha) \cos \alpha}, \quad (25)$$

where $E$ and $l$ represents the Young’s Modulus and second moment of area of the flexure joint respectively, and $\rho$ is a constant equal to $L \sqrt{1.8 + l / (l + l)}$.

The schematics of flexure design in Fig. 8(a) and (b) are simplified and represented in Fig. 8(c) using the Pseudo-Rigid-Body (PRB) model [7]. For each beam-based flexure, two torsional springs with a rigid body are used to model the stiffness as shown in Fig. 8(d). For this one beam in Fig. 8(d), the deflection stiffness in $z$-direction is equivalent to two torsional springs with free ends connected in series. As a result, the deflection stiffness of one beam $K_b$ is calculated as $K_b = K_s / 2$, where $K_s$ denotes the stiffness of torsional spring in PRB model, that can be predicted effectively as

$$K_s = \frac{F_z}{\Delta s / 2}, \quad (26)$$

where $F_z$ is calculated according to (25).

The stiffness $K_{b2s}$ between the Base and one side Secondary Platform can be calculated as $K_{b2s} = 2K_b$, since two beams are located in parallel. As all beams are of same dimension, similarly, the stiffness $K_{2p}$ between the Secondary Platform and Primary Platform is within same calculation as $K_{2p} = 2K_b$. Consider the series relationship between $K_{b2s}$ and $K_{2p}$ and the symmetry of this flexure design in Fig. 8(c), the total deflection stiffness of flexure mechanism $K_f$ is predicted as

$$K_f = 2K_b. \quad (27)$$

From the simulation, it suggests that the less damped resonant mode corresponds to the deeper notch in sensitivity transfer function. As a result, standard aluminum is selected as the flexure material and no additional damper is used, which also reduces the system complexity. According to the stiffness modeling, the physical parameters of the flexure beams are selected as length $L = 21$ mm, width of 0.3 mm and height of 5 mm, which results the stiffness of 2063 N/m theoretically, that is close to $k_f = 2054$ N/m.

The designed flexure is verified by commercial Finite Element Analysis (FEA) software ANSYS. The stiffness is calculated as $K_{FEA} = 2099$ N/m, which is of 1.7% error compared with the analytical prediction. FEA shows the first resonant mode occurs at 30.7 Hz. Furthermore, FEA also indicates the second and higher resonant modes occur at frequency above 340 Hz, which is 10 times higher than the first resonant mode.

IV. Experimental Results and Discussions

A prototype of flexure-based Lorentz motor fine stage is fabricated, and experimental investigation is presented in this section. Fig. 9 shows the experiment setup of implemented dual-stage positioning system. For the fine stage, a Renishaw RLE fibre optic laser interferometer is employed to measure the displacement of fine stage with a resolution of 39.6 nm.
and a target mirror is mounted above the fine stage. The controller employs a National Instruments PXI platform to achieve a sampling rate of 10 kHz, and a Trust Automation TA115 current amplifier is utilized to actuate the fine stage. System identification is performed on the fine stage, and the frequency response is measured by frequency sweeping up to 1000 Hz, as plotted in Fig. 10. The measured frequency response is identified, denoted as identified model, with the following structure

\[
\frac{G_f}{\omega_n} = \frac{2 - Ts}{2 + Ts} \sum_{n=1}^{3} \frac{k_n}{s^2 + 2\zeta_n\omega_n s + \omega_n^2} \text{A/m},
\]

(28)

where the parameters in each mode are listed in Table I, and \( T = 2.25 \times 10^{-8} \) s is introduced to model the phase lag caused by time delay. Fig. 10 indicates that the low frequency characteristics of the fabricated fine stage consists well with the nominal plant model \( G_f \), that the first resonant mode appears at 34.02 Hz. It is also noted two out-phase resonant modes appear in high frequency range around 400 Hz, which is similar as the FEA predicted.

Combined with the controller \( C_f \) (23) from the design approach, the open loop transfer functions of both the nominal model (22) and real identified model (28) are plotted in Fig. 11. It is noted that with the designed controller (23), although the two high frequency resonant modes peak above 0 dB in the open loop transfer function, their phase properties satisfy the condition of phase-stabilization [24], that the phase of open loop transfer function should be kept within \(-360^\circ \pm 90^\circ\) at each resonant mode frequency. As a result, the closed-loop system is stable that the two high frequency resonant modes are phase-stabilized. The Nyquist plot of open loop transfer function with identified model (28) is shown in Fig. 12, and it indicates the system possesses primary phase margin of 38.6° and secondary phase margin of 44.5°, which follows the suggestion given in [24], that primary phase margin should be kept at 30° or more, and the second phase margin should be kept at 40° or more to maintain the robust stability.

The sensitivity and complementary sensitivity transfer functions of closed-loop fine stage are measured experimentally by frequency sweeping up to 1000 Hz, and the results are plotted in Fig. 13 against the simulation with both the nominal model and identified model. The sensitivity transfer function notch appears in 33.6 Hz, which is of 1.23% variation against the identified model, and 4.12% variation against the nominal model. Furthermore, the bandwidth of fine stage is about 40 Hz, which is largely improved comparing with the coarse stage.

To analyze the dual-stage performance, the sensitivity transfer function of dual-stage system is predicted using the obtained sensitivity transfer functions of coarse and fine stages, as plotted in Fig. 14. It shows that the sensitivity peak occurs at 32.2 Hz in coarse stage, while the notch appears in 33.6 Hz in fine stage, which is of 4.34% variation. The sensitivity

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{n} & \textbf{k_n} & \textbf{\zeta_n} & \textbf{\omega_n (rad/s)} \\
\hline
1 & \(2.82 \times 10^{-4}\) & \(4.41 \times 10^{-4}\) & \(2\pi 34.02\) \\
2 & \(-7.20 \times 10^{-4}\) & \(7.50 \times 10^{-4}\) & \(2\pi 354.80\) \\
3 & \(-8.96 \times 10^{-4}\) & \(4.00 \times 10^{-4}\) & \(2\pi 431.50\) \\
\hline
\end{tabular}
\caption{Identified Parameters of Fine Stage}
\end{table}
transfer function peak is reduced from 14.5 dB in the coarse stage to 7.1 dB in the predicted dual-stage system, and the frequency range with disturbance attenuation capability is enhanced from frequency below 22.7 Hz to below 36.7 Hz.

Fig. 15 shows the tracking performance of the individual fine stage when track 1 Hz and 33 Hz sinusoidal reference signals, which demonstrates that the fine stage can compensate the tracking error of coarse stage effectively in both low frequency and the frequency of coarse stage sensitivity peak, and the fine stage is able to achieve submicron accuracy. The dual-stage positioning performance is also shown in Fig. 16 when track a sinusoidal reference of 1 Hz and 1 mm magnitude. It demonstrates that the maximal tracking error is reduced significantly from about 20 µm in coarse stage to less than 2 µm in dual-stage positioning.

V. CONCLUSION

In this paper, a flexure-based Lorentz motor is designed as the fine stage for dual-stage positioning system, where coarse
stage employs a MMLM. To carry out the fine stage design, an integrated servo-mechanical design approach is proposed that systematically design both the plant and controller of fine stage, considering both mechanical and control specifications. Through this approach, it is demonstrated that, by utilizing suitable mechanical plant design, the fine stage can fulfill various control specifications by only using a PID controller, e.g. to compensate sensitivity peak in coarse stage, etc. Experimental investigation also indicates that the designed PID controller works well with the real fine plant in regarding of robust stability, and the fine stage can compensate the tracking error of coarse stage effectively. It also demonstrates that the sensitivity peak is reduced effectively from 14.5 dB in coarse stage to only 7.1 dB through dual stage, and the maximal tracking error is reduced significantly from about 20 µm in coarse stage to less than 2 µm in dual-stage positioning.

**Appendix**

**KYP and GKYP Lemmas**

**Lemma 1 (KYP Lemma [21]):** Consider \( G(s) = C_g(I - A_g)^{-1}B_g + D_g \), the positive realness condition \( G(jω) \in \mathbb{S} \) for all \( ω \in (0, +∞) \) is guaranteed if and only if there exist Hermitian matrices \( P, S \), such that

\[
\begin{bmatrix}
A_g & B_g & 0 & P \\
I & 0 & 0 & 0 \\
0 & 0 & C_g & T \\
D_g & 0 & 0 & S \\
\end{bmatrix} < 0.
\]

(29)

**Lemma 2 (GKYP Lemma [21]):** Consider \( G(s) = C_g(I - A_g)^{-1}B_g + D_g \), the bounded realness condition \( |G(jω)| < ρ \) for all \( ω \in Ω \) is guaranteed if and only if there exist Hermitian matrices \( P, Q > 0 \), such that

\[
\begin{bmatrix}
A_g & B_g & 0 & 0 \\
I & 0 & 0 & -ρ \\
0 & 0 & C_g & D_g \\
0 & -ρ & 0 & S \\
\end{bmatrix} < 0,
\]

(30)

where \( Σ = [Φ ⊗ P + Θ ⊗ Q] \), \( Φ \) and \( Θ \) are the matrices to characterize the frequency range \( Ω \) as given in [21].

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**References**


Haiyue Zhu (S’13) received the B.Eng. degree in automation from the School of Electrical Engineering and Automation and the B. Mgt. degree in business administration from the College of Management and Economics, Tianjin University, Tianjin, China, in 2010, and the M.Sc. degree in electrical engineering from the National University of Singapore (NUS), Singapore, in 2013, where he is currently pursuing the Ph.D. degree with the Department of Electrical and Computer Engineering.

He joined the Singapore Institute of Manufacturing Technology (SIMTech)–NUS Joint Laboratory on Precision Motion Systems in 2013, and is an Attached Research Student with the Agency for Science, Technology, and Research (A*STAR), SIMTech. His current research interests include integrated design and control of ultraprecision mechatronics and magnetic levitation technology.

Chee Khiang Pang (S’04–M’07–SM’11) received the B.Eng.(Hons.), M.Eng., and Ph.D. degrees in 2001, 2003, and 2007, respectively, all in electrical and computer engineering, from National University of Singapore (NUS).

In 2003, he was a Visiting Fellow in the School of Information Technology and Electrical Engineering (ITEE), University of Queensland (UQ), St. Lucia, QLD, Australia. From 2006 to 2008, he was a Researcher (Tenure) with Central Research Laboratory, Hitachi Ltd., Kokubunji, Tokyo, Japan. In 2007, he was a Visiting Academic in the School of ITEE, UQ, St. Lucia, QLD, Australia. From 2008 to 2009, he was a Visiting Research Professor in the Automation & Robotics Research Institute (ARRI), University of Texas at Arlington (UTA), Fort Worth, TX, USA. Currently, he is an Assistant Professor in Department of Electrical and Computer Engineering (ECE), NUS, Singapore. He is also an A*STAR Singapore Institute of Manufacturing Technology (SIMTech) Associate, Faculty Associate of A*STAR Data Storage Institute (DSI), a Senior Member of IEEE, and a Member of ASME. His research interests are on ultra-high performance mechatronic systems, with specific focus on advanced motion control for nanopositioning systems, precognitive maintenance using intelligent analytics, and energy-efficient task scheduling considering uncertainties.


Tat Joo Teo (M’09) received the B.Eng. degree in mechatronics engineering from the Queensland University of Technology, Brisbane, Qld., Australia, in 2003, and the Ph.D. degree in mechanical and aerospace engineering from Nanyang Technological University, Singapore, in 2009.

In 2009, he joined the Mechatronics Group, Singapore Institute of Manufacturing Technology, Singapore, as a Researcher. He has four grant patents and two provisional patents filed. His research interests include ultraprecision system, compliant mechanism theory, parallel kinematics, electromagnetism, electromechanical system, thermal modeling and analysis, energy-efficient machine, and topological optimization.

Dr. Teo is currently serving as the Associate Editor for the Nanoscience and Nanotechnology Letters and the IEEE International Conference on Robotics and Automation 2015. He also serves as the Technical Reviewer for the IEEE/ASME TRANSACTIONS ON MECHATRONICS, the IEEE TRANSACTIONS ON ROBOTICS, MECHANISM AND MACHINE THEORY, and the IFAC Mechatronics Journal. He received the Best Session Paper Award in the 39th Annual Conference of the IEEE Industrial Electronics Society in 2013. One of his patents received the 2014 R&D 100 Award.