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Numerical and experimental study on silicon microresonators based on phononic crystal slabs with reduced central-hole radii

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Abstract
In this paper, we report the numerical and experimental study on micromechanical resonators which are made by introducing defects on an otherwise perfect two-dimensional (2D) silicon phononic crystal (PnC) slab. The 2D PnC slab is made by etching a square array of cylindrical air holes in a free-standing silicon plate with a thickness of 10 μm, while the defects are created by reducing the radii of three rows of air holes at the centre of the 2D PnC slab. Three resonators with different values of reduced radii, i.e., 2 μm, 4 μm and 6 μm, are included in this study. The finite-element-modelling method is used to calculate the band structure of the perfect 2D PnC slab and to analyse the different mode shapes of the structure. The design, numerical modelling, fabrication process, as well as characterization results and discussions of the three PnC resonators are also included. Due to its CMOS-compatibility, aluminium nitride is chosen to be the piezoelectric material of the inter-digital transducers, which are used to generate and detect acoustic waves. Testing is done to characterize the resonant frequency (f), quality factor (Q), as well as insertion loss of each of the three microfabricated PnC resonators and the results are discussed by analysing the simulated transmission spectra, the defected band structures, and the steady-state displacement profiles of the structures at their respective resonant frequencies. The experimental results show that the designed PnC resonators with reduced central-hole radii have higher resonant frequency and higher quality factors as compared to their normal Fabry–Perot counterpart, thanks to the higher-frequency modes supported within the cavity and slow sound effect in the lateral direction introduced by the central holes with reduced radii, respectively. As a result, the achieved (f-Q) product can be as high as 2.96 × 10^{11}, which is among the highest for silicon resonators operating in air.

(Some figures may appear in colour only in the online journal)
1. Introduction

During the past two decades, the propagation of acoustic waves in phononic crystals (PnCs) has received a great deal of research attention because of their renewed physical properties and potential applications [1–11]. PnCs are also referred to as phononic band gap (PnBG) materials or acoustic band gap (ABG) materials, whereby a periodic array of scattering inclusions located in a homogeneous background material leads to the formation of PnBG or ABG, prohibiting elastic waves of certain frequencies from travelling in any direction. From the view of the material compositions, PnCs can be classified into three main groups, i.e., solid scattering inclusions in a solid background [5, 10, 12], air scattering inclusions in a solid background [7, 13, 14], and vertical pillars on top of a substrate [15–17]. From the view of the geometry of the structures, three-dimensional (3D) PnC substrate [4], 2D PnC slabs [5, 7, 18] and one-dimensional (1D) PnC strips [19–21] have also been reported, of which the 2D PnC slab is gaining more and more research interest because of the better capability to trap the elastic energy within the structure provided by the 2D nature of the PnC slabs. Various configurations of PnC slabs, such as air holes drilled on a 2D silicon membrane [7], cylindrical rods deposited on the top of the membrane [12], cylindrical rods inserted into the air holes [5, 22], as well as inverse acoustic band gap (IABG) structure [23], have been reported by various research groups. When various types of defects are introduced to an otherwise perfect PnC structure, devices of different functionalities like resonators and waveguides have been reported as well [24–34]. For example, a line defect can also be introduced along the direction of wave propagation to form a waveguide [26], a cavity-mode resonator can be formed by adding a line defect perpendicular to the direction of wave propagation in the form of a Fabry–Perot structure [34], and Bloch-mode resonators which are formed by adding an extra row of air holes [33] or introducing alternate defects [31, 32] to the Fabry–Perot resonant structure are also proposed to solve the trade-off [35] between quality factor and motional impedance. Such trade-off is typically reported in capacitive-based [36–41] and piezoelectric-based [42–44] devices.

From the aspect of the dispersion properties, PnBG is the frequency range which no elastic vibrating modes can be supported within. Analogically to the well-known photonic crystals (PhCs) [45, 46] which are formed by a periodic dielectric in electromagnetic medium, PnCs are inhomogeneous materials with periodic variations in the acoustic medium. In PhC, researchers have already demonstrated that the scattering loss in Fabry-Perot micro/nano cavity structure [47, 48] is high due to the abrupt terminations of the two edges of the Fabry–Perot resonant cavity, which subsequently renders a lower quality factor (Q) achieved as the Q is inversely related to the amount of energy loss. To overcome this problem, Lalanne et al proposed that the resonant cavity should be created by modifying portions of the lattice structure, instead of completely removing the scattering inclusions [49]. This is because the wave penetrates some distance into the edge of the cavity instead of being reflected exactly at the edge. It is crucial that the supported mode does not abruptly terminate at the cavity boundary but penetrates and exponentially decays outside the cavity, as abrupt termination would lead to a significant amount of scattering loss [49]. By analogy, if the air holes in the PnC are partially modified instead of being completely removed, the scattering loss can also be reduced and Q can be enhanced.

In this paper, we report numerically and experimentally the micromechanical resonators formed by reducing the radii of three rows of air holes at the centre of the 2D silicon PnC slab of square lattice with a thickness of 10 μm. This design approach of reducing the central-hole radii is adopted in order to reduce the scattering loss and enhance Q. Three resonators with different values of reduced radii, including 2 μm, 4 μm and 6 μm, are investigated in this study. A Fabry–Perot PnC resonator which was reported previously [34] with three rows of air holes completely removed from the centre of the 2D silicon PnC slab is also included in this paper for reference. Prior to the resonator design, several silicon/air square lattice PnCs with different parameters are analysed by finite-element-modelling (FEM) and their performances are optimized between the width of the band gap and the challenge to the microfabrication capability. The detailed fabrication process is discussed and the microfabricated resonators with high-Q resonant peaks in the hundred MHz range are characterized in terms of the resonant frequency (f), quality factor (Q), as well as insertion loss (IL). The experimental results are discussed by using different FEM approaches to analyse the transmission spectra, the defected band structures, and the steady-state displacement profiles of the structures at their respective resonant frequencies.

2. Modelling and design

In the phononic structure, a phononic band gap can be formed when scattering inclusions arranged periodically in a homogeneous host material cause waves in certain frequencies to be completely reflected by the structure. When defects are introduced to the otherwise perfect PnC structure by making the radii of the central rows of inclusions different from the inclusions of the surrounding PnC structure, a resonant peak can then appear within the stop band to form a PnC resonator.

2.1. PnC band gap optimization

The band structure of the perfect 2D PnC slab is analysed by FEM using the eigenfrequency solver in the ‘piezo solid’ model of COMSOL Multiphysics software. The main principle governing the FEM of the band structure is derived from first-principles physics, i.e., the combination of Newton’s Second Law of motion and a constitutive relation. The detailed equations which govern the calculation of the band structure are reported in our previous work [33] and the FEM approach is similar to the modelling approach described in [33] and [34], which starts with the construction of a unit cell of the 2D PnC slab as depicted in figure 1(a). In the subdomain settings of the COMSOL FEM, parameters which describe the material properties, such as elasticity matrix, coupling
After optimizing the band structure, the optimized parameters, i.e., \( r = 8 \, \mu m \) and \( a = 18 \, \mu m \), are used as the basis for the
Figure 2. Band gap optimization by (a) keeping $d/a = 0.5$ and varying $r/a$ (b) keeping $r/a = 0.475$ and varying $d/a$. $d$ is fixed at 10 μm for both cases.

Figure 3. Schematic drawings of the supercell of the (a) the Fabry–Perot PnC resonator with three rows of air holes completely removed at the centre of the PnC region (b) the PnC resonator with reduced radii of $r' = 2$ μm for the central three rows of air holes (c) the PnC resonator with reduced radii of $r' = 4$ μm for the central three rows of air holes (d) the PnC resonator with reduced radii of $r' = 6$ μm for the central three rows of air holes. The supercells repeat themselves in the $y$ direction in the microfabricated devices and the acoustic waves travel along the $x$ direction. For all the four cases, since the number of rows of air holes being removed or modified is three, the cavity length ($L$) is $3a$, which means three times the lattice constant. As all the resonators have the same basic parameters ($r = 8$ μm, $a = 18$ μm, and $d = 10$ μm) and the same cavity length of $3a$, the differences in their performance will be solely caused by the configurations of the central defected region.

3. Microfabrication

The top-down CMOS-compatible microfabrication process which requires four photolithography masks (figure 4(a)) follows partially from our previous work [33], but improvements and modifications in the detailed fabrication steps were made to further optimize the CD control and other fabrication qualities for the new batch of fabricated devices.

The process starts with silicon-on-insulator (SOI) wafers with a device layer thickness of 10 μm, the buried oxide (BOX) layer thickness of 1 μm and silicon handle layer thickness of 750 μm. First, AlN of 1 μm thickness is deposited using radio frequency (RF) sputtering. Next, a layer of SiO$_2$ with 1 μm thickness is deposited using plasma enhanced chemical vapour deposition (PECVD) and dry etched to serve as a hard mask to define the subsequent etching of AlN. After the PR is stripped, the deposited AlN layer is then patterned subsequently with a SiO$_2$ hard mask, using dry etch with a combination gas of Cl$_2$/BCl$_3$/Ar (the ratio of the gas flow is 100:55:15) in an AMAT Centura ECR (Electro Cyclotron Resonance) etcher (model: 5200, chamber pressure: 11 mT, bias voltage: $-1000$ V, RF power: 1300 W), using an endpoint based recipe to detect etch completion with an endpoint detection mechanism, so that over etching of silicon device
layer is minimized. The etched AlN layer is to serve as the piezoelectric layer for the Inter-Digital Transducer (IDT) (figure 4(a)(iii)). The etching is done in an STS Inductively Coupled Plasma (ICP) etcher with an 11 s etching cycle (etch gas: SF6, flow rate: 110 sccm, RF power: 15 W, chamber pressure: 10 mT, helium cooled chuck) and a 5 s passivation cycle (passivation gas: C2F6, flow rate: 80 sccm, RF power: 600 W, chamber pressure: 10 mT, helium cooled chuck). As the radius of the central holes for some PnC resonator (r′) is much smaller than the radius of the PnC structure (r), e.g. the case of r′ = 2 μm, 120% over-etching using reduced RF power (RF etching power: 2W, RF passivation power: 400 W) is done to minimize the micro-loading effect. We also use a slower recipe for silicon etching by reducing the RF power to ensure a straight sidewall profile. Lastly, the wafer is flipped and the whole structure becomes a free-standing 2D phononic slab by DRIE on the backside (figure 4(a)(iv)) after grinding the wafer from backside to reduce the thickness of the wafer to 500 μm. At this step, it is better not to grind the wafer too thin for easy handling purposes. The releasing process involves two steps: DRIE of grinded silicon substrate and Reactive Ion Etching (RIE) of the BOX layer, both from the backside. The RIE of the BOX layer is done in a Plasma-Therm RIE etching chamber (etching gas: CF4/O2, flow rate: 15 sccm/3 sccm, RF power: 200 W, chamber pressure: 15 mT, helium cooled chuck). The schematic drawing of a typical PnC resonator structure with reduced central-hole radii after all the fabrication process is shown in figure 4(b), with IDT formed by piezoelectric AlN layer and an Al electrode on the two sides of the air/silicon phononic structure.

Figure 5(a) shows the SEM image of the Fabry–Perot PnC resonator from our previous work [34] which is formed by completely removing three rows of air holes at the centre of the PnC region. We include it here to serve as a reference for the comparison between the performance of the Fabry–Perot PnC resonator and the PnC resonators with reduced central-hole radii. Figures 5(b)–(d) show the SEM images of the designed PnC resonators with reduced central-hole radii of r′ = 2 μm, r′ = 4 μm and r′ = 6 μm, respectively. Four periods (rows) of air holes are patterned on each side of the defect region for all the PnC resonators, and IDT are formed by a piezoelectric AlN layer and an Al electrode on the two sides of the air/silicon phononic structure. The IDT employed for this study has a periodicity of 15 μm and a finger width of 7.5 μm, rendering the metallization ratio of the transducer to be 50%. The length of the IDT is 150 μm. The PnC region inside the solid red square in figure 5(b) is the typical supercell structure shown in figure 3(b) and the supercell is also used for the FEM simulation which will be discussed later in this manuscript. From the SEM images, it can be shown that the actual microfabricated PnC structures are a repetition of various supercells along the y direction and acoustic waves travel along the x direction. Due to the piezoelectric properties of the AlN film, acoustic waves are launched toward the PnC structure along the x direction when an AC signal is applied on the IDT of the input port. As the piezoelectric AlN film can convert the displacements caused by transmitted acoustic
waves into electrical signals, the transmitted acoustic waves through the PnC structure could be picked up by the IDT of the output port on the other side of the phononic structure, after their interaction with the PnC structure. Figure 6 shows the close-up views of the defected regions at the centre of the PnC, which are the areas enclosed by the dashed squares in all sub-figures in figure 5.

Figure 7 shows an even closer look at the air holes after the silicon device layer DRIE process, in order to examine the effects of over etching and micro loading. For over etching, we reduced the RF etching power to 2 W and the RF passivation power to 400 W, in order to minimize the micro-loading effect and the unintended etching to the underlying SiO₂ layer. As the RF etching power is reduced very significantly while the passivation power is reduced only slightly, the passivation cycle during the over-etching process offers excellent protection of the side wall against etching during the etching cycle. Furthermore, our silicon DRIE recipe has a selectivity of 330:1 over the BOX layer underneath the silicon device layer. Therefore, the effect of over etching on the diameter of various holes is negligible. Also, the unintended etching to the underlying SiO₂ layer is also minimized. From the SEM image shown in figure 7, the side wall profile is very straight, showing excellent protection of the side wall against over etching. Nevertheless, the scallop pattern in the sidewall during the DRIE process can still be observed and it causes the drift in CD control, which will lead to the drift in the performance of the resonators from the desired value. In addition, as the releasing DRIE process is done on the backside of the wafer, we deposited a layer of SiO₂ on the front side of the wafer to protect the silicon device layer with PnC structures on it from breaking during the releasing DRIE process. After the DRIE process, the layer of protecting SiO₂ is then etched away by vapour hydrofluoric acid (VHF) to expose the underneath silicon device layer. This additionally deposited SiO₂ layer serves as a protection layer which not only protects the underneath silicon device layer from breaking after the structure was released, it also protects the front side of the silicon device layer from scratching and damaging by the machine during the DRIE process on the back side.

Another common concern for the fabrication of the 2D suspended membrane structure is the curvature of the membrane after releasing. To access the curvature of our PnC resonators, we also measured the surface profiles of the PnC structure and the free-standing silicon slab without any PnC structure using Veeco Wyko Non-contact Profilometer and the measured profiles are shown in the figure 8, with the
Figure 6. Close-up views of the central defected regions of (a) the Fabry–Perot PnC resonator, (b) the PnC resonator with reduced central-hole radii of $r' = 2 \mu\text{m}$, (c) the PnC resonator with reduced central-hole radii of $r' = 4 \mu\text{m}$, (d) the PnC resonator with reduced central-hole radii of $r' = 6 \mu\text{m}$. The scale bar in each of the SEM image represents 20 $\mu\text{m}$.

Figure 7. More zoomed-in view of the air holes after the silicon device layer DRIE process, in order to examine the effects of over etching and micro loading.

The surface profile of the free-standing silicon slab without any PnC structure shown in figure 8(a) and the surface profile of the PnC structure shown in figure 8(b). From the two figures, we can see that the height differences for both the free-standing PnC membrane and the silicon slab without any PnC structure are around 100 nm. Given that the thickness of the Si device layer is 10 $\mu\text{m}$, the height difference is only 1% of the device layer thickness, which can be safely neglected. Therefore, the curvature of the free-standing slab after releasing is also negligible, which also indicates that the stress level of the released membrane film is very low. Due to the low stress level, the DRIE released membrane is not easily broken.

Therefore, for our fabricated resonator structures, the curvature for the free-standing slab after releasing is negligible because the area of the silicon handle layer which is exposed for etching during the releasing process is only the area directly under the phononic structure. At the chip level or the wafer level, the area exposed for releasing, which is actually the area of free-standing slab, only occupies less than 10% of the total wafer or chip area. In other words, the free-standing slab is actually a small area of silicon membrane which is bounded by a large area of surrounding unreleased silicon. Moreover, our Si DRIE process requires that the total area exposed for Si etching should be less than 15% of the total wafer area, in order to achieve a straight side wall profile and good etching uniformity across the wafer. Also, if the area exposed for Si etching is too large, the whole wafer becomes fragile after releasing as the device layer is only 10 $\mu\text{m}$. Therefore, the free-standing silicon slab is fixed by the surrounding unreleased silicon to prevent any curvature or breakage.

4. Device characterization and discussions

To experimentally characterize the microfabricated PnC devices, a network analyser (Model: Agilent E8364B) is
used after performing a standard Short-Open-Load-Through calibration. The testing setups and testing procedures are the same as those reported in our previous work \[33, 34\], so they will not be repeated for simplicity. We also include a free-standing silicon slab without any PnC structure and with the same length of wave propagation and in the same frequency range for normalization purposes. Therefore, when the frequency-response transmission spectra of various PnC resonators are normalized with respect to the pure silicon slab (the normalization structure), the influences of the air surrounding the structure, together with other parasitic effects caused by the IDTs, are eliminated. In this case, the resultant transmission spectra of various designed resonators after normalization are solely due to the effects of the PnC structure.

Figure 9(a) shows the measured transmission spectrum of the free-standing silicon slab without any PnC structure and figure 9(b) shows the transmission spectrum of the perfect PnC structure after the optimization of the lattice parameters. We observed that the measured stopband of 140 MHz < f < 195 MHz agreed quite well with the frequency range of the band gap in the calculated band structure shown in figure 1(c). We believe the discrepancies are due to the drift of CD control in microfabrication process, in which the radii of the air holes in microfabricated devices are larger than designed. The measured stopband leads to the centre frequency of 167.5 MHz and gap-to-midgap frequency ratio of 32.8%. Figure 9(b) also shows a maximum attenuation of 30 dB within the stopband.

Figure 10(a) reproduces the measured transmission spectrum of the PnC Fabry–Perot resonator (figure 5(a)) from our previous work \[34\], with three rows of air holes completely removed (L = 3a) at the centre of the PnC region. Figures 10(b)–(d) show the measured transmission spectra of the PnC resonator with a reduced radii (r’) of the central three rows of air holes (L = 3a) of (b) r’ = 2 μm (figures 5(b)) and (c) r’ = 4 μm (figures 5(c)), and (d) r’ = 6 μm (figure 5(d)), respectively. From the measured transmission spectra shown in figure 10, it can be shown that for the same cavity length, all the PnC resonators with reduced central-hole radii have higher f and higher Q as compared to their Fabry–Perot counterpart. The Fabry-Perot PnC resonator shown in figure 5(a) can also be considered as the PnC resonator with a reduced central-hole radii of r’ = 0 μm. When the central-hole radius increases from 0 μm to 2 μm, the resonant frequency increases sharply from 152.46 MHz to 182.1 MHz, the quality factor also jumps from 1016 to 1624, and the insertion loss decreases from 13 dB to 11 dB. This indicates that the pure Fabry–Perot resonator changed to a completely different mode of operation when additional air holes are introduced in the central cavity region. When r’ is further increased to 4 μm, both the resonant frequency and the quality factor drop, to a value of 179.9 MHz and 1086, respectively, while the insertion loss increases to 13.1 dB. This means the optimized condition for the performance parameters as achieved in the case of r’ = 2 μm is destroyed. When r’ is further increased to 6 μm, the resonant frequency drops further to 176.22 MHz but the quality factor increases to 1311, while the insertion loss remains almost unchanged at 13.2 dB.

From the measured transmission spectra, we can calculate the f-Q product, a common figure-of-merit for micromechanical resonators, for the three PnC resonators with a reduced central-hole radii to be 2.96 × 10\(^{11}\), 1.95 × 10\(^{11}\) and 2.31 × 10\(^{11}\), for the case of r’ = 2 μm, r’ = 4 μm, and r’ = 6 μm, respectively. These values are among the highest f-Q products for silicon micromechanical resonators operating in air. It can also be noticed that the f-Q product of the three PnC resonators with reduced central-hole radii are all larger than the case of the pure Fabry-Perot resonator with the same cavity length (L = 3a), which is 1.5 × 10\(^{11}\). Since all the PnC resonators reported here have the same basic parameters (r = 8 μm, a = 18 μm, and d = 10 μm) and the same cavity length, i.e., L = 3a, it can be concluded that the differences in their performance will be solely caused by the configurations of the central defected region. Table 1 below summarizes the performance parameters of all the resonators reported in this manuscript.

The transmission spectra of the designed PnC resonator structure is also analysed by FEM, with a supercell (figure 3) constructed first. Subdomain settings are applied to simulate
Figure 9. Measured transmission spectra of (a) the free-standing silicon slab without any PnC structure, (b) the optimized PnC structure.

Figure 10. Measured transmission spectra of (a) the Fabry–Perot PnC resonator, (b) the PnC resonator with reduced central-hole radii of $r' = 2 \mu m$, (c) the PnC resonator with reduced central-hole radii of $r' = 4 \mu m$, (d) the PnC resonator with reduced central-hole radii of $r' = 6 \mu m$. All the PnC resonators with reduced central-hole radii have a higher $f$ and a higher $Q$ as compared to their Fabry–Perot counterpart.

Table 1. Performance parameters of various resonators.

<table>
<thead>
<tr>
<th>Performance parameters</th>
<th>$r'(\mu m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$ (MHz)</td>
<td>0 2 4 6</td>
</tr>
<tr>
<td>$Q$</td>
<td>152.46 182.1 179.9 176.22</td>
</tr>
<tr>
<td>$IL$ (dB)</td>
<td>1016 1624 1086 1311</td>
</tr>
<tr>
<td>$f - Q$ product (Hz)</td>
<td>$1.5 \times 10^{11}$ $2.96 \times 10^{11}$ $1.95 \times 10^{11}$ $2.31 \times 10^{11}$</td>
</tr>
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</table>

air holes in the silicon background matrix. For boundary settings, zero charge/symmetry is chosen and force is applied from the left edge of the structure to simulate the acoustic waves generated. Periodic boundary conditions are then applied along the $y$ direction. A frequency response solver is selected as the solving mechanism and the frequency of the input acoustic waves is swept from the lower edge to the upper edge of the band gap of the pure PnC structure (figure 1(c)). The acoustic waves after interaction with the phononic structure is collected by a probing tool at the right edge of the structure to calculate the total displacement of the right edge. A flat silicon plate without any PnC structure and with the same length of wave propagation is also constructed for the purpose of normalization. The simulated transmission spectra for the case of the Fabry-Perot resonator is shown in figure 11(a), while figures 11(b)–(d) show the simulated transmission spectra for the PnC resonators with reduced central-hole radii of $r' = 2 \mu m$, $r' = 4 \mu m$, and $r' = 6 \mu m$, respectively. For all four cases, the resonant frequencies in the simulated transmission spectra agree quite well with the experimental results, expect
Figure 11. Simulated transmission spectra of (a) the Fabry–Perot PnC resonator, (b) the PnC resonator with reduced central-hole radii of \( r' = 2 \mu m \), (c) the PnC resonator with reduced central-hole radii of \( r' = 4 \mu m \), (d) the PnC resonator with reduced central-hole radii of \( r' = 6 \mu m \). The resonant frequencies in the simulated transmission spectra are in very good agreement with the measured data.

for small deviations which are caused by the fabrication error. However, from our previous experience, the simulated transmission spectra are a good estimation of the resonant frequencies but not a good estimation of the quality factors due to the following two reasons. First, the scattering loss, which is a major source of energy loss, is not considered when periodic boundary conditions were applied in the simulation and we assumed the whole resonator is formed by the duplication of the repeating unit in the \( y \) direction. Second, the loss due to electro-acoustic coupling is not considered when loads are directly applied to the structure in our simulation. However, acoustic waves are launched towards the phononic structure through the piezoelectric effect of IDT in the actual experiment. Therefore, the simulated transmission spectra are good indicators of only the resonant frequencies of various PnC resonators.

In order to analyse the reasons for the different quality factors obtained from different resonators by accounting the scattering loss, the band structures of various PnC resonators are also calculated using similar approach as described in section 2.2. Again, supercells shown in figure 3 are constructed first and periodic boundary conditions are applied along the \( y \) direction only instead of both the \( x \) and \( y \) directions for the case of the perfect PnC structure. The calculated eigenfrequencies are combined to generate the defected band structures for various PnC resonators. In the band structure calculated for the PnC structure, the first order derivative of frequency against wave vector represents the group velocity of the guided mode of the acoustic waves along the waveguide in the lateral direction, which is the \( y \) direction in our case. Therefore, flatter bands lead to slower group velocities and steeper bands lead to higher group velocities, since the first order derivative of frequency against wave vector is reflected as the slope of that particular band in the band structure diagram. As a result, the group velocity for a completely flat band is equal or close to zero and the modes associated with the flat band have a strong spatial localization [50]. Elastic waves with a small group velocity are called slow sound. Therefore, the phenomenon of the strong spatial localization of elastic waves due to a small group velocity is also called the 'slow sound effect'. On the other hand, a steep band means that the group velocity in the \( y \) direction is high. From the SEM images shown in figure 5, we can see that for the PnC resonators, the resonant cavity in the \( x \) direction is essentially a waveguide in the \( y \) direction with the two ends at the top and the bottom of the resonant cavity being open. In this case, when elastic waves enter the resonant cavity, its associated elastic energy can leak through the open ends at the top and the bottom of the resonant cavity. Therefore, the rate of energy leakage is high when the band is steeper as the group velocity of elastic waves along the \( y \) direction is high, leading to higher energy loss. On the other hand, when the band is flatter as the group velocity of elastic waves along the \( y \) direction is low or even zero, elastic waves hardly travel along the \( y \) direction and stay inside the resonant cavity to be reflected back and forth for a longer period of time, resulting in lower energy loss. Conceptually, the quality factor of a resonator is a direct indicator of the energy loss and is defined as

\[
Q = \frac{2\pi \times \text{Energy Stored}}{\text{Energy Disipated}}
\]

Therefore, the resonator with a flatter band (lower group velocity of elastic waves along the \( y \) direction) should have a higher quality factor, due to lower energy loss.

The calculated band structure for the case of the Fabry–Perot resonator is shown in figure 12(a), while figures 12(b)–(d) show the calculated band structures for the PnC resonators with a reduced central-hole radii of \( r' = 2 \mu m \), \( r' = 4 \mu m \), and \( r' = 6 \mu m \).
and $r' = 6 \mu m$, respectively. The bands of interest which are selected are the bands which correspond to the resonant frequencies shown in figure 11 when $k_y = 0$. In other words, the frequency of the bands of interest when $k_y = 0$ should be the same as the simulated resonant frequencies in figure 11 for all four PnC resonators. As such, the selected bands of interest which correspond to the resonant peaks shown in figure 11 are labelled using blue arrows in all the sub-figures of figure 12. From the slopes of the bands, we can see that the selected bands in the calculated band structures agree quite well with the experimental results. For example, the PnC resonator with $r' = 2 \mu m$ has the highest quality factor among all the cases, as shown in figure 10(b). Correspondingly, the case of $r' = 2 \mu m$ also has the flattest band when $k_y = 0$ among the four cases. Since it has the flattest band, the group velocity of the elastic waves along the $y$ direction is slowest due to the slow sound effect. As a result, the amount of energy loss is smallest and thus the highest quality factor achieved. On the contrary, for the case of the Fabry–Perot resonator, it has the steepest band among the four cases as shown in figure 12(a), thus the quality factor for this case is also the lowest since the amount of energy loss is largest, as shown in figure 10(a). For the PnC resonators with $r' = 4 \mu m$ and $r' = 6 \mu m$, the slope of the selected bands which correspond to the simulated resonant frequencies also agree well with the experimental results.

With the supercells which were built for the analysis of resonant frequencies, the steady-state displacement profiles of the structures at their respective resonant frequencies in the $x$, $y$ and $z$ directions, respectively. The colour bar indicates the amplitude of displacements in an arbitrary unit, with extreme red representing the maximum displacement (maximum displacement amplitude in the positive direction) and extreme blue representing the minimum displacement (maximum displacement amplitude in the negative direction).

The simulated steady-state displacement profiles for the case of the Fabry-Perot resonator is shown in figure 13(a), while figures 13(b)–(d) show the simulated steady-state displacement profiles for the PnC resonators with reduced central-hole radii of $r' = 2 \mu m$, $r' = 4 \mu m$, and $r' = 6 \mu m$, respectively, under their respective resonant frequencies. The $u_x$, $u_y$ and $u_z$ represent the displacement vector components in the $x$, $y$, and $z$ directions, respectively. The colour bar
indicates the amplitude of displacements in an arbitrary unit, where extreme red represents the maximum displacement (maximum displacement amplitude in the positive direction) and extreme blue represents the minimum displacement (maximum displacement amplitude in the negative direction). One point to note here is the displacement which the colour bar represents is in an arbitrary unit, which means the $u_{\text{max}}$ and $u_{\text{min}}$ for the four cases are different. Therefore, from the steady-state displacement profiles we can only get information about the energy distribution along the single structure.

From the steady-state displacement profiles shown in figure 13, it can be easily seen that the steady-state displacement profile for the Fabry-Perot PnC resonator (figure 13(a)) is completely different from the PnC resonators with reduced central-hole radii (figures 13(b)–(d)). For the case of the Fabry–Perot resonator (figure 13(a)), the displacement profiles are concentrated in the $x$ and $z$ directions and almost no displacements can be observed along the $y$ direction. This is actually the displacement profiles of a typical Fabry–Perot resonant mode. However, for the cases of PnC resonators with reduced central-hole radii (figures 13(b)–(d)), the Fabry-Perot resonant mode is destroyed and the displacement profiles are concentrated along the $y$ direction and the displacement along the $x$ and $z$ directions is minimum.

For the case whereby the displacement vector components along the $y$ direction are mainly distributed at the central defected region (e.g., figure 13(b)), the energy of the structures are concentrated at the central defect region. As such, the energy is better confined in the central defect region and a large amount of energy is lost, resulting in a lower quality factor expected. As compared to the cavity-mode Fabry–Perot resonators and Bloch-mode resonators, the defects are introduced by partially modifying the perfect PnC structures instead of completely removing several rows of air holes, the quality factor can be improved as a result of the significantly reduced scattering loss. The measured quality factors agree excellently with the calculated defected band structure and the phenomenon can be explained very well using the slow sound effect along the lateral direction, as indicated by the flat bands in the calculated defected band structure.

5. Conclusion

In this paper, micromechanical PnC resonators with various reduced central-hole radii have been explored and characterized. Experimental data for three designs of PnC resonators with central-hole radii of $\mu m$, $r' = 2 \mu m$, $r' = 4 \mu m$, and $r' = 6 \mu m$ are reported. These designed PnC resonators are made from a 2D PnC slab of square lattice, which is fabricated from a free-standing silicon plate of $10 \mu m$ thickness using a CMOS-compatible process. The microfabricated PnC resonators with reduced central-hole radii are characterized in terms of the resonant frequency, quality factor, as well as insertion loss and the experimental results are discussed by analysing the simulated transmission spectra, the defected band structures, and the steady-state displacement profiles of the structures at their respective resonant frequencies. Good agreement was found between the simulation results and experimental data. The designed PnC resonators with reduced central-hole radii have higher resonant frequencies and higher quality factors as compared to their normal Fabry-Perot counterpart, with the $(f/Q)$ product of as high as $2.96 \times 10^3$ achieved which is among the highest for silicon resonators operating in air.

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