SIMULTANEOUS ESTIMATION OF MUTUAL COUPLING MATRIX AND DOAS USING STRUCTURED LEAST SQUARE METHOD

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ABSTRACT
A structured Least Square (LS) method for simultaneous estimation of the mutual coupling matrix (MCM) and direction of arrival (DOA) of signal source is proposed in this paper. Mutual coupling effects are modelled in the form of a complex Toeplitz matrix. The DOAs and MCM can be simultaneously estimated by the proposed method when the observations of at least two different DOAs are available. This method is especially useful for the calibration of uniform linear array (ULA) and uniform circular array (UCA). Simulation results confirm the efficiency of the proposed method.

KEY WORDS
Mutual coupling, direction of arrival, least square method.

1 Introduction
Adaptive array signal processing is critical for many applications, such as radar, sonar, and wireless mobile communications. The performance of an adaptive array signal processing is considerably affected by the electromagnetic characteristics of the antenna array [1]. The effects of the mutual coupling among antenna elements are significant and become more drastic as the interelement spacing drops below half a wavelength [2]. The performance of an array signal processing method may degraded significantly when the effect of the mutual coupling is ignored. To relieve the effects of the mutual coupling in DOA estimation, various methods were proposed [3]-[6]. However, most of them were not so practical in real applications. In [3], the degradation of the performance was observed in the direct data domain algorithm. The method of moment (MOM) was used to compute the mutual impedance matrix of an antenna array. In [4], the effects of the mutual coupling were analyzed by using the open circuit voltages method in adaptive antennas. In [5], [6], the authors applied a more accurate method to compute the mutual impedance matrix based on an estimated current distribution carrying a direction reference of the incoming signal. It was shown that this method can significantly reduce the mutual coupling effect and thus lead to an improved performance of MUSIC [7] and ESPRIT [8] DOA estimations. However, in the above mentioned methods, the computation of mutual coupling matrix (MCM) is very intensive and time-consuming. In this paper, a new method is proposed to simplify MCM computation. The main advantage of the proposed method is that the MCM and DOAs can be simultaneously estimated when two or more DOAs are available. This method is simple, easy to implement, and attractive in practical applications.

The structure of an MCM can be transformed into the product of two matrices. One is a diagonal matrix in which the diagonal elements are the gains of the individual array elements. The other is a matrix whose elements are the mutual coupling coefficients between array elements. In the case of absence of gain and phase mismatching between the array elements and assuming identical impedances at the output of each elements, it has already been known that for some arrays with special geometries such as the well studied uniform linear array (ULA) and uniform circular array (UCA), their MCMs have special structures, symmetric Toeplitz matrix for ULA and circular matrix for UCA [4]. These properties of MCMs can be well exploited in the computation process so as to increase the efficiency of estimation procedure. The gain matrix of each antenna is easy to be measured. Therefore, in this paper, we assume that the gain matrix of an array is known.

We formulate the problem and propose a new method to obtain mutual coupling matrix, and then conduct numerical experiments to illustrate the performance of the proposed method in conjunction with the DOA estimation.

2 Proposed Method
Consider a uniform linear array with \( M \) elements and assume one narrow band source impinging on the array from \( \theta \) direction.

It can be proved that for ULA, the MCM denoted by \( A \) is a symmetric Toeplitz matrix [9], which can be represented by (1)

\[
A = \begin{bmatrix}
a_1 & a_2 & \cdots & a_N \\
a_2 & a_1 & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
a_N & \cdots & a_2 & a_1
\end{bmatrix} = \sum_{i=1}^{N} a_i E_i
\]  

(1)

where \( E_i \) is a symmetric Toeplitz matrix. Its \( i \)th row vector...
is \([0 \cdots 1 0 \cdots 0]\), where the \(i\)th element is one, others are all zero.

The signal received by the array is

\[
y(n) = x(n)A\bar{s}(\theta) + w(n)
\]

where \(w(n)\) denotes the noise vector at each of the loaded antenna element, \(x(n)\) the incident signal on the elements of the array from the direction \(\theta\), \(s(\theta)\) the array steering vector (ASV),

\[
s(\theta) = \left[ 1, e^{-j\frac{\pi}{2}\sin(\theta)}, \ldots, e^{-j(\frac{M-1}{2})\sin(\theta)} \right]^T
\]

In the controlled environment of calibration phase, the observed ASV \(\bar{s}(\theta)\) is easy to be measured using the antenna output voltages. For \(L\) different calibration sources, we have \(L\) observed ASVs. From \(L\) DOAs, \(\theta_i\) \((i = 1, \ldots, L)\), we can obtain the estimated ASVs,

\[
\hat{s}(\theta_i) = A\bar{s}(\theta_i) + e_i, \quad i = 1, \ldots, L
\]

where \(e_i\) is the measurement error.

If the exact \(\theta_i\)'s in the experiment are already known, with sufficient number of \(L\) experiments, the MCM \(A\) can be estimated by using LS method to solve the following optimization problem,

\[
\hat{A} = \arg \min_A \sum_{i=1}^{L} || \hat{s}(\theta_i) - A\bar{s}(\theta_i) ||_2^2
\]

where, \(|| \cdot \||_2\) is Euclidean norm.

Since there are \(M\) unknown parameters in MCM, and with one more experiment performed, one more unknown parameter \(\theta_i\) will be included in optimization problem. Therefore, if \(L\) experiments are performed, there are \(M + L\) unknown parameters in (5). For one experiment performed, there are \(M\) linear equations, same as (3), which can be obtained. For \(L\) experiments, there are \(M \times L\) linear equations available. Therefore, if \(M \times L > (M + L)\), it is possible to determine the unknown parameters uniquely. That means, if \(L \geq (M/(M-1))\) experiments are carried out, we can obtain \(L\) independent steering vector \(s(\theta_i)\), then the estimate of MCM \(A\) can be uniquely determined. The smallest number of \(L\) is 2.

However, for some practical experiments, the direction of signal source \(\theta_i\) can not be accurately known, or totally unknown. In such cases, the estimates of MCM have errors as well as the estimated DOAs. In this paper, we estimate DOAs and MCM simultaneously with the proposed method.

Our problem can be stated as follows. Given a uniformly spaced linear array in the presence of mutual coupling among array elements, the mutual coupling matrix \(A\) and direction of signal sources \(\{\theta_i\}\) are all unknown. The following optimization method is used to find the estimates of MCM and DOAs.

\[
J(A, \theta_i) = \sum_{i=1}^{L} || \hat{s}_i - A\bar{s}_i ||_2^2
\]

\[
\{\hat{\theta}_i, \hat{A}\} = \arg \min_{A, \theta_i} J(A, \theta_i)
\]

It can be proved that for fixed DOAs, the optimization problem (6) to \(\hat{A}\) is a quadratic problem. The closed form of the solution can be easily derived. However, the optimization problem to \(\hat{\theta}_i\) is nonlinear and not quadratic, so it is difficult to get its closed form solution. To simplify the optimization problem, we proposed to solve (6) using two steps - first assuming that \(\hat{\theta}_i\) is known and then obtaining the solution for \(A\). By substituting estimated \(A\) into (6), this optimization problem is simplified and only have unknown parameters \(\theta_i\). In case more accurate DOAs are estimated, iterative approach can be used to estimate the MCM.

Substituting (1) into (6), we have

\[
J(A, \theta_i) = \sum_{i=1}^{L} || \hat{s}_i - A\bar{s}_i ||_2^2
\]

\[
= \sum_{i=1}^{L} \bar{s}_i^H \hat{s}_i - 2\sum_{i=1}^{L} \bar{s}_i^H A\bar{s}_i + \sum_{i=1}^{L} A^H A \bar{s}_i
\]

\[
= \sum_{i=1}^{L} \bar{s}_i^H \hat{s}_i - \sum_{i=1}^{L} \sum_{k=1}^{N} \bar{a}_k \bar{s}_i^H E_k \bar{s}_i
\]

\[
+ \sum_{i=1}^{L} \sum_{k=1}^{N} \sum_{l=1}^{L} \bar{a}_k \bar{a}_l \bar{s}_i^H E_k E_l \bar{s}_l
\]

The optimal estimate of \(A\) is obtained by setting the derivative of \(J\) with respect to \(\hat{a}_k^*\) to zero

\[
\frac{\partial J(A, \theta_i)}{\partial \hat{a}_k} = -\sum_{i=1}^{L} \bar{s}_i^H E_k \bar{s}_i
\]

\[
+ \sum_{i=1}^{L} \sum_{k=1}^{N} \bar{a}_k \bar{s}_i^H E_k E_l \bar{s}_l = 0
\]

We have the linear equations for each \(k\),

\[
\sum_{l=1}^{N} \bar{a}_l \sum_{j=1}^{L} \bar{s}_j^H E_k E_l \bar{s}_j = \sum_{j=1}^{L} \bar{s}_j^H E_k \bar{s}_j, \quad k = 1, \cdots, N
\]

With estimated MCM \(\hat{A}\), the DOAs of sources can be estimated using following optimization problem,

\[
\hat{\theta}_i = \arg \min_{\theta_i} \sum_{i=1}^{L} || \hat{s}_i - \hat{A}\bar{s}(\theta_i) ||_2^2
\]

This is achieved through the use of this iterative optimization method. The following is a step-by-step description that needs to be done to obtain the estimated DOAs in order to obtain the estimated mutual coupling matrix iteratively.
Step 1: Using the observed ASVs to estimate the DOAs, and considering these DOAs as initial DOAs $\theta_i^0$.

Step 2: Using the estimated DOAs $\theta_i^{(m-1)}$ ($m - 1$ iteration) to construct the ASVs $s(\theta_i^{(m-1)})$ at $m$th iteration.

Step 3: Using (9) to estimate $\hat{A}^m$. 

Step 4: Using (10) to estimate $\theta_i^m$ with the estimated $\hat{A}^m$. 

Step 5: If the algorithm converges, then stop. Otherwise, go to Step 2 to continue.

3 Numerical Study

In this section, numerical examples are presented to demonstrate the proposed approach. We consider a uniformly spaced linear array consisting of 8 monopoles in the $Z$ direction placed along $x$ axis. The dimensions of the monopole elements are: length = 3.0 cm and wire radius = 0.3 mm. They are placed over a large ground plane and connected to a 50 $\Omega$ load and inter-element spacing is $d = 6.25$ cm (half wavelength at 2.4 GHz). The array is aligned along the $x$ axis with the monopole elements parallel to the $z$ axis. The incoming signal is sinusoidal plane wave with vertical polarization, amplitude = 1 V, frequency = 2.4 GHz, and additive thermal noise about $-20$ dB. In the experiment, the mutual coupling matrix is constructed using the a simulated Toeplitz matrix which is obtained from simulations in our previous work [6]. Assuming that we have two observations available, the true DOAs are $30^\circ$ and $40^\circ$, respectively. In Fig. 1, the DOA estimates at each iteration is shown. It is shown that the converged DOA estimates have very small estimation errors. The elements of the estimated MCM are shown in Fig. 2 and the estimation errors of MCM elements are also very small.

In the second experiment, we use 4 signals whose
DOAs vary from $5^\circ$ to $35^\circ$ with step $10^\circ$. The estimated DOAs and MCM elements are shown in Fig. 3 and Fig. 4, respectively. Compared with the simulation results for two observations, one may find that the algorithm converges faster with more observations available.

4 Conclusion

A structured Least Square (LS) method is proposed for simultaneous estimations of the mutual coupling matrix and the direction of arrival in the presence of mutual coupling and thermal noise. The geometry of the antenna array is limited to uniformly spaced ULA or UCA. This method can be used to estimate the DOA and MCM simultaneously when at least two DOAs were available. The proposed method is also useful in calibration of ULA and UCA when the DOAs of calibration sources were unknown. By comparing two examples, one can conclude that the method converges faster with more DOAs available.

References


