Increasing the Signal-to-Noise Ratio by Using Vertically Stacked Phased Array Coils for Low-Field Magnetic Resonance Imaging

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Abstract—A new method is introduced to increase the signal-to-noise ratio (SNR) in low-field magnetic resonance imaging (MRI) systems by using a vertically stacked phased coil array. It is shown theoretically that the SNR is increased with the square root of the number of coils in the array if the array signals are properly combined to remove the mutual coupling effect. Based on this, a number of vertically stacked phased coil arrays have been designed and characterized by a numerical simulation method. The performance of these arrays confirms the significant increase of SNR by increasing the number of coils in the arrays. This provides a simple and efficient method to improve the SNR for low-field MRI systems.

Index Terms—Magnetic resonance imaging (MRI), mutual coupling, numerical method, phased coil array, signal combining, signal-to-noise ratio (SNR).

I. INTRODUCTION

Research on the acquisition of high-quality images with good spatial resolution and fast scanning has always been of great interest to the development of magnetic resonance imaging (MRI). The image resolution in MRI depends crucially on the signal-to-noise ratio (SNR) available from the system transducers, so the focus lies on the design of radio frequency (RF) coils, which are the key hardware components in an MRI system for the transmission of RF signal pulses to the tissues being interrogated and the reception of the returning MR signal information to construct the image [1]. The theory of phased array coils was first proposed in MRI for achieving a higher SNR over a wider field-of-view normally associated with body imaging without increase in the scanning time by simultaneously acquiring and subsequently combining data from a multitude of closely positioned receiving surface coils [2]. Over the recent years, the potential and application of phased array coils and multichannel receivers have been actively explored [3]–[6]. In this paper, we introduce a new method based on the theory of phased array coils to improve the SNR in low-field MRI systems, where the circuit noise is the dominant noise source [7]–[10].

II. THEORETICAL BASIS OF THE METHOD

It is known that a conventional solenoid can increase the SNR by the square root of the number of turns, but the phase cancelation problem along the helical wire and the mutual coupling effect among different helical turns make it a very poor receiving coil [11]. In our proposed design, we turn each turn of a conventional solenoid into a coil element, of which the output voltage is combined together with the voltages of other turns in a proper way. That is, we effectively turn the conventional solenoid into a vertically stacked phased coil array, in which the phases of the coil signal voltages can be arbitrarily controlled. In this way, the phase cancelation problem and the mutual coupling effect problem can both be solved in an articulated way. The result is that, if the coil signals are properly combined through a careful design of the combiner coefficients, the SNR of the output signal from the phased coil array can be significantly increased. This provides an efficient and simple method to increase the SNR in MRI compared with other proposed methods, such as the use of a stronger dc magnetic field and the use of a high-temperature superconductor to design the RF receiver coils [12]. In the next section, the working principle of a vertically stacked phased coil array is first explained theoretically, followed by a detailed description of the numerical design and characterization procedure of a number of vertically stacked phased coil arrays in Section III. The conclusions are given in Section IV.

Fig. 1 Proposed stacked phased coil array with a combiner for increasing the SNR in MRI.

The configuration of the proposed vertically stacked phased coil array with m coils is shown in Fig. 1. The coils are closely...
stacked together in the vertical direction and the terminal volt-ages of coils are weighted and combined by a combiner to form a vertical phased coil array (in comparison to the horizontal phased coil arrays in [2]). The vertical alignment of coils is to optimize the alignment of the sensitivity profiles of the coils for the reception of the MRI signal originating from the location immediately under the coil array. The output voltage of each coil consists of the signal voltage and the noise voltage, in which the noise voltage comprises the circuit noise voltage and the sample noise voltage. The circuit noise voltage is assumed to be mainly originating from the low-noise amplifier connected to the coil [13] and the sample noise is mainly the random electric-field radiations from the imaged samples [2]. Since the sample noise is dependent on the frequency, and in turn, on the field strength of the system, it is usually substantially weaker than the circuit noise in the low-field systems \((B < 1 \text{ T})\) [7]–[10].

So for a low-field MRI system with negligible sample noise, \(V_N(t)\) is the vector of the combiner coefficients yet to be determined, \(V_c\) is the vector of the combiner coefficients already determined, the circuit noise voltage is assumed to be totally uncorrelated [13]. The output voltage of each coil \(V_i\) is the terminal voltage vector \(V\) and the circuit noise voltage vector \(N_c\) as:

\[
V_i = V + N_c
\]

where the aforementioned three vectors are all with a dimension of \(m \times 1\). Note that, same as the phased surface array coils in [2], the vertically stacked phased array coils in Fig. 1 are assumed to be connected to the input stages of the LNAs through lossless phase shifters and transformers. The output voltage \(V_{out}\) from the combiner is then

\[
V_{out} = W^H V_t
\]

where \(W\) is the combiner weight vector and the superscript \("H"\) denotes the Hermitian operation. We can write the signal and the noise voltages in terms of the coupled and uncoupled components as follows:

\[
V = V' + C_v V
\]

\[
N_c = N'_c + C_v N_c
\]

where \(C_v\) and \(C_c\) are, respectively, the signal and circuit noise coupling coefficient matrices, and \(V'\) and \(N'_c\), are, respectively, the ideally uncoupled signal and circuit noise voltage vectors without mutual coupling. The SNR at the combiner output is [14]

\[
\text{SNR} = \frac{|W_i^H V|}{\sqrt{|W_i^H (N'_c N'_c^H) W_i|}}.
\]

Due to the close separation between the neighboring coils in the array (this requirement is necessary as shown in the results in next section), we shall investigate the SNR in (5) in two cases: with mutual coupling between the coils and without mutual coupling between the coils.

\section*{A. Without Mutual Coupling Between the Coils}

When there is ideally no mutual coupling between the coils, i.e., \(V = V'\) and \(N_c = N'_c\), the SNR at the combiner output can be simply expressed as

\[
\text{SNR}_{m} = \frac{|W_i^H V|}{\sqrt{|W_i^H (N'_c N'_c^H) W_i|}}.
\]

where the subscript "mc" signifies the state of "isolated" operation of the coils. We consider the SNR for the case with a simple combiner vector as

\[
W_i = [1, 1, \ldots, 1]^T.
\]

This is the simplest and the most easily realizable combiner, which just adds the coil voltages together without any modification. Other combiners, such as the optimum combiner, perform as well but require more complicated circuit designs.

By using the combiner weight vector in (7), the voltage output of the vertically stacked phased coil array is

\[
|W_i^H V'| = |V'_1 + V'_2 + \cdots + V'_m| = |mV'|
\]

where \(\sigma^2\) is the variance of the circuit noise and \(I\) is the \(m \times m\) identity matrix. The identity matrix \(I\) in (9) indicates that circuit noise, which is assumed to be generated independently by the respective LNA input impedances, is totally uncorrelated [13].

By putting (8) and (9) into (6), the SNR can be derived as

\[
\text{SNR}_{m} = \frac{\sqrt{m} |V'|}{\sqrt{\sigma^2 m}}.
\]

Equation (10) shows that the SNR is proportional to \(\sqrt{m}\) and this indicates that the SNR can be increased by increasing the number of coils in the array.

\section*{B. With Mutual Coupling Between the Coils}

When there is a mutual coupling between the vertically stacked coils as in a practical case, the result in (10) cannot be obtained. By using (3) and (4), the signal and noise voltages \(V\) and \(N_c\) can be expressed in terms of their corresponding uncoupled voltages as

\[
V = (I - C_v)^{-1} V' = T_v V'
\]

\[
N_c = (I - C_c)^{-1} N'_c = T_c N'_c
\]

where \(T_v = (I - C_v)^{-1}\) and \(T_c = (I - C_c)^{-1}\) are, respectively, the transformation matrices for the signal and circuit noise voltages which relate the coupled voltages and the uncoupled voltages. By using (11) and (12), the SNR in (5) for the mutual coupling case can then be written as

\[
\text{SNR}_{mc} = \frac{|W_i^H T_v V'|}{\sqrt{|W_i^H (N'_c N'_c^H) W_i|}}.
\]

where \(W_{mc}\) is the vector of the combiner coefficients yet to be determined and the subscript "mc" in (13) is to signify the state
of operation of the coils being affected by “mutual coupling.” Note that by using a physical consideration, the total circuit noise power at the combiner output for the case with mutual coupling must be less than that for the case without mutual coupling because noise will experience loss in the coupling paths. That is, the trace of the matrix $\langle N_c N_c^H \rangle$ in the denominator of (13) is smaller than the trace of the matrix $\langle N'_c N'_c^H \rangle$ in the denominator of (6). So if the combiner coefficients are chosen such that

$$W_{mc} = T_{v}^{-H} W_i \quad (14)$$

then the SNR in (13) has the following lower bound:

$$\text{SNR}_{mc} \geq \frac{\sqrt{m} |V'|}{\sqrt{\sigma^2_c}}. \quad (15)$$

This lower bound is the same as (10), i.e., for the case without mutual coupling. Therefore, we can obtain a similar conclusion as for (10), i.e., the SNR in mutual coupling case can be increased by using lossless transformers and phase shifters to realize the combination coefficients as in (14) for the output voltage of the vertically stacked phased coil array.

### III. Numerical Experiments and Demonstrations

In order to demonstrate the previously proposed scheme, we have carried out a series of numerical experiments using commercial simulation software, FEKO [15]. The numerical experiments were performed in a simulated MRI system with the key electromagnetic components being modeled in a way that enables the investigation of the electromagnetic response and interactions of vertically stacked phased coil array.

#### A. Simulation of the Signal and Noise in the Numerical Experiments

As shown in Fig. 2, a homogenous cuboid with dimensions of $30 \text{ cm} \times 16 \text{ cm} \times 0.2 \text{ cm}$ is built to simulate the active slice (the signal source) in an MRI scanning process. The dielectric properties of the slice are set to be $\varepsilon_r = 48.6$ and $\sigma = 0.6 \text{ S/m}$ to simulate an active slice of a typical head phantom being imaged. A large number of magnetic dipoles are evenly distributed inside the slice to simulate the generation of the magnetic field by the active slice [16]. The magnetic dipoles are made to radiate at 38.3 MHz which corresponds to a low-field MRI at 0.9 T. The proposed vertically stacked phased coil array with $m$ coils worked as the receiver and was placed below the active slice.

In our simulation, the first coil element was placed at a distance of 5 cm below the active slice and the coil separation of the array is denoted by $d$, which was varied in order to investigate the effect of mutual coupling. Each coil was designed to have a resonant frequency of 38.3 MHz. The reflection coefficient of the coil was $-27.9$ dB at the resonant frequency. The schematic diagram of a typical coil element used in the simulation is depicted in Fig. 3. Each coil is terminated by a load impedance $Z_L = 0.0012 - j13.3568$, which is the equivalent impedance of the matching network connected to an LNA. Because the gain of the LNA is common to the signal and noise, we will omit the gain of the LNA and consider the voltage across the equivalent load impedance $Z_L$ as the primary quantity of interest for the collection of signal and noise voltages [13].

In the numerical simulation, the signal voltages were first generated by exciting the coil array with the active slice above it. The voltages picked up by the coil elements are then the coupled signal voltages represented by the voltage vector $V$ in (11).

To simulate the noise voltages in the numerical experiments, we used the method in [13], [17], and [18] by modeling the LNA’s internal noise into two voltage and current noise generators, as illustrated in Fig. 4. Note that in Fig. 4, the coil’s
input impedance $Z_a$ and the LNA’s equivalent impedance $Z_L$ are noiseless because the sample noise is assumed to be negligible (in a low-field MRI system) while the LNA’s internal noise has been represented by the two noise generators $v_n$ and $i_n$. Typically, $v_n$ and $i_n$ are white noise sources and can be represented by two Gaussian random variables in our numerical simulation. The variances and correlation of these two noise sources are given by [13]

\[
\begin{align*}
\langle |v_n|^2 \rangle &= 4K_0 B R_n \\
\langle |i_n|^2 \rangle &= 4K_0 B G_n \\
\langle v_n i_n^* \rangle &= 4K_0 B Y_n^* R_n
\end{align*}
\]

where $R_n$, $G_n$, and $Y_n$ are the noise resistance, conductance, and admittance of the LNA, respectively. These three parameters are given by or can be interpreted from the data provided by the manufacturer. For the other three parameters in (16)–(18), $K$ is Boltzmann’s constant, $T_0 = 290$ K is the standard noise temperature, and $B$ is the noise bandwidth. The noise of the LNAs connected to different coils is assumed to be uncorrelated [13].

To generate the noise voltages in our simulation, we removed the active slice from the coil array and excited each coil element with a unit voltage source. The coil terminal currents $I_{nc}$ were then obtained in FEKO by

\[
I_{nc} = Z^{-1} [1, 1, \ldots, 1]^T
\]

where $Z^{-1}$ is the impedance matrix (including the self-impedances, mutual impedances, and load impedances) of the coil array and is calculated by simulation using FEKO. To account for the random noise of the noise generators $v_n$ and $i_n$, we multiplied the elements of the unit voltage vector $[1, 1, \ldots, 1]^T$ on the right-hand side of (19) by $m$ noise generators each with two Gaussian random variables in the form of $v_n + i_n Z_L$. Hence, the random noise voltages [i.e., $N_c$ in (12)] were then calculated as

\[
N_c = Z_L Z^{-1} \left[ v_{n,1} + i_{n,1} Z_L, v_{n,2} + i_{n,2} Z_L, \ldots, v_{n,m} + i_{n,m} Z_L \right]^T
\]

and the noise correlation matrix $\langle N_c, N_c^H \rangle$ was then obtained as (21), shown at the bottom of the next page.

**B. Determination of the Combiner Coefficients**

From the theoretical description of the method given in Section I, in order to achieve the SNR increase by the proposed vertically stacked phased coil array for an MRI system, the crucial prerequisite is to realize the combiner coefficients in (14). This requires the knowledge of the inverse of the signal transformation matrix $T_v^{-1}$, which is to decouple the coil terminal voltages $V$, yielding the uncoupled signal voltages $V'$ [see (11)] for the array coils. From the perspective of electromagnetic analysis, the elements of $T_v^{-1}$ are the mutual impedances of the coil antennas. More specifically, these mutual impedances are the receiving mutual impedance [19] as the phased array coils are in the receiving mode of operation. A standard measurement method to determine the receiving mutual impedances for a two-element array was introduced in detail in [20]. In our current study, we applied this standard measurement method to each pair of coils in the array to find the receiving mutual impedance of the coils and thus obtain the elements of $T_v^{-1}$. For example, the receiving mutual impedance $Z_i^j$ between the $i$th coil and the $j$th coil is calculated by the following formula:

\[
Z_i^j = -\frac{V_i - V'_j}{V_j} Z_L
\]

where $V'_i$ is the ideal voltage of the $i$th coil without mutual coupling, $V_i$ and $V_j$ are the coupled voltages of the $i$th coil and the $j$th coil, respectively, when only coil $i$ and coil $j$ are receiving and the other coil elements are removed. Thus $T_v^{-1}$ can be written as

\[
T_v^{-1} = \begin{bmatrix}
1 & Z_1^2 & \cdots & Z_1^m \\
Z_2^1 & 1 & \cdots & Z_2^m \\
\vdots & \vdots & \ddots & \vdots \\
Z_m^1 & Z_m^2 & \cdots & 1
\end{bmatrix}
\]

and thereby the combiner coefficients can be determined as in (14). These coefficients can be realized by a set of lossless phase shifters and transformers as the method used in [2].

In Fig. 5, we plot the combiner output signal voltage in comparison with the summation of the ideal uncoupled signal voltages and the summation of the coupled signal voltages. The coil separation $d$ is set to be 5 mm. It can be seen that the combiner output signal voltage increases almost exactly the same as the summation of the ideal uncoupled signal voltages, indicating that the mutual coupling between the coils has been effectively removed. On the other hand, the summation of the coupled signal voltages actually decreases drastically with the number of coils, indicating the detrimental effect of the mutual coupling. A quite large sudden drop in the combined coupled signal is also seen when there are ten coils in the array. The reason for this is that the coupled coil signals combine excessively destructive at this number of coils and coil separation due to the strong mutual coupling effect which changes the phases and magnitudes of the coil signals. Fig. 5 shows the importance of the accurate design of the combiner for the phased coil array to work properly. Note that there is no need to calculate the noise transformation.
matrix $T_c$ because we have already generated the coupled noise voltages $N_c$ by the method described in the last section (see Section III-A). Another reason is that the noise voltages are to be decoupled and combined by the same combiner coefficient $W_{mc}$ as the signal [see (13) and (14)].

C. Calculation of the SNR

Once $V, N_c,$ and $W_{mc}$ are known, the SNR at the combiner output can be calculated by (13). In the calculation of $N_c$ in (20), we have considered a typical LNA, MAX9632, of which the noise parameters [21] are $R_n = 55.2 \, \Omega, G_n = 19.0 \, \text{mS},$ and $Y_\gamma = 18.1 \, \text{mS}.$

The variation of the combiner output SNR with the increasing number of coils in the coil array and at different coil separation $d$ is shown in Fig. 6. For comparison, the corresponding case with the signal and noise voltages just summed together is shown in Fig. 7. Note that the signal levels in Figs. 6 and 7 have been adjusted to make the SNR $= 20 \, \text{dB}$ for the single coil case. From Fig. 6, it can be seen that the SNR increases with the number of coils. The smaller the coil separation, the greater is the rate (slope) of the increase. The reason for this is rather obvious because a larger coil separation means that the coils are farther away from the signal source, the active slice, and thus the signals picked up by the respective coils are weaker than the case with a smaller coil separation. This is confirmed by the plot of normalized magnetic field intensity received by a single coil against its distance from the active slice as shown in Fig. 8. This figure shows that the magnetic field intensity drops almost half when the coil moves from a position at $z = 10 \, \text{mm}$ to a position at $z = -50 \, \text{mm}$ along the line of $x = 0, y = 0.$ This tells that coils located farther away from an active slice receive a signal with a

$$
\langle N_c N_c^H \rangle = [Z_L]^2 Z^{-1} \begin{bmatrix}
\langle |v_{n,1} + i_{n,1} Z_L|^2 \rangle & 0 & \cdots & 0 \\
0 & \langle |v_{n,2} + i_{n,2} Z_L|^2 \rangle & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \langle |v_{n,m} + i_{n,m} Z_L|^2 \rangle 
\end{bmatrix} Z^{-H}. \tag{21}
$$
much weaker strength than those coils closer to the active slice. This explains the decreasing speed of SNR increase in Fig. 6 as the coil separation increases from \( d = 3 \text{ mm} \) to \( d = 9 \text{ mm} \). Thus, it can be concluded that in order to obtain a desirable increase in SNR, a small coil separation has to be used.

Another observation from Fig. 6 is that the increase in SNR experiences a big jump from one coil in the array to two coils in the array, and thereafter the increase is much slower. This is seemingly inconsistent with our theoretical prediction in (10), in which the SNR is predicted to be increased with the square root of the number of coils \( \sqrt{m} \) if there is no mutual coupling. Indeed, this theoretical prediction is also plotted in Fig. 6 (the dashed line). The big difference from one coil to two coils is easily discernible. Actually, this is an extra big advantage because the extra gain in SNR is about 8-dB strong. The reason for this extra gain in SNR can be understood from (13) and (14), in that the noise voltages are decoupled and combined in the same way as the signal voltages. That is, the noise voltages are decoupled by the signal transformation matrix \( T_{s}^{-1} \) whereas they should be decoupled by the noise transformation matrix \( T_{c}^{-1} \). The consequence of this unmatched decoupling method actually helps to suppress the noise power, resulting in a much higher SNR (as shown in Fig. 6 for the case going from one coil to two coils) compared to what it should be if the correct decoupling method (i.e., \( T_{c}^{-1} \)) was used for the noise voltages.

As for the performance of the SNR in Fig. 7 calculated by coupled signal and noise voltages, it shows that if the signal voltages are not properly decoupled, the increase in SNR is obtainable only for the first few coils. When more coils are added, the SNR drops sharply. For example, with \( d = 5 \text{ mm} \), the SNR drops drastically from nine coils to ten coils, at which the SNR is even worse than the case of one coil. Apparently, the mutual coupling effect has led to a noncoherent addition (or phase cancelation) of the coil voltages, resulting in a severe cancelation of some of the coil voltages.

IV. Conclusion

A new concept of using a vertically stacked phased coil array to increase the SNR in low-field MRI systems is introduced. A detailed theoretical description of the working principle of the vertically stacked phased coil array has been given. A number of typical arrays have been designed by numerical simulations, followed by a series of numerical experiments to demonstrate the performance of the proposed method. It is shown that through a proper design of the signal combining method of the coil signals, the SNR of the array output can be significantly increased. This provides a simple and efficient method to improve the SNR for low-field MRI systems. Although the numerical characterization method has been conducted in a simplified MRI environment in this preliminary study, we believe that the proposed method is realizable for practical MRI operations. With the widespread availability of multichannel receivers for MRI in recent years, the proposed vertically stacked phased coil array has the potential of being further developed into a commercial use.

REFERENCES


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