EE5403 RF Circuit Design II

RF Mixer

Dr Guo Yongxin
RF and Optical Department
Institute for Infocomm Research (I2R)
20 Science Park Road, #02-21/25 TeleTech
ParkScience Park II, Singapore 117674
Tel: 6870-9165
Email: guoyx@i2r.a-star.edu.sg
Web: http://www1.i2r.a-star.edu.sg/~guoyx
Outline

• Introduction to mixer
• Nonlinear operation mechanism
  - Single-ended
  - Single-balanced
  - Double-balanced
• Specifications
  - Image rejection, Conversion gain, Isolation
  - Large signal performance: gain compression: $P_{1dB}$, intermodulation distortion: third-order intercept (TOI)
  - Small signal performance: noise figure
  - Operating range: Spurious-free dynamic range
Why study mixers?

• ** Receivers  
  – up or down conversion  
  – demodulation  
  – input must support large dynamic range

• ** Transmitters  
  – up conversion  
  – modulation: amplitude and phase  
  – input has optimum signal level for high overall SNR output performance
Learning Objectives

Choosing the right mixer for the task...

- Understand operating principles of the mixers
- Choices: Switching / Nonlinear; single / double balance; active / passive
- What makes a good mixer?
- Specify performance: Gain, NF, $P_{1dB}$, TOI, SFDR, isolation, image rejection.
What is a mixer?

- An ideal mixer is usually drawn with a multiplier symbol. Doesn’t “mix” or “sum”; it multiplies
\[(A \sin \omega_1 t)(B \sin \omega_2 t) = AB/2. \left[\cos(\omega_1 - \omega_2) t - \cos(\omega_1 + \omega_2) t\right]\]

- A real mixer cannot be driven by arbitrary inputs. Instead one port, the “LO” port, is driven by a local oscillator with a fixed amplitude sinusoid.

- In an up-conversion mixer, the other input is the IF signal and the output is the RF signal, usually for a transmitter.

- In a down-conversion mixer, the other input port is driven by the “RF” signal, and the output is at a lower IF frequency, usually for a receiver.
Mixer + Filter

• \((V_{Lo}\sin \omega_{Lo}t)(V_{RF} \sin \omega_{RF}t) = V_{Lo} \cdot V_{RF}/2 \cdot [\cos(\omega_{Lo} - \omega_{RF})t - \cos(\omega_{Lo} + \omega_{RF})t]\)

• We see that the modulation is indeed translated to two new frequencies, LO+ RF and LO-RF. We usually select either the upper or lower “sideband” by filtering the output of the mixer.

• Note that the LO can be below the RF (lower side injection) or above the RF (high side injection).

• Also note that for a given LO, energy at LO±IF is converted to the same IF frequency. This is a potential problem!
Images

• Two inputs (RF & Image) will mix to the same output (IF) frequency.
• The image frequency must be removed by filtering
• $f_{IF}$ and $f_{LO}$ must be carefully selected
• Image rejection ratio: dB ($P_{IF}$ desired/$P_{IF}$ image)

Even in an ideal multiplier, there are two RF input frequencies ($F_{RF}$ and $F_{IM}$) whose second-order product has the same difference IF frequency.

$$F_{RF} - F_{LO} = F_{LO} - F_{IM} = F_{IF}$$

The two results are equally valid. One is generally referred to as the “image” and is undesired. In the example above, the lower input frequency is designated the image.
Images in a Receiver

- Example: Down-conversion Mixer
  RF = 1000MHz
  IF = 100MHz
Let’s say we choose a low-side injection:
  LO = 900 MHz
That means that any signals or noise at 800 MHz will also be down-converted to the same IF

- The image frequency is the second frequency that also down-converts to the same IF. This is undesirable because the noise and interference at the image frequency can potentially overwhelm the receiver.

- One solution is to filter the image band. This places a restriction on the selection of the IF frequency due to the required filter Q.
Image Rejection – RF Filtering

- Suppose that RF = 1000MHz, and IF = 1MHz. Then the required filter bandwidth is much smaller than 2MHz to knock down the image.

- In general, the filter Q is given by \( Q = \frac{RF}{BW} \)

- Let’s design a filter with \( f_0 = 1000 \) MHz and \( f_{\text{cutoff}} = 1001 \) MHz.

- But the Q for such a filter is \( Q = \frac{10^3 \text{ MHz}}{1 \text{MHz}} = 1000! \)
• The image filtering problem can be relaxed by using multi-IF stages. Instead of moving to such a low IF where the image filtering is difficult (or expensive and bulky), we down-convert twice, using successively lower IF frequencies.

• The IF and LO frequencies must be carefully selected to avoid image frequencies that are too close to the desired RF frequency.
Image Reject Mixer

- An image-rejection mixer could be designed which suppresses one of the input sidebands by phase and amplitude cancellation. This approach requires two mixers and some phase-shifting networks.
Mixer operating mechanisms

- **Nonlinear transfer function**
  - use device nonlinearities
  - useful at mm-wave frequencies
- **Switching or sampling**
  - a time-varying process
  - preferred; fewer spurs

Three techniques have proven to be effective in the implementation of mixers:
1. Use a device that has a known and controlled nonlinearity.
2. Switch the RF signal path on and off at the LO frequency.
3. Sample the RF signal with a sample-hold function at the LO frequency.

The nonlinear mixer can be applicable at any frequency. It is the only approach available at the upper mm-wave frequencies.

When frequencies are low enough that good switches can be built, the switching mixer is preferred because it generates fewer spurs. In some cases, sampling has been substituted for switching.
Nonlinear Characteristics

- Production of a new frequency requires a nonlinear device
- Two common semiconductor nonlinear characteristics
  - PN junction diodes or bipolar junction transistors
  - Field effect transistors

The nonlinearity can be expressed as a Taylor series:

\[ I_{out} = I_0 + a[V_i(t)] + b[V_i(t)]^2 + c[V_i(t)]^3 + \ldots \]

Taking the squared term:

\[ b(V_1 + V_2)^2 = b(V_1^2 + 2V_1V_2 + V_2^2) \]

Only the second-order product term produces the desired output.
Single-Ended Switching mixers

Let $V_{IN}(t) = V_R \cos (\omega_{RF} t)$ multiply by the LO switching function $T(t)$

This simple switch is operated by the LO. If the LO is a square wave with 50% duty cycle, it is easily represented by its Fourier Series. The symmetry causes the even-order harmonics to drop out of the LO spectrum.

When multiplied by a single frequency cosine at $\omega_{RF}$ the desired sum and difference outputs will be obtained. There will be harmonics of the LO present at $3\omega_{LO}, 5\omega_{LO}$, etc. that will also mix to produce outputs called “spurs” (an abbreviation for spurious signals). **Note that everything is single-ended; there is no balancing on this design.**
The product of $V_{RF}(t)T(t)$ produces the desired output frequencies at RF-LO and RF+LO from the second order product. Odd harmonics of the LO frequency are present since we have a square wave LO switching signal. These produce spurious 4th, 6th, … products with outputs at $n \omega_{LO} - \omega_{RF}$ and $n \omega_{LO} + \omega_{RF}$ where $n$ is odd.

We also get RF feedthrough directly to the output. None of the LO signal should appear in the output if the mixer behaves according to this equation. But, if there is a DC offset on the RF input, there will be a LO frequency component in the output as well. Many mixer implementations require some bias current which leads to a DC offset on the input.
Unbalanced diode mixer output

We can see that there are a lot of spurious outputs generated. Ideally, we would like to see outputs only at 10 MHz and 210 MHz.

\[ F_{RF} = 110 \text{ MHz} \quad |V_{RF}| = 0.1V \]
\[ F_{LO} = 100 \text{ MHz} \quad |V_{LO}| = 0.2V \]
Conversion gain or loss

conversion gain or loss

Generally expressed as a voltage gain or as a transducer power gain

Conversion gain is usually defined as the ratio of the IF output power to the available RF power. If the source and load impedances are different, the power gain must account for this.

\[
A_v = \frac{V_{IF}}{V_{IN}}
\]

\[
\text{ConvGain} = \frac{\text{Output power at } F_{IF}}{\text{RF available input power}} = \frac{v_{IF}^2}{2R_L} \frac{\sin[\omega_{RF}t]}{8R_S}
\]

We see that the simple switching mixer has low conversion gain because the voltage gain \( A_v \) is only \( 1/\pi \). Also, the RF feedthrough problem and in most instances, an LO feedthrough problem exist. All of these deficiencies can be improved by the use of balanced topologies which provide some cancellation of RF and LO signals as well as increasing conversion gain.
The RF feedthrough can be eliminated by using a differential IF output and a polarity reversing LO switch.
LO Switching Function $T(t)$

\[ T_1(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \sin(\omega_{LO}t) + \frac{1}{3} \sin(3\omega_{LO}t) + \ldots \right] \]

\[ T_2(t) = -\frac{1}{2} + \frac{2}{\pi} \left[ \sin(\omega_{LO}t) + \frac{1}{3} \sin(3\omega_{LO}t) + \ldots \right] \]

\[ T(t) = T_1(t) + T_2(t) \]

- When added together, the DC terms cancel. The DC term was responsible for the RF feedthrough in the unbalanced mixer since the $\cos(\omega_{RF}t)$ term was multiplied only by $T_1(t)$.

\[ V_{IF}(t) = g_m R_L V_R \cos(\omega_{RF}t) \frac{4}{\pi} \left[ \sin(\omega_{LO}t) + \frac{1}{3} \sin(3\omega_{LO}t) + \frac{1}{5} \sin(5\omega_{LO}t) + \ldots \right] \]

Second-order term: \[ \frac{2g_m R_L V_R}{\pi} \left[ \sin(\omega_{RF} + \omega_{LO})t + \sin(\omega_{RF} - \omega_{LO})t \right] \]

Here we see that the ideal conversion gain $(V_{IF}/V_{RF})^2 = (2/\pi)^2$ is 6 dB greater than for the unbalanced design (if $g_m R_L = 1$).
LO Feedthrough

\[ V_{IF}(t) = R_L[I_{DC} + g_m V_R \cos(\omega_{RF}t)] \times \]
\[ \frac{4}{\pi} \left[ \sin(\omega_{LO}t) + \frac{1}{3} \sin(3\omega_{LO}t) + \frac{1}{5} \sin(5\omega_{LO}t) + \ldots \right] \]

\[ = \frac{4R_L}{\pi} \{ I_{DC} \sin(\omega_{LO}t) + \]
\[ \frac{1}{2} g_m V_R \left[ \sin(\omega_{RF} + \omega_{LO})t + \sin(\omega_{RF} - \omega_{LO})t \right] \}

But, we can still get LO feedthrough if we take a single-ended output or if there is a DC current in the signal path. There is often DC present since the output of the transconductance amplifier will have a DC current component.
As you can see, the output spectrum of the single-balanced switching mixer is much less cluttered than the nonlinear mixer spectrum.

- Transient analysis using an ideal switch.
- The behavioral switch model has an on-threshold of 2V and an off-threshold of 1V.
- The LO was generated with a 4V pulse function and the duty cycle was set to 50%.
- The output is taken differentially as $V_{IF} = V1 - V2$.

Note the strong LO feedthrough component in the output. This is present because of the DC offset on the RF input.
An ideal double balanced mixer consists of
(1) a switch driven by the local oscillator that reverses the polarity of the RF input at the LO frequency
(2) a differential transconductance amplifier stage.

- The polarity reversing switch and differential IF cancels any output at the RF input frequency since the DC term cancels.
- The double LO switch cancels out any LO frequency component.
Double-balanced mixer

Two single-balanced mixers – difference cancels LO feedthrough

\[
V_o = R_L \left[ I_{DC} + I_R \cos(\omega_{RF}t) \right] \frac{4}{\pi} \left[ \sin(\omega_{LO}t) + \frac{1}{3} \sin(3\omega_{LO}t) + \cdots \right] - \\
R_L \left[ I_{DC} - I_R \cos(\omega_{RF}t) \right] \frac{4}{\pi} \left[ \sin(\omega_{LO}t) + \frac{1}{3} \sin(3\omega_{LO}t) + \cdots \right]
\]

From - gm side of mixer (desired output adds)

Single balancing got rid of the RF feedthrough which was caused by the average DC value of the switching function.

Double balancing removes the LO feedthrough as well, since the DC term cancels.
Output Spectrum: DB mixer

There is no LO or RF feedthrough in this ideal DB mixer.

In real mixers, there is always some imbalance. Transistors and baluns are never perfectly matched or balanced. These nonidealities will produce some LO to IF or RF to IF feedthrough (thus, isolation is not perfect).

Secondly, the RF to IF path is not perfectly linear. This will lead to intermodulation distortion. Odd-order distortion (typically third and fifth order are most significant) will cause spurs within the IF bandwidth or cross-modulation when strong signals are present. Also, the LO switches are not perfectly linear, especially while in the transition region. This can add more distortion to the IF output and will increase loss due to the resistance of the switches.
Mixer Performance Specifications

• Image rejection √
• Conversion gain: voltage or power √
• Port-to-port isolation:
• Large signal performance:
  – gain compression: \( P_{1\text{dB}} \)
  – intermodulation distortion spec: third-order intercept (TOI)
• Small signal performance: noise figure
• Operating range: Spurious-free dynamic range
Isolation between ports

- The mixer is not perfectly unilateral leakage between:
  - LO to IF = LO power at LO port / LO power at IF port
  - LO to RF = LO power at LO port / LO power at RF port
  - RF to IF = RF power at RF port / RF power at IF port
- Determine the magnitude of these leakage components at the IF and RF ports using harmonic balance.
- Use the mix function to select frequencies.

Isolation can be quite important for certain mixer applications. For example, LO to RF leakage can be quite serious in direct conversion receiver architectures because it will remix with the RF and produce a DC offset.
Gain Compression

- Gain compression is a useful index of distortion generation. It is specified in terms of an input power level at which the small signal conversion gain drops off by 1 dB.
- Conversion gain degrades at large input signal levels due to nonlinearity in the signal path. Assume a simple nonlinear transfer function:

\[ V_{RF}(t) = v_{in} - a_3 v_{in}^3 \]

\[ V_{RF}(t) = V_R \left( 1 - \frac{3a_3 V_R^2}{4} \right) \sin(\omega_{RF} t) + \frac{1}{4} a_3 V_R^3 \sin(3\omega_{RF} t) \]

The example above assumes that a simple cubic function represents the nonlinearity. When we substitute \( v_{in}(t) = V_R \sin(\omega_{RF} t) \) and use trig identities, we see a term that will produce gain compression as the first term. If we knew the coefficient \( a_3 \), we could predict the 1 dB compression input voltage. Typically, we obtain this by measurement of gain vs. input voltage.

We also see a cubic term that represents the third-order harmonic distortion (HD). It is the intermodulation distortion that results from multiple signals that is more troublesome.

Note that in this simple example, the fundamental is proportional to \( V_R \) whereas the third-order HD is proportional to \( V_R^3 \). Thus, if \( P_{out} \) vs. \( P_{in} \) were plotted on a dB scale, the HD power will increase at 3 times the rate that the fundamental power increases with input power.
Gain Compression: $P_{1\text{dB}}$

The RF mixer in ADS has been used to illustrate the gain compression phenomenon. Note that the IF is selected by using the mix function. In this example, LO\_freq = 855 MHz and RF\_freq = 900 MHz. If we are interested in the downconverted IF frequency, 45 MHz, we can select it from: IF\_pwr = dbm(mix(Vout,{-1,1})). The indices in the curly brackets are ordered according to the HB fundamental analysis frequencies. Thus, {-1,1} selects -LO\_freq+RF\_freq.

On the left, we see the simulated IF\_gain against RF\_pwr. On the right, two traces are shown. (1) IF\_pwr at each RF\_pwr and (2) a line that extrapolates the linear value. Insert an equation for the Line = RF\_pwr+gain[0]. Here we can identify $P_{1\text{dB}} = -31$ dBm.

The gain compression power characterization provides a good indication of the signal amplitude that the mixer will tolerate before really bad distortion is generated.
Use Harmonic Balance Simulation

- Harmonic balance is the method for simulation of mixers. By specifying the number of harmonics for the LO and RF input frequencies and the maximum order (highest order of sums and differences), you get the frequency domain result of the mixer at all relevant frequencies. Maximum order corresponds to the highest order mixing product \((n + m)\) to be considered \((\pm nf[1] \pm mf[2])\). The simulation will run faster with lower order and fewer harmonics of the sources, but may be less accurate. You should test this by checking if the result changes significantly as you increase order or number of harmonics.

- The frequency with the highest power level (the LO) is always the first frequency in the harmonic balance controller. Other inputs follow from highest to lowest power.

- Highest order of IM products
- Fundamental Frequencies (Put highest power source first)
- Number of harmonics of sources [1] and [2]
- Sweep RF\_pwr from -50 dbm to -20 dbm in 1 db step
Intermodulation Distortion

Let’s consider again the cubic nonlinearity. When two inputs at $\omega_1$ and $\omega_2$ are applied simultaneously to the RF input, the cubing produces many terms, some at the harmonics and some at the IMD frequency pairs.

We will be mainly concerned with the third-order IMD. This is especially troublesome since it can occur at frequencies within the IF bandwidth. For example, suppose we have 2 input frequencies at 899.99 and 900.01 MHz. Third order products at $2f_1 - f_2$ and $2f_2 - f_1$ will be generated at 899.97 and 900.03 MHz. Once multiplied with the LO frequency, these IMD products may fall within the filter bandwidth of the IF filter and thus cause interference to a desired signal. IMD power will have a slope of 3 on a dB plot.

In addition, the cross-modulation effect can also be seen. The amplitude of one signal (say $\omega_1$) influences the amplitude of the desired signal at $\omega_2$. This cross-modulation can have error generating effects at the IF output.
IMD simulation

The IMD simulation is performed with a two-tone generator at the RF input. The frequency spacing should be small enough so that both fall within the IF bandwidth. You should keep in mind that both of the generator tones are in phase, therefore the peak voltage will add up periodically to twice the peak of each source independently. **Because of this, you will expect to see some reduction in the P1dB on the order of 6 dB.**

Often accurate IMD simulations will require a large maximum order and LO harmonic order when using harmonic balance. In this case, a larger number of spectral products will be summed to estimate the time domain waveform and therefore provide greater accuracy. This will increase the size of the data file and time required for the simulation. Increase the orders and watch for changes in the IMD output power. When no further significant change is observed, then the order is large enough.
Note that the third-order \((m3)\) and fifth-order products are quite close in frequency to the desired signal \((m1)\). This means that they are often impossible to remove by filtering.

The two IMD sidebands should be approximately of equal power if the simulation is correct. If not, increase the order of the LO in the HB controller and see if this makes the sidebands more symmetric.
A widely-used figure of merit for IMD is the third-order intercept (TOI) point. This is a fictitious signal level at which the fundamental and third-order product terms would intersect. In reality, the intercept power is 10 to 15 dBm higher than the $P_{1dB}$ gain compression power, so the circuit does not amplify or operate correctly at the $IIP3$ input level. The higher the TOI, the better the large signal capability of the mixer.

It is common practice to extrapolate or calculate the intercept point from data taken at least 10 dBm below $P_{1dB}$. One should check the slopes to verify that the data obeys the expected slope = 1 or slope = 3 behavior. In this example, we can see that this is true only at lower signal power levels. $OIP3 = (P_{IF} - P_{IMD})/2 + P_{IF}$. Also, the input and output intercepts are simply related by the gain: $OIP3 = IIP3 + \text{conversion gain}$.
Measuring IMD Performance

- Set the amplitude of generators at $f_1$ and $f_2$ to be equal.
- Start at a very low input power using the variable attenuator, then increase power in steps until you begin to see the IMD output on the spectrum analyzer.
- The resolution bandwidth should be narrow so that the noise floor is reduced. This will allow visibility of the IMD signal at lower power levels.
- Plot the IMD power vs. input power and verify that the slope is close to 3. Then, you can calculate the IP3 as described previously.

- $P_{\text{in}}$ must be more than 10 dB below $P_{1\text{dB}}$
- Plot data to confirm slope = 3 on IMD
- Use narrow RBW on SA to reduce noise floor
Noise figure & SFDR

- We have been concentrating on the large signal limitations of the mixer. Noise determines the other end of the mixer dynamic range.
- Spurious-free dynamic range: 
  \[ \text{SFDR} = \frac{2}{3} [\text{IIP3} - (10 \log(kT\Delta f) + \text{NF})] \]

The minimum detectable signal (MDS) power is determined by noise and corresponds to a signal whose strength just equals the noise. MDS (dBm) = 10 log(kT\Delta f) + NF. Noise figure (NF) is defined as the ratio between the input and output S/N ratio. The thermal noise power in bandwidth \( \Delta f \) is 10 log(kT\Delta f). The maximum signal power is limited by distortion, which we describe by IIP3.

The spurious-free dynamic range (SFDR) is a commonly used figure of merit to describe the dynamic range of an RF system. The SFDR is calculated from the geometric 2/3 relationship between the IIP3 and the MDS. It is important to note that the SFDR depends directly on the bandwidth \( \Delta f \). It has no meaning without specifying bandwidth.