

**Computational Methods
for Electromagnetic Inverse Scattering**

Computational Methods for Electromagnetic Inverse Scattering

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To Lin, Yuexin, Yide, and my parents.

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Foreword

I am thankful to Dr. Xudong Chen for asking me to write a Foreword to his book on *Computational Methods for Electromagnetic Inverse Scattering*. This book comes at an opportune time as the field of inverse scattering has been studied for several decades now. I feel that this field is about to enter a new era, just as the field of artificial intelligence has evolved in the last three decades. To recount the history of artificial intelligence briefly, it started out as a field in computer science to emulate human intelligence with computers. However, to emulate human intelligence with the computers of three decades ago was a tall order. Very quickly, the field evolved to a less ambitious goal of developing expert systems to replace humans. Expert systems found applications in many machines that can perform quasi-intelligent menial tasks for humans. When the field of artificial neural networks was conceived, it again aroused much excitement in the computer science community: it portended great potential for machines to emulate the inner workings of the human brain. However, the excitement period subsided gradually, as many of the algorithms were too slow, and it was too difficult and time consuming to train neural networks of high complexity. Nevertheless, neural networks re-emerged later in the new field of machine learning. This was especially significant when machines were trained to beat humans in a game as complicated as the ancient oriental board game *go* in Japanese, or *weiqi* (*weichi* in Wade–Giles phonetics) in Chinese.

Three main reasons precipitate this breakthrough in artificial intelligence: (1) Computers have become at least 10 million times faster in the last three decades. (2) Computer memories are a lot cheaper compared to three decades ago, due to the compounding effect of Moore's Law. (3) Algorithms for information propagation through neural nets have become cleverer and faster.

Inverse scattering is facing the same juncture at this point as it shares many similar features with artificial intelligence; for instance, one of the bottle-necks of the inverse scattering algorithm is its computational cost or labor. But after several decades, computer technologies have grown a lot more powerful and cheaper. The clever use of modern computer technologies in massively parallel computations, the use of *a priori* data in inverse scattering and imaging, and

the development of compressive sensing knowledge can be the game changers in this field. Moreover, the dogged pursuit of more efficient inverse scattering algorithms by many researchers makes the time ripe for this field to undergo a major revolution, as has been witnessed in the field of artificial intelligence.

Another reason that this field has become very interesting is that it is a field that is highly inter-disciplinary, drawing upon knowledge from mathematics, wave physics, and signal processing, as well as computer science. The confluence of various forms of knowledge and their judicious synergy are important to stimulate the next generation of technology that can follow from inverse scattering: for instance, in various forms of imaging, detection, and identification applications. This book will become an excellent resource for researchers and students who wish to learn the relevant knowledge needed for studying inverse scattering and related topics. Dr. Chen has started from the fundamentals of electromagnetic scattering theory and guides the readers slowly into the advanced form of scattering and inverse scattering theory. He also gives comprehensive coverage of the major inverse scattering techniques, plus pertinent signal processing methods. It is pleasing to see that both perfect electric conductor inversion and dielectric object inversion are discussed, as well as the complicated case when the background is inhomogeneous. Small-scatterer inversion is discussed alongside with large-scatterer inversion. The issue of phaseless imaging (or reconstruction) as well as imaging with phase information have been discussed. Phase imaging has been done at microwave frequency but is becoming increasingly popular at optical frequency as optical measurements become more precise. The manner the book is organized makes this knowledge accessible to researchers who are not in mainstream electromagnetic physics. Also, topics are added to ease the learning of computational mathematics and signal processing.

In summary, Dr. Chen should be lauded for spending the effort to write this book, which will become an important resource for researchers and students in this field.

September 2017

Weng Cho Chew
Purdue University

Preface

This book is dedicated to presenting computational methods for solving electromagnetic inverse scattering problems. The intended audience includes graduate students and researchers in electrical engineering and physical sciences who are interested in inverse scattering and related imaging or who may encounter this subject in their work. Researchers in applied mathematics might also find the book useful.

There are two main reasons that motivated me to write this monograph. First, despite the fact that a rapidly expanding number of research articles on inverse scattering have been published, thanks to its wide range of real-world applications as well as the availability of powerful and cheaper computational resources, few research textbooks have been written on the subject. In particular, there has not yet been a book dedicated to solving electromagnetic inverse scattering problems without making linearization approximations. The lack of a suitable reference book has been an inconvenience for many researchers who are either in this area or are interested in entering into this subject. Second, although progress in the research into inverse scattering would not be possible without the confluence of various forms of knowledge, researchers in the engineering community usually have little knowledge on the theories and tools that have been developed in the applied mathematical community. Although there are excellent textbooks on the topic in applied mathematics, these books are usually inaccessible to engineering readers due to a lack of sufficient training in mathematics.

Based on my research experiences in the subject during 2006–2016, I wrote this monograph, keeping in mind these two concerns. The book mainly addresses inverting exact wave equations, without making linearization approximations, which results in a highly nonlinear problem. The book is written in such a way that it presents the following features:

- 1) Most of the major inversion algorithms are reviewed and, in particular, their strengths and weakness are discussed, as well as their relationships to other algorithms.

- 2) Important mathematical concepts, such as existence, uniqueness, and stability, are introduced. A general introduction to ill-posed problems and regularization is provided in the Appendix. Some inversion algorithms that prevail in the applied mathematical community are also introduced, such as the well-established linear sampling method. All these mathematical topics are presented in a way accessible to engineering readers.
- 3) The book is highly oriented to the practical implementation of algorithms. The details of solving the forward problem and the implementation steps of individual inversion algorithms are presented such that readers can practice them without a long learning curve. Along the same pragmatic direction, several important tools are provided in Appendices.

To summarize, the book presents inverse scattering for an engineering audience in a well-balanced way; that is, emphasizing pragmatism of computational methods but still with the right formal rigor.

Keeping in mind that the research into the inverse problem requires a deep or fairly good understanding of the corresponding forward problem, I always hesitate to directly apply a general optimization method to a high-dimensional nonlinear problem, where the original forward problem is iteratively evaluated. I am convinced that insights and intuitions, no matter whether they are mathematical, physical, or engineering, potentially help us to solve the problem in a more efficient and elegant way. In inverse scattering problems, induced source plays an essential role. The analysis of induced source, such as its degrees of freedom, multipole expansion, Fourier series, and expansion with respect to singular vectors, provides deep insights into solving inverse scattering problems, which is demonstrated throughout this book.

Supplementary materials, such as the MATLAB m-files used to generate many of the examples and figures, can be found on my personal website. These materials help readers make rapid progress in learning the subject and comparing the various solution methods.

I am indebted to my Ph.D. supervisor Professor Jin Au Kong who taught me electromagnetic wave theory and to my Masters supervisors Professor Guangzheng Ni and Professor Shiyong Yang who introduced me to the field of optimization and taught me the importance of physical insight. Their passion and enthusiasm in teaching greatly influenced my view on education. I am very grateful to Professor Weng Cho Chew who was so generous in writing the Foreword to the book and provided me with valuable suggestions on my writing. The depth and width of his knowledge, as well as his interest in learning whenever and wherever possible, have deeply impressed and influenced me. I would like to thank my close collaborators Dr. Dominique Lesselier, Professor Colin Sheppard, Professor Lixin Ran, and Professor Zhi-Xun Shen, together with whom I worked on various inverse problems and imaging projects. I appreciate my friendship with many mathematicians; in particular, Professor

Gunther Uhlmann, Professor Jun Zou, Professor Hongkai Zhao, Professor Jenn-Nan Wang, and Professor Gen Nakamura, who have helped me in various ways, taught me mathematics, and influenced my style of research.

I have been very fortunate to work with brilliant Ph.D. students and postdoctoral fellows on this subject, in particular, Yu Zhong, Krishna Agarwal, Li Pan, Xiuzhu Ye, Rencheng Song, Rui Chen, and Zhun Wei. Dr. Zhong and Dr. Agarwal, my first two Ph.D. students, started working on inverse scattering almost at the same time as I did. I cherish the time and effort we spent together in embarking on a new journey in inverse scattering. Special thanks go to Dr. Wei and Dr. Chen who generated many of the figures and provided a lot of editorial assistance to the book. I would also like to thank Dr. Maokun Li, who read most of chapters and provided many suggestions for improvements.

Finally, I am deeply grateful to my wife, Lin, my children, Yuexin and Yide, and my parents, for their tremendous support, patience and love during this project.

September 2017, Singapore

Xudong Chen