Bio-inspired sensing and imaging

Habib Ammari

Department of Mathematics, ETH Zürich
Bio-inspired sensing and imaging

- Mimic electrolocation by weakly electrical fish and echolocation by bats.
- Enhance the resolution, the robustness, and the specificity of tissue property imaging modalities.

Long-nosed elephant fish  Long-eared bat
Bio-inspired sensing and imaging

- **Biological vision**: two types of retina-brain pathways in the visual system.
  - Transient magno-cellular pathway and the sustained parvo-cellular pathway.
  - **Magno-system**: sensitive to changes and movements; detect dangers that arise in the peripheral vision.
Bio-inspired sensing and imaging

- Key concepts:
  - **Resolution**: smallest distance between two point reflectors that can be resolved; limited by half the operating wavelength.
  - **Robustness**: stability of the image formation with respect to model uncertainty and medium and electronic noises.
  - **Specificity**: physical nature (for tumors: benign or malignant).
Tissue property imaging

- Tissue property imaging: electromagnetic and elastic waves play a key role in visualizing contrast information on the electrical, optical, mechanical properties of tissues.

- Tissue contrasts:
  - Highly sensitive to physiological and pathological tissue status.
  - Depend on the cell organization and composition.
  - Overall parameters, averaged in space over many cells.

- Recognize the microscopic cell organization and composition from measurements at the macroscopic level.
Tissue property imaging

- **Diagnosis** and **staging** of cancer disease.
- Help surgeons to make sure they removed everything unwanted around the **margin** of the cancer tumor.
- Perform **biopsy** in the operating room.
Tissue property imaging

• Electrical tissue properties:
  • Electrical conductivity: tissue’s ability to transport charges;
  • Electrical permittivity (dielectric constant): tissue’s ability to
    trap or to rotate molecular dipoles; determined by the
    polarization under an external electric field;
  • Frequency-dependent or dispersive; anisotropic;
  • Capacitive effect generated by the cell membrane structure;
  • Macroscopic parameters; represent the electrical properties of
    the tissue averaged in space over many cells.
Bio-inspired dictionary matching based approach

- Electrolocation for weakly electric fish\(^1\):
  - **Electric organ**: generates a stable, high-frequency, weak electric field.
  - **Electroreceptors**: measure the transdermal potential modulations caused by a nearby target.
  - **Nervous system**: locates the target, perceives its shape, determines its physical nature.

Shape perception

Mechanism for mimicking shape perception:

• Form an image from the perturbations of the field due to targets.
• Identify and classify the target, knowing by advance that it belongs to a learned dictionary of shapes.
  • Extract the features from the data.
  • Construct invariants with respect to rigid transformations and scaling.
  • Compare the invariants with precomputed ones for the dictionary.
• Biological targets: frequency dependent electrical properties (capacitive effect generated by the cell membrane structure).
• ⇒ Spectroscopic measurements of the target's polarization tensor.
Weakly electric fish

- **Wave-type electric signal**: $f(x, t) = f(x) \sum_n a_n e^{in\omega_0 t}; \omega_0$: fundamental frequency.

- **Skin**: very thin ($\delta \sim 100\mu m$) and highly resistive ($\sigma_s/\sigma_0 \sim 10^{-2}$); $\sigma_b/\sigma_0 \sim 10^2$ (highly conductive).
Weakly electric fish

\begin{itemize}
  \item Target \( D = z + \delta' B \); \( z \): location; \( \delta' \): characteristic size of the target; 
  \( k(\omega) = (\sigma(\omega) + i \omega \varepsilon(\omega))/\sigma_0 \); \( k, \sigma, \) and \( \varepsilon \): the admittivity, the conductivity, and the permittivity of the target; \( \omega_n = n\omega_0 \): the probing frequency.
  \item \( u_n \): the electric potential field generated by the fish:
    \[
    \begin{cases}
    \Delta u_n = f, & x \in \Omega, \\
    \nabla \cdot (1 + (k(\omega_n) - 1)\chi(D))\nabla u_n = 0, & x \in \mathbb{R}^2 \setminus \overline{\Omega}, \\
    \frac{\partial u_n}{\partial \nu} \big|_+ = 0, & x \in \partial \Omega, \\
    [u_n] = \xi \frac{\partial u_n}{\partial \nu} \big|_+ \\
    |u_n(x)| = O(|x|^{-1}), & |x| \to \infty.
    \end{cases}
    \]
  \item \( \xi := \delta \sigma_0 / \sigma_s \) effective thickness.
  \item \( \lambda(\omega_n) = (k(\omega_n) + 1)/(2(k(\omega_n) - 1)) \).
\end{itemize}
Weakly electric fish

- **Dipole approximation:** \( u_n(x) - U(x) \simeq \mathbf{p} \cdot \nabla G(x - z) \).
  - \( G \): Green’s function associated to Robin boundary conditions.
  - **Dipole moment** \( \mathbf{p} = - M(\lambda(\omega_n), D) \nabla U(z) \).

**Polarization tensor**

- **Neumann-Poincaré operator:**
  \[
  \mathcal{K}_D^*[\varphi](x) = \frac{1}{2\pi} \int_{\partial D} \frac{\langle x - y, \nu_x \rangle}{|x - y|^2} \varphi(y) \, ds(y), \quad x \in \partial D.
  \]

- **Matrix** \( M(\lambda(\omega_n), D) = \int_{\partial D} x(\lambda(\omega_n)I - \mathcal{K}_D^*)^{-1}[\nu](x) \, ds(x) \).
Weakly electric fish

- $\mathcal{K}_D^*$: compact operator on $L^2(\partial D)$; Spectrum of $\mathcal{K}_D^*$ lies in $(-\frac{1}{2}, \frac{1}{2}]$ (Kellog).

- Spectral decomposition formula in $H_0^{-1/2}(\partial D)$,

$$\mathcal{K}_D^*[\psi] = \sum_{j=0}^{\infty} \lambda_j(\psi, \varphi_j)_{\mathcal{H}^*} \varphi_j.$$ 

- $(\lambda_j, \varphi_j), j = 0, 1, 2, \ldots$: eigenvalue and normalized eigenfunction pair of $\mathcal{K}_D^*$ in $\mathcal{H}^*(\partial D)$; $\lambda_j \in (-\frac{1}{2}, \frac{1}{2}]$ and $\lambda_j \to 0$ as $j \to \infty$;

- $\mathcal{H}^*(\partial D) = H_0^{-\frac{1}{2}}(\partial D)$ equipped with

$$(u, v)_{\mathcal{H}^*} = -(u, S_D[v])_{-\frac{1}{2}, \frac{1}{2}}; \quad S_D : \text{single layer potential}.$$
Weakly electric fish

• **Space-frequency response matrix**: \((V_{sr}^n)_{rn}\)

\[ V_{sr}^n = \left( \frac{\partial u_n}{\partial \nu}(x_r) \bigg\rvert_+ - \frac{\partial U}{\partial \nu}(x_r) \bigg\rvert_+ \right), \]

\(x_s\): position of the electric organ; \((x_r)\): receptors on the skin of the fish.

• **Space-frequency location search algorithm**.

• **Movement**: Fish takes measurement at different positions around the target \(\Rightarrow\) can use only one frequency.
Weakly electric fish

- Dipole approximation:

\[ V_{sr}^n \simeq -\nabla U(z) \cdot M(\lambda(\omega_n), D) \cdot \left( \nabla \frac{\partial G}{\partial \nu_x} (x_r - z) \right); \]

- \( z^S \) in the search domain; Vector field \( g(z^S) \) given by

\[ \left( \nabla U(z^S) \cdot \nabla \left( \frac{\partial G}{\partial \nu_x} \right) (x_1 - z^S), \ldots, \nabla U(z^S) \cdot \nabla \left( \frac{\partial G}{\partial \nu_x} \right) (x_L - z^S) \right)^T; \]

- Subspace imaging functional:

\[ \mathcal{I}(z^S) := \frac{1}{| (I - P) g(z^S) |}; \]

\( P \): orthogonal projection onto the first singular vector of \((V_{sr}^n)_n; \)

- \( \mathcal{I}(z^S) \): large peak at \( z^S = z \).
Weakly electric fish

Number of frequencies: 10; number of receptors: 64.

- $\sigma, \varepsilon$: determined by minimizing a quadratic misfit functional.
Dictionary matching based approach
Weakly electric fish

- **Multi-frequency** approach: $\omega \mapsto M(\lambda(\omega), D)$.
  - Invariance with respect to **translation**, **rotation**, and **scaling**.
  - $\tau_j(\omega)$: eigenvalues of $\Im m M(\lambda(\omega), D)$; $\omega_\infty$: highest probing frequency. Plot
    $$\omega \mapsto \frac{\tau_j(\omega)}{\tau_j(\omega_\infty)},$$
    for $j = 1, \ldots, d$. 
Dictionary matching based approach

Probability of detection in terms of the noise level. Stability of classification based on differences between ratios of eigenvalues of $\Im m M(\lambda(\omega), D)$. 

Bio-inspired sensing and imaging

Habib Ammari
Weakly electric fish

Nonbiological targets (frequency-independent electrical parameters):

- Use multipolar approximation:

\[ u_n(x) - U(x) \approx \sum_{\alpha, \beta} \partial^\alpha G(x - z) M_{\alpha\beta}(\lambda, D) \partial^\beta U(z). \]

- \( M_{\alpha\beta}(\lambda, D) \): high-order polarization tensors.

\[ M_{\alpha\beta}(\lambda, D) := \int_{\partial D} x^\beta (\lambda I - K_D^*)^{-1} [\partial x^\alpha / \partial \nu](x) \, ds(x). \]
Weakly electric fish

Properties of high-order polarization tensors:

- Recover high-frequency information on the shape;
- Separate topology;
- Determine uniquely the shape and the material parameter.
Weakly electric fish

- Positivity and symmetry properties on harmonic coefficients; optimal bounds.
- Harmonic coefficients:
  \[(x_1 + ix_2)^m = \sum_{|\alpha|=m} a_{\alpha}^{m} x^{\alpha} + i \sum_{|\beta|=m} b_{\beta}^{m} x^{\beta} .\]
- Translation, rotation, and scaling formulas.
- Construct shape descriptors invariant with respect to translation, rotation, and scaling.
Weakly electric fish

- Reconstruction of high-order polarization tensors from the data by a least squares method.
- Instability:
  \[ M_{\alpha\beta}(k, D) = O(|D|^{\alpha+\beta+d^{-2}}), \partial^\alpha G(x - z) = O(|x|^{-\alpha})(|x| \to +\infty). \]
- Resolving power = number of high-order polarization tensors reconstructed from the data: depends on the signal-to-noise ratio (SNR) in the data.
- \( \epsilon = \) characteristic size of the target/ the distance to the fish.
- \( \text{SNR} = \epsilon^2/\)standard deviation of the measurement noise (Gaussian).
- Formula for the resolving power \( m \) as function of the SNR:
  \[ (m\epsilon^{1-m})^2 = \text{SNR}. \]
Weakly electric fish

Stability of classification based on Shape Descriptors.
Spectroscopic electrical tissue property imaging

- Differentiate between normal, pre-cancerous and cancerous tissues from electrical measurements at tissue level.
- Frequency dependence of the (anisotropic) homogenized admittivity: $\omega \mapsto K^*(\omega)$.
- Relaxation times:
  - $1/\arg \max_\omega$ eigenvalues of $\Im m K^*(\omega)$;
  - Classification: invariance properties;
  - Measure of anisotropy: ratios of the eigenvalues of $\Im m K^*(\omega)$.
Spectroscopic electrical tissue property imaging

The effective admittivity of a periodic dilute suspension\(^2\):

\[ K^* = k_0 \left( I + f M \left( I - \frac{f}{2} M \right)^{-1} \right) + o(f^2). \]

- \( f \): volume fraction; \( \xi \): effective thickness of the membrane; \( \partial D \): cell membrane; \( \tilde{D} = D / \sqrt{f} \): rescaled cell.
- \( M \): membrane polarization tensor

\[ M = - \left( \xi \int_{\partial \tilde{D}} \nu_j (I + \xi L_{\tilde{D}})^{-1} [\nu_i](y) ds(y) \right)_{i,j=1,2}. \]

- \( L_{\tilde{D}}[\varphi](x) = \frac{1}{2\pi} \text{p.v.} \int_{\partial \tilde{D}} \frac{\partial^2 \ln |x - y|}{\partial \nu(x) \partial \nu(y)} \varphi(y) ds(y), \quad x \in \partial \tilde{D}. \)

Spectroscopic electrical tissue property imaging

- Properties of the membrane polarization tensor:
  - $M$: symmetric; invariant by translation;
  - $M(sC, \xi) = s^2 M(C, \frac{\xi}{s})$ for any scaling parameter $s > 0$.
  - $M(\mathcal{R}C, \xi) = \mathcal{R} M(C, \xi) \mathcal{R}^t$ for any rotation $\mathcal{R}$.
  - $\Im M$ is positive and its eigenvalues, $\lambda_1 \geq \lambda_2$, have one maximum with respect to $\omega$.

- Relaxation times for the arbitrary-shaped cells:
  \[
  \frac{1}{\tau_i} := \arg \max_\omega \lambda_i(\omega).
  \]

- $\tau_i$ for $i = 1, 2$: invariant by translation, rotation and scaling.

- Concentric circular-shaped cells: Maxwell-Wagner-Fricke formula ($\lambda_1 = \lambda_2$).

- Nondilute regime: Assume $f$ known $\Rightarrow$ Classification based on relaxation times.
• Dictionary matching based approach for target classification in echolocation\(^3\).

• \(u^i\): incident wave; \(\kappa\): bulk modulus; \(\rho\): density. Helmholtz equation:

\[
\begin{align*}
\nabla \cdot \left( \chi(\mathbb{R}^2 \setminus \bar{D}) + \frac{1}{\rho} \chi(D) \right) \nabla u + \omega^2 \left( \chi(\mathbb{R}^2 \setminus \bar{D}) + \frac{1}{\kappa} \chi(D) \right) u = 0,
\end{align*}
\]

\(u^s := u - u^i\) satisfies the outgoing radiation condition.

Bats

- **Scattering coefficients**:  
  \[ W_{mn}(D, \kappa, \rho, \omega) = \int_{\partial D} \psi_m(y) J_n(\omega |y|) e^{-i \theta y} \, ds(y). \]

- **\( \psi_m \)**:  
  \[ J_m(\omega |x|) e^{im \theta x} + S_D^\omega [\psi_m] \quad x \in \mathbb{R}^d \setminus \overline{D}; \]

  *cylindrical wave*

- **\( J_m \)**: *Bessel* function.

---

Bats

Properties of the scattering coefficients:

- $W_{mn}$ decays rapidly:

\[
|W_{mn}| \lesssim \frac{C|m|+|n|}{|m||m||n||n|}, \quad m, n \in \mathbb{Z}.
\]

- For any $z \in \mathbb{R}^2, \theta \in [0, 2\pi), s > 0$,
  - Translation:
    \[
    W_{mn}(D^z) = \sum_{m', n' \in \mathbb{Z}} J_{n'}(\omega |z|) J_{m'}(\omega |z|) e^{i(m'-n')\theta z} W_{m-m', n-n'}(D);
    \]
  - Rotation:
    \[
    W_{mn}(D^\theta) = e^{i(m-n)\theta} W_{mn}(D);
    \]
  - Scaling:
    \[
    W_{mn}(D^s, \omega) = W_{mn}(D, s\omega).
    \]
Bats

- **Scattering amplitude:**

\[ u^s(x) = -ie^{-\frac{\pi i}{4}} \frac{e^{i\omega|x|}}{\sqrt{8\pi\omega|x|}} A_\infty[D, \kappa, \rho, \omega](\theta, \theta') + o(|x|^{-\frac{1}{2}}), \]

| | → ∞; \( u^i \): plane wave; \( \theta, \theta' \): incident and scattered directions.

- **Graf’s formula:**

\[ A_\infty[D, \kappa, \rho, \omega](\theta, \theta') = \sum_{n,m \in \mathbb{Z}} (-i)^n i^m e^{in\theta'} W_{nm}(D, \kappa, \rho, \omega) e^{-im\theta}. \]
Bats

Feature extraction:

- $V$: measurements; $W$: features; $L$: linear operator.
- Extract $W$ by solving a least-squares method

$$W = \arg \min_W \| L(W) - V \|.$$  

- $L$: ill-conditioned.
- Formula for the resolving power as function of the SNR: Maximum resolving order $K$ satisfies

$$K^{K+1/2} = C(\omega)SNR.$$
Shape descriptor matching in a multi-frequency dictionary.
Acoustic cloaking

- Make a target **invisible** when **probed** by acoustic waves.
- Cloaking: scattering coefficient cancellation\(^5\):
  - Small layered object with vanishing first-order scattering coefficients.
  - Transformation optics:
    \[
    (F_\eta)[\phi](y) = \frac{DF_\eta(x)\phi(x)DF_\eta(x)^t}{\det(DF_\eta(x))}, \quad x = F_\eta^{-1}(y).
    \]
  - Change of variables \(F_\eta\) sends the annulus \([\eta, 2\eta]\) onto a fixed annulus.

Acoustic cloaking

- Scattering cross-section:
  \[
  Q_s[D, \kappa, \rho, \omega](\theta') := \int_0^{2\pi} \left| A_{\infty}[D, \kappa, \rho, \omega](\theta, \theta') \right|^2 d\theta.
  \]

- Scattering coefficients vanishing structures of order \(N\):
  \[
  Q_s[D, (F_\eta)_*(\rho \circ \Psi_{1/\eta}), (F_\eta)_*(\kappa \circ \Psi_{1/\eta}), \omega](\theta') = o(\eta^{4N}).
  \]

- \(\eta\): size of the small object; \(N\): number of layers; \(\Psi_{1/\eta}(x) = (1/\eta)x\).
- Anisotropic density and bulk modulus distributions.
- Invisibility at \(\omega \Rightarrow\) invisibility at all frequencies \(\leq \omega\).
Acoustic cloaking

- Cloaking: scattering coefficient cancellation

Cancellation of the scattered field and the scattering cross-section: 4 orders of magnitude (with wavelength of order 1, $\eta = 10^{-1}$, and $N = 1$).
Differential imaging

- Ultrasound-modulated optical tomography
- Cross-correlation techniques.
- Hybrid imaging modality: one single imaging system based on the combined use of different imaging modalities.
Differential imaging

- Acoustically modulated optical tomography\(^6\):

\( \Omega \)

\( y \)

Contrasted anomaly

Fused acoustic beam

Spherical acoustic pulses

Light source

Light detectors

\( \text{Acoustic source} \)

\( \text{Record the variations of the light intensity on the boundary due to the propagation of the acoustic pulses.} \)

Differential imaging

- $g$: the light illumination; $a$: optical absorption coefficient; $l$: extrapolation length. Fluence $\Phi$ (in the unperturbed domain):

\[
\begin{cases}
-\Delta \Phi + a\Phi = 0 \quad \text{in } \Omega, \\
\left(l \frac{\partial \Phi}{\partial \nu}\right) + \Phi = g \quad \text{on } \partial \Omega.
\end{cases}
\]

- Acoustic pulse propagation: $a \to a_u(x) = a(x + u(x))$.
- Fluence $\Phi_u$ (in the displaced domain):

\[
\begin{cases}
-\Delta \Phi_u + a_u\Phi_u = 0 \quad \text{in } \Omega, \\
\left(l \frac{\partial \Phi_u}{\partial \nu}\right) + \Phi_u = g \quad \text{on } \partial \Omega.
\end{cases}
\]

- $u$: thin spherical shell growing at a constant speed; $y$: source point; $r$: radius.
- Cross-correlation formula:

\[
M(y, r) := \int_{\partial \Omega} \left( \frac{\partial \Phi}{\partial \nu} \Phi_u - \frac{\partial \Phi_u}{\partial \nu} \Phi \right) = \int_{\Omega} (a_u - a) \Phi \Phi_u \approx \int_{\Omega} u \cdot \nabla a |\Phi|^2 .
\]

\[\text{Taylor+Born}\]
Differential imaging

- Helmholtz decomposition: $\Phi^2 \nabla a = \nabla \psi + \nabla \times A$.

- Spherical Radon transform: $\nabla \psi = -\frac{1}{c} \nabla R^{-1} \left[ \int_0^r \frac{M(y, \rho)}{\rho^{d-2}} d\rho \right]$.

- System of nonlinearly coupled elliptic equations: $\nabla \cdot \Phi^2 \nabla a = \Delta \psi$ and $-\Delta \Phi + a\Phi = 0$.

- Fixed point and Optimal control algorithms.

- Convergence result for the fixed point scheme provided that $\|\Delta \psi\|_{L^\infty(\Omega)}$: small.

- Convergence result for the optimal control algorithm assuming a good initial guess.
Differential imaging

- Reconstruction for a **realistic** absorption map: proof of convergence for **highly discontinuous absorption maps** (bounded variation)\(^7\).
- **Minimal regularity** assumption on the absorption coefficient: \(a \in S\)BV\(^\infty\).

Nanoparticle imaging

- **Gold nano-particles**: accumulate selectively in tumor cells; bio-compatible; reduced toxicity.
- Detection: localized enhancement in radiation dose (strong scattering).
- Ablation: localized damage (strong absorption).
- Functionalization: targeted drugs.
Nanoparticle imaging

- $D$: nanoparticle; $\varepsilon_c(\omega)$: complex permittivity of $D$; $\varepsilon_m > 0$: permittivity of the background medium;
- Permittivity contrast: $\lambda(\omega) = (\varepsilon_c(\omega) + \varepsilon_m)/(2(\varepsilon_c(\omega) - \varepsilon_m))$.
- $G_{km}$: outgoing fundamental solution to $\Delta + k_m^2$; $k_m := \omega/\sqrt{\varepsilon_m}$.
- Quasi-static far-field approximation\(^8\): $|D| \to 0$,

$$u^s = -M(\lambda(\omega), D) \nabla_z G_{km}(x - z) \cdot \nabla u^i(z) + O\left(\frac{|D|^{3/2}}{\text{dist}(\lambda(\omega), \sigma(K_D^*))}\right).$$

- Spectral decomposition: $(l, m)$-entry

$$M_{l,m}(\lambda(\omega), D) = \sum_{j=1}^{\infty} \frac{(\nu_m, \varphi_j)_{\mathcal{H}^*}(\nu_l, \varphi_j)_{\mathcal{H}^*}}{(1/2 - \lambda_j)(\lambda(\omega) - \lambda_j)}.$$

- $(\nu_m, \varphi_0)_{\mathcal{H}^*} = 0$; $\varphi_0$: eigenfunction of $K_D^*$ associated to $1/2$.
- Quasi-static plasmonic resonance: $\text{dist}(\lambda(\omega), \sigma(K_D^*))$ minimal $(\Re \varepsilon_c(\omega) < 0)$.

• Enhancement of the scattering and absorption cross-sections $Q^s$ and $Q^a$ at plasmonic resonances$^9$: 

\[ Q^a + Q^s (= \text{extinction cross-section } Q^e) \propto \Im \text{Trace}(M(\lambda(\omega), D)); \]

\[ Q^s \propto |\text{Trace}(M(\lambda(\omega), D))|^2. \]

Nanoparticle imaging

- **Single nanoparticle imaging**\(^{10}\):
  \[
  \max_{z^S} I(z^S, \omega)
  \]

- \(I(z^S, \omega)\): imaging functional; \(z^S\): search point.
- **Resolution**: limited only by the signal-to-noise-ratio.
- **Cross-correlation techniques**: robustness with respect to medium noise.

Nanoparticle imaging

- **Blow-up** of the electric field in the gap at the plasmonic resonances\(^{11}\):

\[
\nabla u = O\left(\frac{1}{(\delta/R)^{3/2} \ln(R/\delta)}\right).
\]

\(^{11}\)with S. Yu, SIAM Rev., 2018.
Nanoparticle imaging

- Reconstruction from plasmonic spectroscopic data\textsuperscript{12}.

Concluding remarks

• Resolution, stability, and specificity bio-inspired enhancement techniques:
  • Physics-based learning approach.
  • Multi-frequency imaging.
  • Differential imaging.
  • Nanoparticle imaging.

• Other applications: autonomous robotics
  • Equip autonomous robots with a "electric and acoustic sense perception".
  • Provide autonomous robots, by mimicking weakly electric fish and bats, with detection and classification capabilities in dark or turbid environments.
  • Complex targets; tracking of the position and orientation of mobile targets by extended Kalman filtering; autonomous navigation, ....