Optimal Parameter Selection for Discrete-Time Throughput-Optimal MAC Protocols

Huijiang Li and Biplab Sikdar
Department of ECSE, RPI,
Troy NY 12180 USA

Abstract—**Distributed, throughput-optimal medium access control (MAC) protocols have recently been proposed for wireless networks. While the performance of these protocols is well understood in terms of the throughput, simulation studies have shown that their delay performance is very sensitive to the protocol's parameter values. To address the problem of selecting parameters that minimize the average packet delay, this paper first develops a queueing model to evaluate the delay of a class of discrete-time, throughput-optimal MAC protocols. This model is then used to derive the optimal parameter settings for the MAC protocol. Simulation results are presented to validate the delay model and the parameter selection methodology.**

I. Introduction

The development of practical, throughput-optimal MAC protocols for wireless networks has been an area of considerable importance to the research community. Recent advances in this area have led to the development of methodologies, primarily based on the application of Glauber dynamics, for achieving throughput-optimality while maintaining an acceptable level of complexity of the protocol [1]. The performance of these protocols is provably throughput-optimal (in the sense that they maintain queue stability for all arrival rates in the stability region). However, the packet delays associated with these protocols can be significant, and vary widely depending on the choice of the protocol parameters. The focus of this paper is on developing a queueing model for a discrete-time throughput-optimal MAC protocol and then use the model to derive the optimal parameter settings that minimize the average packet delay.

Initial solutions for throughput-optimal scheduling in wireless networks were maximum weight schedulers that used the queue lengths at nodes to determine the channel access rights [2]. These approaches usually have limited applicability in real life scenarios due to their overheads associated with collecting neighbor information, and computational complexity resulting from the need to solve a combinatorial optimization problem in each slot. More recently, distributed throughput-optimal schedulers have been proposed for operation in both discrete and continuous time. Continuous time carrier sense multiple access (CSMA) protocols that are throughput-optimal have been proposed in [3], [4]. However, the protocols necessitate considerable information exchange between the nodes and due to their idealistic operation in continuous time, avoid the question of collisions that may arise in real systems. Discrete time, CSMA based distributed MAC protocols have been proposed in [5], [6]. Under these protocols, the throughput-optimality is achieved by letting the nodes use their queue lengths to control the transition probabilities of the Markov chain (representing the network state) over the space of all feasible schedules.

978-1-4799-3083-8/14/$31.00

The focus of the proposed protocols on achieving throughput-optimality comes at the price of packet delays. It has been shown that the delay performance of the Glauber dynamics based schemes depends on the mixing time of the underlying Markov chain [6], [1]. Existing work on the delays experienced by throughput-optimal MAC protocols has also shown that the delay is affected by the access probabilities [7]. However, selecting the parameters that minimize the delays remains an open problem for throughput-optimal MAC protocols such as the one in [6]. Also, none of the existing works provide queueing models to evaluate the delays associated with the throughput-optimal MAC protocols. This paper addresses the problem of developing a mathematical framework for the optimal selection of the channel access probability for a discrete-time, throughput-optimal MAC protocol. The MAC protocol considered in this paper is different from the one in considered in [7].

To develop a framework for determining the optimal parameter settings, this paper first develops an analytic model to evaluate the average delay associated with the throughput-optimal MAC protocol. Since the delays are more significant as the traffic or packer arrival rates at the nodes become large, the focus of this paper is on the moderate and high utilization regions. To model the delays, we propose a cycle-time based approach where the time between two successive transmissions from a node is used to define a cycle. This delay model is then used to evaluate the optimal channel access parameter for the given network settings. The proposed delay model and the parameter selection mechanism have been verified through simulations.

The rest of the paper is organized as follows. Section II introduces the throughput-optimal MAC protocol considered in this paper. Section III presents the model for evaluating the average packet delay and the optimal parameter settings. Section IV presents simulation results to verify the proposed models and Section V concludes the paper.

Algorithm 1 A Throughput Optimal MAC Protocol [6]

```
# node i's operation in time slot t
```
if node i's transmission was successful at previous slot $t-1$, **then**

$$\theta_i(t) = \begin{cases} 1 & \text{w.p. } 1-\frac{1}{W_i(t)} \\ 0 & \text{otherwise} \end{cases};$$

else if no neighbor of i attempted to transmit at $t-1$, **then**

$$\theta_i(t) = \begin{cases} 1 & \text{w.p. } 0.5 \\ 0 & \text{otherwise} \end{cases};$$

else

$\theta_i(t) = 0.$

end if

Algorithm 2 Procedure for Estimating Weights of Neighbors

Initialize: $A_j^i(0) = B_j^i(0) = 0.$

if node $j \in \mathcal{N}_i$ attempted transmission at time slot $t-1$, **then**

$A_j^i(t) = A_j^i(t-1)$

$B_j^i(t) = B_j^i(t-1) + 1;$

else if $B_j^i(t-1) \geq 2$ **then**

$$A_j^i(t) = \begin{cases} A_j^i(t-1)+1 & \text{if } B_j^i(t-1) \geq g(A_j^i(t-1)) \\ A_j^i(t-1)+1 & \text{otherwise} \end{cases}$$

else

$A_j^i(t) = A_j^i(t-1)$

$B_j^i(t) = 0.$

end if

II. Background

This section presents an overview of the assumptions, system model, and the MAC protocol considered in this paper.

A. System Model and Definitions

We consider a discrete-time network of N nodes. The positions of the nodes are arbitrary and assumed to be such that the network formed is a co-located network, i.e., a network where each link interferes with all the other links and the resulting conflict graph is complete. The interference graph is denoted by $G = (V, E)$ where $(i, j) \in E$ denotes that transmissions from nodes i and j interfere with each other and thus may not transmit simultaneously. The set of independent sets of G is denoted by $\mathcal{I}(G)$. λ_i denotes the average traffic arrival rate at node i and the set of arrival rates of all nodes is denoted by $\boldsymbol{\lambda} = [\lambda_i]$. Our analysis assumes a homogeneous traffic process of rate λ at each node that is independent and identically distributed. Each node is assumed to have an infinite buffer (in the simulations we relax this assumption).

At each slot, at most one packet may be transmitted by any given node. Let $\sigma_i(t) \in \{0, 1\}$ denote whether node i is successfully transmitting in time slot t and the vector $\sigma(t) = [\sigma_i(t)]$ be the set of the outcomes of the transmission of all nodes at time slot t. The *capacity region* of a network is defined as the set of all arrival rates λ for which there exists a scheduling algorithm that results in stable or bounded queues at all nodes. From [6], the capacity region of a network may be defined as

$$\boldsymbol{\Lambda} = \left\{ \mathbf{y} \in \mathbb{R}_+^N : \mathbf{y} < \sum_{\sigma \in \mathcal{I}(G)} \gamma_\sigma \sigma \text{ with } \gamma_\sigma \geq 0, \sum_{\sigma \in \mathcal{I}(G)} \gamma_\sigma \leq 1 \right\}. \tag{1}$$

A MAC protocol or scheduler is thus throughput-optimal if it ensures that the queues at all nodes in the network are finite, for any $\boldsymbol{\lambda} \in \boldsymbol{\Lambda}$.

B. MAC Protocol

In this paper, we consider the discrete-time throughput-optimal MAC protocol proposed in [6]. In each slot, a node with data to send attempts to transmit based on: (i) whether it successfully transmitted in the previous slot or whether any of its neighbors attempted to transmit in the previous slot, and (ii) the queue length of the node and an estimate of the queue lengths of its neighbors. Each node uses its current queue length and the estimate of the queue lengths of its neighbors to calculate its "weight". If the node successfully transmitted in the previous slot, it does not transmit in the current slot with a probability that is inversely proportional to the node's weight. Else, if none of the node's neighbors transmitted in the previous slot, a node transmits in the current slot with probability 0.5 (*the channel access probability*). Otherwise, the node does not transmit in the current slot with probability 1. Algorithm 1 shows the exact protocol. Here, $\theta_i(t)$ denotes the scheduling decision of node i in time slot t, with $\theta_i(t) \in \{0, 1\}$. The procedure for calculating the quantities that allow an estimate of the weight of the neighboring nodes is shown in Algorithm 2. In this algorithm, $\mathcal{N}(i) = \{j \in V : (i, j) \in E\}$ is the set of neighbors of node i. The counter $A_j^i()$ serves as node i's long term estimate of the $W_j(\cdot)$ at node j while the counter B_j^i is node i's short term estimate of the $W_j(\cdot)$ at node j. The initial values of the two counters are $A_j^i(0) = B_j^i(0) = 0$.

In time slot t, the node i calculates its weight as

$$W_i(t) = \max\left\{ \log Q_i(t), \max_{j \in \mathcal{N}(i)} \exp\left(\sqrt{\log g(A_j^i(t))} \right) \right\} \tag{2}$$

where $Q_i(t)$ represents the queue length at node i in time slot t; $\log$ and $\log\log$ mean $[log]_+$ and $[\log\log]_+$ respectively; the function g is defined as $g(x) = \exp(\log\log x)^4$; and $\mathcal{N}(i) = \{j \in V : (i, j) \in E\}$ is the set of neighbors of node i. By definition, $W_i(t) \geq 1$ for all t.

III. Framework for Optimal Parameter Selection

In this section we first present a model to evaluate the average per packet delay associated with the MAC protocol. This delay model is used to determine the optimal channel access probability that minimizes the packet delay. Our analysis is based on a cycle-time model, as described below.

A. Node Utilization Rates

Under the MAC protocol described in the previous section, nodes with data to send spend their time in two states: channel contention and data transmission. During the course of their

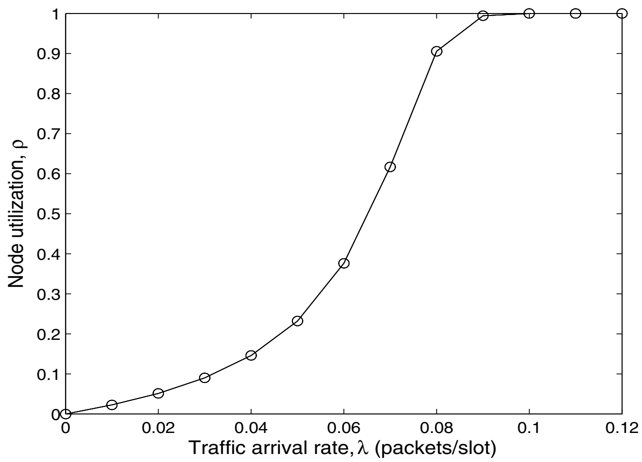

Fig. 1. Utilization rates in a 4 node network for stable arrival rates.

operation, all nodes alternate between these two states. In the MAC protocol under consideration, once a node successfully accesses the channel, it continues to hold the channel (i.e. it keeps transmitting in consecutive slots) with a probability that it based on its current queue length, as well as its estimate of the weights of all its neighbors. Thus as the packet arrival rate increases, on average, a node holds on to the channel for longer (since the average queue lengths are longer).

One of the most significant aspects of the operating characteristics of the MAC protocol under consideration is that as the traffic arrival rate increases, for a range of arrival rates, the queue utilization rate (denoted by ρ) of each node is almost equal to 1, while the queue is stable. This is a consequence of the fact that the MAC protocol adjusts the duration of its busy period in response to the arrival rate (indirectly, by looking at the queue lengths). At high loads, the MAC protocol serves just enough packets to keep the queue stable and may release the channel even though not all packets in the queue have been transmitted. This results in a situation where the queue is almost always non-empty but the queue length is stable value and does not grow unboundedly.

Simulation results for the average queue utilization rate, defined as the fraction of time where the queue at a node is non-empty, for a network of 4 nodes is shown in Figure 1. The buffer size of each node was 10,000 packets. For all the arrival rates shown in Figure 1, there were no packet drops in the network and the queue at each of the nodes was stable. Note that for arrival rates from 0.09-0.12, the queue utilization was greater than 0.995. Thus for this setting, around 25% of the stable region (assuming equal arrival rates at all nodes) leads to utilization rates close to 1. We call this range of arrival rates the *high load* scenario. The high load region corresponds to the arrival rates where the packets experience the highest delays. The focus of this paper is on developing a queueing model for this region and use the model to derive the optimal channel access parameter. Note that the high load scenario needs a special model because standard queueing theoretic models do not hold for cases where the utilization rates are 1.

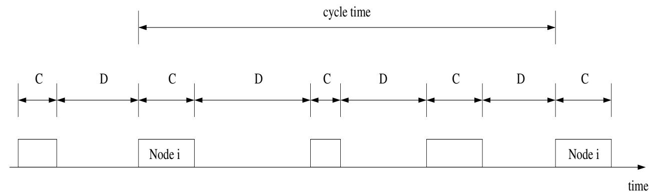

Fig. 2. System operation in terms of the busy and contention periods.

B. Queueing Model

Consider the operation of a network of N nodes as a whole. Once a node releases the channel, all nodes with non-empty queues start contending for the channel. We denote this *contention period* by D. Each contention period consists of idle slots and slots with collisions. Once a node successfully accesses the channel, it continues with its transmissions and we denote this *busy period* by C. Once the node releases the channel, the busy period ends and is followed by a contention period. Figure 2 shows this behavior of the system for a network with 4 nodes. The figure shows the alternation between the contention and access periods. Consider an arbitrary tagged node (node i in the figure). We define the *cycle time* as the time from the start of the tagged node's busy period to the start of the next busy period of the tagged node.

Our delay model and the associated analysis is based on the calculation of the cycle time. We first note that the MAC protocol under consideration is fair in the long term for co-located networks since all nodes use the same channel access probability, denoted by α ($\alpha = 0.5$ in Algorithm 1), the same mechanism for calculating the weights, and we are considering the case where the arrival rates are same at all nodes. Thus between two successive busy periods of the tagged node, *on average*, we expect to see the busy periods of the remaining $N-1$ nodes. Similar arguments have been used for delay models for IEEE 802.11 [8]. The average length of a cycle, $\bar{T}_c$, is then given by

$$\bar{T}_c = N\bar{D} + N\bar{C} \tag{3}$$

where $\bar{D}$ and $\bar{C}$ denote the average length of a contention and busy period, respectively.

Consider an arbitrary node in the network and call it the tagged node. The expected duration of the tagged node's busy period in a cycle $\bar{C}$ and thus the average number of packets transmitted by the tagged node in a cycle is also $\bar{C}$. The average packet arrival rate at the tagged node is λ. Thus the expected number of packet arrivals at the tagged node during a cycle is $\lambda\bar{T}_c$ (i.e. $\lambda N(\bar{D}+\bar{C})$). For a stable system, the expected number of arrivals over a period of time equals the expected number of departures. We then have

$$\begin{aligned} \bar{C} &= \lambda N(\bar{C}+\bar{D}) \\ &= \frac{\lambda N\bar{D}}{1-\lambda N}. \end{aligned} \tag{4}$$

Next, we consider the channel access time D. As per the MAC protocol in Algorithm 1, if the channel is currently busy, all nodes not involved with the ongoing transmission must wait for an idle slot before transmitting their data. Thus

$$P[D=0]=0. \tag{5}$$

Now consider the case of $D=1$ slot. As per the protocol definition, the slot following the end of a busy period, say slot t, is always idle. The contention period equals one if and only if a node successfully transmits in the next slot, i.e. in slot $t+1$. Since each node with data to send accesses the channel independently with probability α ($\alpha=0.5$ in Algorithm 1), the number of nodes that access the channel in a given slot is binomially distributed with parameters $B[N,\rho\alpha]$. Since in the heavy load case considered here $\rho\approx 1$, we have

$$P[D=1]=N\alpha(1-\alpha)^{N-1}. \tag{6}$$

Similarly, $D=2$ if and only if no node transmits in slot $t+1$ while exactly one node transmits in slot $t+2$ and this occurs with probability

$$P[D=2]=(1-\alpha)^N N\alpha(1-\alpha)^{N-1}. \tag{7}$$

For larger channel access times, we also have to consider the case of collisions. Consider the case where $D=i$, with $i>2$. As before, the first slot is always idle. Let j of the remaining $i-1$ slots have collisions. Now each of these j slots with collisions will be followed by an idle slot as per the protocol specifications. In the remaining $i-1-2j$ slots, none of the nodes attempt a transmission. A slot has a collision if more than one nodes transmit in the slot and the probability of a collision is given by

$$p_{col}=1-(1-\alpha)^N-N\alpha(1-\alpha)^{N-1}. \tag{8}$$

Then the probability that j of the i slots have collisions is given by

$$P[D{=}i,j]=N\alpha(1{-}\alpha)^{N-1}\binom{i{-}1{-}j}{j}p_{col}^{j}\left[(1{-}\alpha)^N\right]^{i-1-2j}. \tag{9}$$

Now, in i slots we can have at most $\lfloor\frac{i-1}{2}\rfloor$ collisions (since each collision has to be followed by an idle slot). Unconditioning on j, we have

$$P[D=i]=N\alpha(1-\alpha)^{N-1}\sum_{j=0}^{\lfloor\frac{i-1}{2}\rfloor}\binom{i-1-j}{j}\left[1-(1-\alpha)^N-N\alpha(1-\alpha)^{N-1}\right]^j\left[(1-\alpha)^N\right]^{i-1-2j}. \tag{10}$$

Combining all cases, we have

$$P[D=i]=\begin{cases}N\alpha(1-\alpha)^{N-1} & i=1\\ (1-\alpha)^N N\alpha(1-\alpha)^{N-1} & i=2\\ N\alpha(1-\alpha)^{N-1}\sum_{j=0}^{\lfloor\frac{i-1}{2}\rfloor}\left[\binom{i-1-j}{j}\right. & i\geq 3\\ \left.p_{col}^{j}\left[(1-\alpha)^N\right]^{i-1-2j}\right] & \end{cases}. \tag{11}$$

The expected length of a contention period is then given by

$$\begin{aligned}\bar{D}&=\sum_{i=0}^{\infty}iP[D=i]\\ &=N\alpha\sum_{i=1}^{\infty}i(1{-}\alpha)^{Ni-1}\sum_{j=0}^{\lfloor\frac{i-1}{2}\rfloor}\binom{i{-}1{-}j}{j}p_{col}^{j}(1{-}\alpha)^{2Nj}.\end{aligned} \tag{12}$$

Using the expression for $\bar{D}$ above, $\bar{C}$ can now be obtained by combining Eqns. (4) and (12) while $\bar{T}$ is obtained by combining Eqns. (3), (4) and (12).

1) Average Packet Delay: After each successful transmission, a node's decision to continue transmitting or not depends on its queue length and its estimate of the weights of the other nodes (which in turn depends on their queue length). To obtain the average queue length at a node (and thus the average packet delay), we can then relate the average queue length to the average length of a busy period.

After each transmission by a node in any slot (say node i in time slot t), if the node's queue is non-empty, it transmits another packet in the next slot with probability $1-\frac{1}{W_i(t)}$. Given the fairness of the system for co-located networks and our assumption of equal arrival rates at all nodes, the queue lengths at all nodes are the same on average. Thus we approximate Eqn. (2) as

$$W_i(t)=\log Q_i(t). \tag{13}$$

Let Q_s denote the queue length of the tagged node at the start of a busy period. Since $W_i(t)\geq 1$ by definition, Eqn. (13) implies that if $Q_s\leq 3$, a node releases the channel immediately after the transmission of a single packet. If the starting queue length is greater than 3 (i.e. $Q_s=k>3$), the node releases the channel after one transmission with probability $\frac{1}{\log k-1}$. On the other hand, the probability that the node transmits i packets in the busy period is given by $\prod_{j=1}^{i-1}\left(1-\frac{1}{\log(k-j)}\right)\frac{1}{\log(k-i)}$ for $2\leq i\leq k-3$. Note that a node may transmit at most $k-2$ packets in a busy period (assuming no arrivals) since after that the queue length becomes 2 and weight associated with the node becomes 1. Combining these cases, the length of a busy period conditioned on the starting queue length is given by

$$P[C{=}i|Q_s{=}k]=\begin{cases}1 & i=1,k\leq 3\\ \frac{1}{\log(k-1)} & i=1,k>3\\ \prod_{j=1}^{i-1}\left[1-\frac{1}{\log(k-j)}\right]\frac{1}{\log(k-i)} & 2\leq i\leq k-3, k>3\\ \prod_{j=1}^{i-1}\left[1-\frac{1}{\log(k-j)}\right] & i{=}k{-}2, i{>}1, k{>}3\\ 0 & \text{otherwise}\end{cases}. \tag{14}$$

Obtaining the the average queue length at the start of a busy period $\bar{Q}_s$ directly from Eqn. (14) is not possible since the marginal distribution of C is unknown. As an alternative, we first compute the expected duration of a busy period given the starting queue length:

$$E[C|Q_s = k] = \sum_{i=1}^{k-2} iP[C = i|Q_s = k] \tag{15}$$

where $P[C = i|Q_s = k]$ is given in Eqn. (14). Using the value of $\bar{C}$ from Eqn. (4), we can now use a maximum likelihood estimator for Q_s using Eqn. (15).

Finally, we need to relate the queue length at the start of a busy period (Q_s) to the queue length at an arbitrary instant of time (Q). During a cycle of length $\bar{T}_c$, stability of the queues implies that the number of arrivals equals the number of departures and thus a node has $\bar{C}$ arrivals on average. To relate Q_s and Q, we approximate the arrival process using a discrete fluid model where the $\bar{C}$ arrivals occur at a constant rate of $\frac{\bar{C}}{N(\bar{C}+\bar{D})}$ packets per slot, with each slot's arrivals occurring at the beginning of the slot. The queue length at a node is then affected by two processes: (i) departures at a rate of one packet per slot beginning at the start of cycle and ending at the end of the node's busy period; (ii) arrivals at a rate of $\frac{\bar{C}}{N(\bar{C}+\bar{D})}$ packets per slot, throughput the busy period. Since the node starts the cycle and its busy period with an expected queue length of $\bar{Q}_s$, the average queue length over the entire cycle (and thus the average queue length at an arbitrary instant of time) due to *only the departures* is given by

$$\bar{Q}^d = \frac{1}{N(\bar{C}+\bar{D})}\left[\sum_{i=0}^{\lfloor\bar{C}\rfloor-1}(\bar{Q}_s-i)+(\bar{Q}_s-\bar{C})\left[N(\bar{C}+\bar{D})-\lfloor\bar{C}\rfloor\right]\right] \tag{16}$$

where the second term accounts for the non-integral departure in the last, fractional slot comprising the expected busy period. Similarly, the average queue length over the entire cycle due to *only the arrivals* is given by

$$\begin{aligned}\bar{Q}^a = &\frac{1}{N(\bar{C}+\bar{D})}\Bigg[\frac{\bar{C}}{N(\bar{C}+\bar{D})}+\frac{2\bar{C}}{N(\bar{C}+\bar{D})}+\cdots+\\ &\frac{\bar{C}\lfloor N(\bar{C}+\bar{D})\rfloor}{N(\bar{C}+\bar{D})}+\bar{C}\left(N(\bar{C}+\bar{D})-\lfloor N(\bar{C}+\bar{D})\rfloor\right)\Bigg]\\ = &\frac{\bar{C}}{N(\bar{C}+\bar{D})}\Bigg[\frac{\lfloor N(\bar{C}+\bar{D})\rfloor\left(\lfloor N(\bar{C}+\bar{D})\rfloor+1\right)}{2N(\bar{C}+\bar{D})}+\\ &N(\bar{C}+\bar{D})-\lfloor N(\bar{C}+\bar{D})\rfloor\Bigg].\end{aligned} \tag{17}$$

Combining Eqns. (16) and (17), the overall average queue length at an arbitrary instant of time is given by

$$\begin{aligned}\bar{Q} = &\bar{Q}^d+\bar{Q}^a\\ = &\frac{1}{x}\Bigg[\frac{\lfloor x\rfloor\left(\lfloor x\rfloor+1\right)}{2x}+x-\lfloor x\rfloor-\frac{\lfloor\bar{C}\rfloor\left(\lfloor\bar{C}\rfloor+1\right)}{2}+\\ &\bar{Q}_s x-\bar{C}\left(x-\bar{C}\right)\Bigg]\end{aligned} \tag{18}$$

where $x = N(\bar{C}+\bar{D})$.

The average packet delay, defined as the total time spent in the system, can then be obtained using Little's law as

$$\bar{V} = \frac{\bar{Q}}{\lambda} \tag{19}$$

C. Optimal Channel Access Rate

From Eqn. (18), the average packet delay at a node depends on $\bar{C}$, $\bar{D}$ and $\bar{Q}_s$. From Eqn. (4) we note that $\bar{C}$ is an increasing function of $\bar{D}$. Also, $\bar{C}$ and $\bar{Q}_s$ are related as per Eqns. (14) and (15) and we note that $\bar{Q}_s$ is an increasing function of $\bar{C}$. Thus to minimize $\bar{Q}$, it suffices to minimize $\bar{D}$.

$\bar{D}$ is related to the channel access rate α as per Eqn. (12). Differentiating Eqn. (12) with respect to α, we have

$$\begin{aligned}\frac{d\bar{D}}{d\alpha} &= N\sum_{i=1}^{\infty} i\frac{d}{d\alpha}\alpha(1-\alpha)^{Ni-1}\sum_{j=0}^{\lfloor\frac{i-1}{2}\rfloor}\binom{i-1-j}{j}f(\alpha)\\ &= N\sum_{i=1}^{\infty} i\Bigg[\alpha(1-\alpha)^{Ni-1}\sum_{j=0}^{\lfloor\frac{i-1}{2}\rfloor}\binom{i-1-j}{j}f'(\alpha)\\ &\quad+(1-\alpha)^{Ni-1}\left[1-\frac{(Ni-1)\alpha}{(1-\alpha)}\right]\sum_{j=0}^{\lfloor\frac{i-1}{2}\rfloor}\binom{i-1-j}{j}f(\alpha)\Bigg]\end{aligned} \tag{20}$$

where $f(\alpha) = p_{col}^{j}(1-\alpha)^{2Nj}$. Equating Eqn. (20) to 0 and solving for α gives us the optimal channel access rate. It can be shown that the optimal value of α is less than $1/N$. While we do not have a closed form solution for α, it can be solved numerically.

IV. Simulation Results

This section presents simulation results to validate the analytic models proposed in the previous sections. Since there is no existing work on modeling the queueing delays or determining the optimal parameter settings for the throughput-optimal MAC protocol considered in this paper, no comparison results are provided. The simulator was developed by us and written in the C programming language. We present the results for network sizes of 4 and 6 nodes. Results for other network sizes are similar and omitted because the stability region shrinks rapidly as the number of nodes increases, limiting the range of arrival rates over which results may be shown. Each node has a buffer capacity of 10,000 packets. For each network size, we only report the cases where the arrival rates resulted in a stable system, i.e., an average queue length less than the maximum buffer capacity. For each simulation setting, the results were generated for 10 different runs and the results reported are the average of the 10 runs. The simulation time for each run was 10^8 slots.

Figure 3 shows the results for the average packet delays in a 4 node network for various channel access probabilities. Figure 4 shows the delay values for a 6 node network. There is a close match between the simulation and analytic results. We also note that the average delay tends to decrease as the channel access probability is reduced, specially for higher loads. This is because a lower channel access rate reduces the likelihood

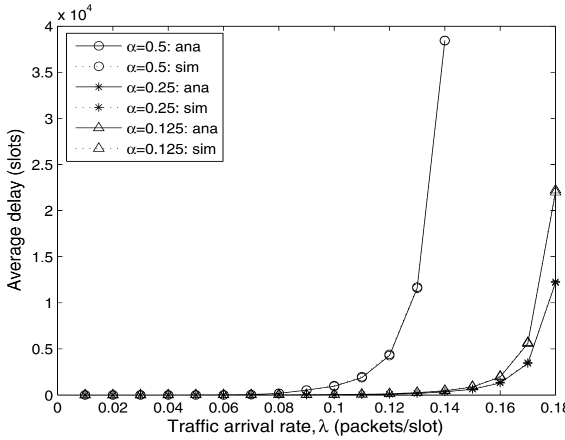

Fig. 3. Average packet delay in a 4 node network for $\alpha = 0.5$, 0.25 and 0.125 for stable arrival rates.

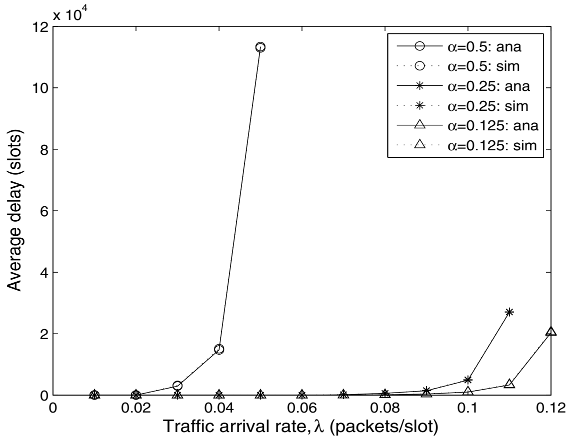

Fig. 4. Average packet delay in a 6 node network for $\alpha = 0.5$, 0.25 and 0.125 for stable arrival rates.

of collisions. However, smaller channel access rates lead to higher delays at very low loads since the nodes now wait longer before they access the channel, even if no contending nodes with traffic are present.

Figure 5 shows the optimal values of α that lead to the lowest average delay in 4 and 6 node networks, for both the simulations and the analysis. The analysis matches closely with the simulation results. There is a small discrepancy at low loads since our model is for high load scenarios. However, the results show that the approximation errors introduced at low loads is not significant. Also, we note that the optimal channel access rate depends on the arrival rate. At lower arrival rates, the queues at the nodes are empty most of the time and thus a higher channel access rate reduces the wait time and does not lead to excessive collisions. However, as the arrival rate increases, queues at the nodes are non-empty for larger fractions of time. As a result, the number of nodes contending for the channel in a given idle slot is higher. A lower channel access rate in this case leads to fewer collisions and compensates for the larger access time (i.e. the number of idle slots before a node decides to transmit).

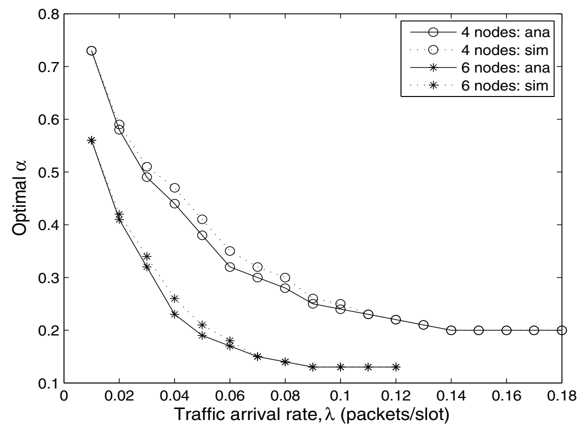

Fig. 5. Optimal channel access rates in 4 and 6 node networks for stable arrival rates.

V. Conclusions

This paper presented a methodology to evaluate the optimal parameter settings in a discrete-time, throughput-optimal MAC protocol. The paper proposed a cycle time based approach to calculate the average packet delays for high load scenarios and this model was used to to determine the channel access rate that leads to the lowest average packet delays. The proposed models have been verified using simulations.

References

[1] L. Jiang, M. Leconte, J. Ni, R. Srikant and J. Walrand, "Fast mixing of parallel Glauber dynamics and low-delay CSMA scheduling," *Proceedings of IEEE INFOCOM Mini-Conference,* Shanghai, China, March 2011.

[2] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximal throughput in multihop radio networks," *IEEE Transactions on Automatic Control,* vol. 37, no. 12, pp. 1936-1948, December 1992.

[3] S. Rajagopalan, D. Shah and J. Shin, "Network adiabatic theorem: An efficient randomized protocol for contention resolution," *Proceedings of ACM SIGMETRICS,* pp. 133144, 2009.

[4] L. Jiang and J.Walrand, "A distributed CSMA algorithm for throughput and utility maximization in wireless networks," *Proceedings of the Allerton Conference on Communication, Control, and Computing,* Urbana-Champaign, IL, 2008.

[5] J. Ni, B. Tan and R. Srikant, "Q-CSMA: Queue length-based CSMA/CA algorithms for achieving maximum throughput and low delay in wireless networks," *Proceedings of IEEE INFOCOM Mini-Conference,* 2010.

[6] D. Shah, J. Shin and P. Tetali, "Medium access using queues," *Proceedings of of IEEE FOCS,* pp. 698-707, Palm Springs, CA, October 2011.

[7] J. Ghaderi and R. Srikant, "Effect of access probabilities on the delay performance of Q-CSMA algorithms," *Proceedings of IEEE INFOCOM,* pp. 2068-2076, Orlando, FL, March 2012.

[8] O. Tickoo and B. Sikdar, "Modeling queueing and channel access delay in unsaturated IEEE 802.11 random access MAC based wireless networks," *IEEE/ACM Transactions on Networking,* vol. 16, no. 4, pp. 878-891, August 2008.

[9] H. Bruneel and B. Kim, *Discrete-Time Models for Communication Systems Including ATM,* Kluwer Academic Publishers, Boston, 1993.