# Evaluation of Spatial Reuse in Wireless Multi-hop Networks

Fengji Ye and Biplab Sikdar
Department of ECSE, Rensselaer Polytechnic Institute
Troy, New York 12180
Email: {yef, sikdab}@rpi.edu

***Abstract*— In wireless multi-hop networks, the spatial reuse determines the number of simultaneous connections allowed in a given region. It has a strong influence on the throughput and delay characteristics of the network and is thus an important metric in performance evaluation. This paper presents an analytic model for the spatial reuse in a wireless multi-hop network with a random access MAC protocol that uses a fair scheduler to accomplish collision avoidance. We employ a purely probabilistic model to derive the closed form solution for the achievable throughput. Using our model we are able to show the maximum saturation throughput obtainable as a function of the node density.**

## I. Introduction

One of the most important metrics which characterize the performance of Medium Access Control (MAC) protocols in wireless ad-hoc networks is their *spatial reuse* which determines the number of simultaneous connections allowed in a given region. This in turn strongly affects the throughput and delay characteristics at each node and thus serves as a fundamental benchmark for the effectiveness and efficiency of MAC protocols. In this paper, we evaluate the spatial reuse in a wireless multi-hop network with a random access MAC protocol that uses a fair scheduler to accomplish collision avoidance. Our work investigates the best throughput obtainable as a function of *node density* in a random access network, using a probabilistic model with practical assumptions on the architecture in the MAC layer. The analysis shows that our framework gives tighter and more realistic bounds on the achievable saturation throughput of the network as compared to the capacity results which assume optimal scheduling, routing and power control.

Latest research efforts on the performance evaluation of ad-hoc networks usually focus with the problem of the capacity and study its relationship with mobility, connectivity and latency [4], [1], [2]. These models consider $n$ identical nodes distributed arbitrarily or randomly on a unit disk, and each node has a randomly chosen destination. The classic problem of network capacity in random networks is formulated by Gupta and Kumar in [4] as to find the maximum throughput in a network where routing, scheduling, and per-node transmission power can all be chosen optimally. The paper proves that the uniform throughput per node scales as $\Theta(\frac{1}{\sqrt{n \log n}})$. In [1], Dousse et al. study the scalability issues in connectivity and capacity in dense ad-hoc networks. The authors define a a dense network as a network deployed on a finite area $s$ with a sufficiently large node density $\lambda$, according to a Poisson point process over the plane. The paper shows that in a dense network, the shape of the power attenuation function strongly affects the connectivity and capacity properties. Efforts have also been made to recompute the capacity under alternate communication models. In [7], Negi et al. assume a Ultra-Wide Band (UWB) physical layer in which each node is constrained to a limited transmit power and but is capable of utilizing an arbitrarily large bandwidth. With this communication model, the authors demonstrate that the per-node throughput increases with node number $n$, under the UWB physical layer assumptions and by using explicit link adaptation.

Existing literature has also incorporated probabilistic model for computing the throughput of wireless networks. Tay and Chua present an analytical model in [9] to obtain closed form approximations for the maximum throughput of the IEEE 802.11 MAC. Using a simplified collision model in 802.11 MAC, the authors have derived the collision probability and the node limit in a wireless cell. In [3] Gobriel et al. establish analytical models for interference and collision analysis in 802.11 MAC from the perspective of power efficiency. Observing the tradeoff in the choice of transmission power, the authors construct the collision model together with the interference model in a unified analysis. In [6] Li et al. employ a simplified spatial reuse model to evaluate the influence of interference range on the network throughput. The paper examines the interaction of the 802.11 MAC and ad hoc forwarding via simulations and analysis from the perspective of spatial reuse. Using a probabilistic model, the authors argue that for the total capacity to scale with network size, the average distance between source and destination nodes must remain small as the network grows.

In this work, we employ a purely probabilistic model to evaluate the spatial reuse of generic, distributed MAC protocols. Our assumptions on the traffic pattern and how MAC protocols regulate the traffic are rather flexible. Consider a sender and receiver within the transmission range of each other which form the *S-R pair* in a transmission. This paper investigates the spatial reuse in static ad-hoc networks under saturated traffic conditions, where simultaneous S-R pairs are tightly packed inside a given region so that no new pair can join in without reorganizing the existing transmission pairs. Our work is based on the observation that the saturation throughput of the whole network varies as a function of the node density. We develop a metric to evaluate the efficiency of spatial reuse in terms of equivalent saturation throughput and obtain its closed form solution.

The rest of the paper is organized as follows. In Section II the preliminaries of our model are presented. In Section III and **??**, we explain our probabilistic model in detail and evaluate the spatial reuse using predefined metrics. Our conclusion is given in Section IV.

## II. PRELIMINARIES

To maximize the spatial reuse, the ideal scheduler is expected to pump as many simultaneous transmissions as possible in an area without causing any interference. Since we seek the best throughput and the scheduler is optimal, no MAC-layer overheads like ACK frames or backoff windows are considered. Also, power control is outside the scope of this paper. All nodes are assumed to be randomly distributed in a network located on an infinite plane. As a MAC-layer analysis, our work focuses on single hop traffic, and assumes no dependency among different transmissions. That is, the analysis does not depend on routing or queueing strategies. All the random settings of the traffic are ergodic and the throughput is evaluated under these conditions.

Spatial reuse assesses the efficiency of channel sharing and the degree of multiple access in an ad-hoc network, and thus it is a MAC-related issue. To simulate the environment of ad-hoc networks we make stochastic assumptions on the network topology and MAC layer traffic pattern. In this section we will explain these assumptions and introduce our metric to evaluate the spatial reuse in a dense ad-hoc network.

### A. Model Overview

To maximize the spatial reuse, a normal scheduler is expected to pump as many simultaneous transmissions as possible in a region while maintaining the interference to a tolerable level. In our probability model we consider a random network deployed in a sufficiently large area under homogeneous and ergodic traffic conditions. Since we study the best-of-effort throughput on the MAC layer, we assume no protocol-specific MAC overheads such as ACK frames, or backoff windows used in the collision resolution phase. All the nodes have the same physical layer characteristics such that they have the same antenna gains and send packets using the same transmission power (we will provide an example of spatial reuse in the power control MAC in Section **??**).

As a MAC layer analysis, our work focuses on single hop traffic, and assumes no dependency among different transmissions. The MAC layer handles traffic on a stand-alone basis and does not depend on routing or queueing strategies. Since an optimal global scheduler is not present, other portions of the network traffic are not known to a local scheduler. We also assume the scheduled traffic exhibits explicit statistical features so that it could fit into the probability framework.

### B. The Assumptions of Random Networks

According to our assumptions, a *random network* is composed of identical nodes randomly located in a planar region of sufficiently large area. The nodes are distributed homogenously so that we can characterize the two-dimensional uniform distribution using Poisson process with intensity parameter $\lambda$, which is also known as *node density*. The assumption of sufficiently large region helps eliminate the edge effects at the boundary and facilitates the use of probability tools.

The MAC layer traffic consists of single hop traffic, which can be characterized as per *S-R pair* (Sender-Receiver pair) transmissions. We assume there is no probabilistic dependency among the occurrence of different transmission pairs. In a S-R pair, the receiver can be located anywhere around the sender. With a *fair* scheduler in the random access MAC protocol, it does not always favor the pairs with shorter one-hop distances for the sake of maximizing spatial reuse. Therefore, under ergodic traffic conditions the receiver is uniformly distributed around the sender within the *transmission range* $R$. Let $r$ denote the one-hop distance in a S-R pair, from Fig. 1(a) we can derive its *probability distribution function* (PDF) as

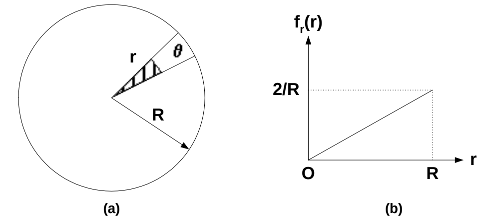


Fig. 1. Probability distribution of one-hop distance $r$. In (a) we illustrate how to derive its probability distribution function from the ratio of the sector area. The shape of its probability density function is shown in (b).

$$F_r(r) = \frac{\frac{1}{2}r^2\theta}{\frac{1}{2}R^2\theta} = \frac{r^2}{R^2},\ 0 \leqslant r \leqslant R \tag{1}$$

Hence its *probability density function* (pdf) is given by (also shown in Fig. 1(b))

$$f_r(r) = \frac{d}{dr}F_r(r) = \begin{cases} \frac{2r}{R^2}, & 0 \leqslant r \leqslant R \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

### C. SINR Model

The spatial reuse in an ad hoc network cannot go unbounded because when a transmitting node produces interference to all other nodes in its vicinity. As the *Signal to Interference and Noise Ratio* (SINR) constraint must be satisfied at each receiver, this prevents the network from supporting an infinite number of simultaneous transmissions. In this paper we use the Physical model as in [4]. Suppose node $S_i$ and $R_i$ are sender and receiver in a S-R pair, respectively, and $S_k$ $(k \neq i)$ denotes any other sender in $R_i$'s neighborhood, then the SINR constraint at $R_i$ is given as

$$\frac{\frac{P}{|S_i - R_i|^\alpha}}{\delta_i + \tau_i} \geqslant \beta \tag{3}$$

where

$$\tau_i = \sum_{k \neq i} \frac{P}{|S_k - R_i|^{\alpha}} \tag{4}$$

Here $\alpha$ ($\alpha > 2$) is the *path loss exponent*, and $\beta$ is the *SINR threshold*. $P$ is the common transmission power at each sender, and $|S_k - R_i|$ is the distance between sender $S_k$ and receiver $R_i$. $\delta_i$ denotes the *ambient noise*, assumed to be *white Gaussian noise* in this analysis. $\tau_i$ is a random variable representing the interference collected at a node from all the existing senders in its vicinity, which we call the *Aggregate Interference*. Since we assume the node distribution is homogenous, the *pdf* of $\tau_i$ at each receiver has an identical form, and we use $f_T(\tau)$ to denote it.

### D. Spatial Reuse Metric

To study the spatial reuse in an ad-hoc network, we assume the network traffic is saturate in that each node always has packets to transmit. Although all the nodes can compete for the channel, only a portion of them can successfully send/receive packets under the arbitration of random access MAC protocol, and we say these nodes are *active*. It is seen that a node can not always stay in the active mode in every transmission/receiving attempt, and thus we define $\eta$, the *Effective Transmission Rate* (ETR), to represent the probability that a node is active in any attempt. Note that each active node is either an *active sender* or an *active receiver*, and senders have the same number as receivers.

Given the node density $\lambda$, Effective Transmission Rate $\eta$ reflects the spatial reuse efficiency brought by the random access MAC protocol. From the probability perspective we can use $\lambda\eta$ to denote the *active node density*. It is valid for both active senders and receivers, which are assumed to have homogenous distributions as well. As one can expect, $\lambda\eta$ reveals the *equivalent saturation throughput* in terms of achievable number of simultaneous S-R pairs that can be contained in unit area, and thus we can use it as our spatial reuse evaluator.

## III. A Probability Model for the Spatial Reuse

In this section we introduce our probability model and establish equations to obtain the closed form solution for the Effective Transmission Rate $\eta$ at each node as a function of node density $\lambda$.

### A. Aggregate Interference

As we have pointed out, the Aggregate Interference $\tau$ in Eqn. (4) is a random variable representing the power level received at a node from all the existing senders around. Hence its probability distribution depends on the active node density $\lambda\eta$. We shall establish a probability model to derive its pdf $f_T(\tau)$ as a function of $\lambda\eta$.

To obtain $f_T(\tau)$ we develop a *concentric ring* model shown in Fig. 2. Consider node $R_i$ surrounded by active senders with the density of $\lambda\eta$. Suppose we have infinite non-overlapping concentric rings centered at $R_i$, and the combination of all the rings gives us the entire unbounded region. Each ring has a radius of $\rho$ and infinitesimal width of $\Delta\rho$, with $\rho$ continuously ranging from 0 to $\infty$. According to the definition of Poisson process [10], for small $\Delta\rho$ the probability that an active sender falls into the ring is proportional to the ring's area. Since we know the active node density is $\lambda\eta$, this probability is given by $\lambda\eta[2\pi\rho\Delta\rho + o(\Delta\rho)]$, where $o(\Delta\rho)$ is used to denote any quantity that goes to zero at a faster than linear rate. For the case that there are more than one active senders falling into the same ring, according to the Poisson process definition, it has a probability of $o(\Delta\rho)$ and is thus negligible in the analysis. We then assign a random variable $\zeta_\rho$ to each ring, representing its contribution to the Aggregate Interference $\tau$ at the central node $R_i$ due to the active node inside the ring, if any. If an active node happens to stay inside, this contribution is $\frac{P}{\rho^\alpha}$, according to Eqn. (4). Therefore $\zeta_\rho$ has the following binary probability distribution:

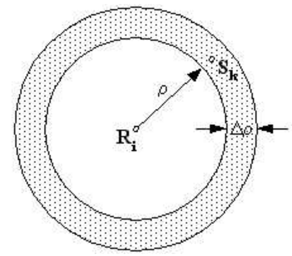


Fig. 2. The concentric ring model. The space is divided into infinite non-overlapping concentric rings centered at $R_i$. Each ring can hold at most one active sender. The aggregate interference at $R_i$ is the summation of the power level received from the active sender inside each ring, if any.

$$\begin{cases} \text{Prob}\,(\zeta_\rho = \frac{P}{\rho^\alpha}) & = & 2\pi\lambda\eta\rho\Delta\rho + o(\Delta\rho) \\ \text{Prob}\,(\zeta_\rho = 0) & = & 1 - 2\pi\lambda\eta\rho\Delta\rho + o(\Delta\rho) \end{cases} \tag{5}$$

We also obtain the *characteristic function* of $\zeta_\rho$ as

$$\begin{aligned} \Phi_{\zeta_\rho}(\omega) &= E[e^{j\omega\zeta_\rho}] \\ &= 1 - 2\pi\lambda\eta\rho\Delta\rho(1 - e^{\frac{j\omega P}{\rho^\alpha}}) + o(\Delta\rho) \\ &= e^{-2\pi\lambda\eta\rho\Delta\rho(1-e^{\frac{j\omega P}{\rho^\alpha}})} + o(\Delta\rho) \end{aligned} \tag{6}$$

Here we have used $e^{-K\Delta\rho} = 1 - K\Delta\rho + o(\Delta\rho)$, valid for any small quantity $\Delta\rho \to 0$ and constant $K$.

It is seen that the Aggregate Interference $\tau$ is the collection of the power received from every individual ring. Thus we have $\tau = \sum_\rho \zeta_\rho$. Therefore the characteristic function of $\tau$ can be derived from that of $\zeta_\rho$:

$$\begin{aligned} \Phi_T(\omega) &= \prod_\rho \Phi_{\zeta_\rho}(\omega) \\ &= e^{-\sum_\rho 2\pi\lambda\eta\rho\Delta\rho(1-e^{\frac{j\omega P}{\rho^\alpha}})} + o(\Delta\rho) \end{aligned} \tag{7}$$

Let $\rho$ continuously change from $\varepsilon$ to $\infty$, we can then rewrite Eqn. (7) in an integral form:

$$\Phi_T(\omega) = \exp[-2\pi\lambda\eta\int_\varepsilon^\infty \rho(1 - e^{\frac{j\omega P}{\rho^\alpha}})\, d\rho] \tag{8}$$

where $\varepsilon$ is a small positive number to avoid the singularity at the origin of power attenuation function when sender and receiver get arbitrarily close.

We can restore the pdf of the Aggregate Interference $\tau$ from its characteristic function in Eqn. (8) using the *Fourier Transform*. As we expect, it is a function of active node density $\lambda\eta$:

$$f_T(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp[-2\pi\lambda\eta\int_{\varepsilon}^{\infty}\rho(1-e^{\frac{j\omega P}{\rho^\alpha}})\,d\rho - j\omega\tau]\,d\omega \tag{9}$$

### B. Closed Form Solution for Effective Transmission Rate

With the pdf of $\tau$, we are able to establish equations to solve for the Effective Transmission Rate $\eta$ at each node. We shall show that $\eta$ can be obtained as a function of node density $\lambda$.

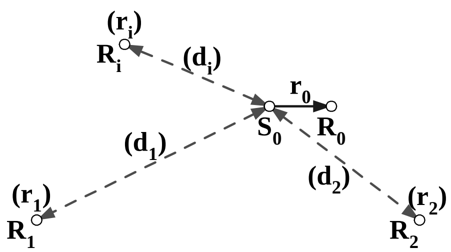


Fig. 3. The node cloud model. $S_0$-$R_0$ is a transmission pair placed with other active nodes in a node cloud. The $r_i$ in parenthesis is the one-hop distance associated with the active receiver $R_i$. The $d_i$ on the edge denotes the distance from $S_0$ to $R_i$.

For illustrative purposes we develop a *node cloud* model shown in Fig. 3. Suppose $S_0$ and $R_0$ are sender and receiver of an S-R pair with a one-hop distance of $r_0$. The pair is placed in a *node cloud* composed of other nodes involved in active transmissions. Now we are interested in the possibility that $S_0$-$R_0$ can collaboratively work with other S-R pairs. Let $R_i$ denote the $i$-th active receiver in the cloud, and $r_i$ is the one-hop distance of the transmission that $R_i$ is involved in. The distance between $S_0$ and the active receiver $R_i$ is $d_i$. $d_i$ complies with some probability distribution such that the occurrences of active receivers are subject to Poisson distribution with active node density $\lambda\eta$.

Now we evaluate the probability that $S_0$-$R_0$ can be accommodated in the node cloud with other transmission pairs, contingent upon all the SINR constraints in the model. We summarize these constraints in the following two principles:

- *Robustness Principle* (RP): It dictates that $S_0$-$R_0$ should be able to endure the interference produced by other transmission pairs in the node cloud.
- *Friendliness Principle* (FP): It dictates that $S_0$-$R_0$ should be cooperative enough not to disrupt the transmission of other S-R pairs.

According to the traffic assumptions, the probability associated with one principle is independent from the other. We can derive them separately and then determine the joint probability.

*1) On the Robustness Principle:* To comply with this principle, the signal received at $R_0$ from $S_0$ must satisfy the SINR constraint in Eqn. (3). With the pdf of the Aggregate Interference $\tau$, the probability associated with this principle can be represented as

$$P_{\text{RP}}(\lambda,\eta) = \text{Prob}\{\tau + \delta_0 - \frac{P}{\beta r_0^\alpha} \leqslant 0\} \tag{10}$$

Here $\delta_0$ is the white Gaussian ambient noise measured at node $R_0$, and the one-hop distance $r_0$ is also a random variable whose pdf is given in Eqn. (2). The probability can be expressed as a joint function of $\lambda$ and $\eta$.

*2) On the Friendliness Principle:* The situation under this principle is much more complicated than the other one. It requires that each active receiver in the node cloud should be able to tolerate the additional interference introduced by the transmission of $S_0$. Let us take the active receiver $R_i$ as an example. Applying the principle is equivalent to re-evaluating the SINR constraint at $R_i$ when the transmission pair $S_0$-$R_0$ joins in the node cloud, given the fact that SINR constraint at $R_i$ has been satisfied without $S_0$-$R_0$. The corresponding probability can be represented as

$$P_{\text{FP}}(\lambda,\eta,d_i) = \text{Prob}\{\frac{\frac{P}{r_i^\alpha}}{\delta_i + \tau_i^{(\text{on})}} \geqslant \beta \mid \frac{\frac{P}{r_i^\alpha}}{\delta_i + \tau_i^{(\text{off})}} \geqslant \beta\} \tag{11}$$

Here we use $\tau_i^{(\text{on})}$ to denote the Aggregate Interference received at $R_i$ when $S_0$-$R_0$ is present in the node cloud, and use $\tau_i^{(\text{off})}$ to denote the case when $S_0$-$R_0$ is absent. $\tau_i^{(\text{on})}$ and $\tau_i^{(\text{off})}$ are random variables, just like $\tau$, whose pdf is given in Eqn. (9). The following equations reveal the relationship among them.

$$\begin{cases} \eta\tau_i^{(\text{on})} + (1-\eta)\tau_i^{(\text{off})} &= \tau \\ \tau_i^{(\text{on})} - \tau_i^{(\text{off})} &= \frac{P}{d_i^\alpha} \end{cases} \tag{12}$$

Note that in Eqn. (12) we treat $\tau_i^{(\text{on})}$ and $\tau_i^{(\text{off})}$ as conditional Aggregate Interference. As a result, they can be linked together as we know the probability that $S_0$-$R_0$ can collaboratively live with other S-R pairs is the Effective Transmission Rate $\eta$.

Now Eqn. (11) becomes

$$\begin{aligned} & P_{\text{FP}}(\lambda,\eta,d_i) \\ = \; & \text{Prob}\{\frac{\frac{P}{r_i^\alpha}}{\delta_i + \tau + \frac{(1-\eta)P}{d_i^\alpha}} \geqslant \beta \mid \frac{\frac{P}{r_i^\alpha}}{\delta_i + \tau - \frac{\eta P}{d_i^\alpha}} \geqslant \beta\} \\ = \; & \frac{\text{Prob}\{\tau + \delta_i - \frac{P}{\beta r_i^\alpha} \leqslant -\frac{(1-\eta)P}{d_i^\alpha}\}}{\text{Prob}\{\tau + \delta_i - \frac{P}{\beta r_i^\alpha} \leqslant \frac{\eta P}{d_i^\alpha}\}} \end{aligned} \tag{13}$$

Here we convert the conditional probability based on the fact that the condition is containable.

To simplify the representations, we define a new random variable:

$$\theta \triangleq \tau + \delta_i - \frac{P}{\beta r_i^\alpha} \tag{14}$$

Recall that we have assumed no probabilistic dependency among different transmission pairs, and $\delta_i$ is the white Gaussian noise. Thus it can be inferred that $\tau$, $\delta_i$ and $r_i$ are independent from each other. As a result, the pdf of $\theta$ can be determined through the convolution of the pdf for the three independent components. Note that with a fair scheduler, the one-hop distance $r_i$ has the same probability distribution as $r$ in Eqn. (2). $\delta_i$ is recognized as a Gaussian random variable with zero mean and some fixed standard deviation. Combining with the pdf of $\tau$ in Eqn. (9), we have all the individual pdf and can accordingly decide the pdf of $\theta$. Suppose its pdf is known, we can then acquire its *probability distribution function* (PDF), denoted by $F_\Theta(\theta)$:

$$F_\Theta(\theta) = \text{Prob}\{\zeta \leqslant \theta\} \tag{15}$$

In this way the probability associated with the two principles above can be simplified with a single probability function $F_\Theta(\theta)$:

$$\begin{cases} P_{\text{RP}}(\lambda,\eta) &= F_\Theta(0) \\ P_{\text{FP}}(\lambda,\eta,d_i) &= F_\Theta(-\frac{(1-\eta)P}{d_i^\alpha})/F_\Theta(\frac{\eta P}{d_i^\alpha}) \end{cases} \tag{16}$$

Note that now $P_{\text{FP}}$ can be expressed as a joint function of $\lambda$, $\eta$ and $d_i$.

It is seen that with a fair scheduler that always tends to maximize the spatial reuse distributively, the Effective Transmission Rate $\eta$ can be obtained by solving the equation

$$\eta = P_{\text{RP}}(\lambda,\eta)\prod_i P_{\text{FP}}(\lambda,\eta,d_i) \tag{17}$$

where the multiplication reflects the joint condition from each active receiver $R_i$ in the Friendliness Principle. Eqn. (17) can be interpreted by the fact that a normal scheduler always tends to pump more S-R pairs into the network as long as it is able to collaboratively live with other existing pairs. In the node cloud model, the feasibility of accommodating a new S-R pair is captured by the probability associated with the Robustness and Friendliness principles. Therefore, from a probabilistic perspective, a potential sender transmits its packet with the probability governed by the two principles, which is revealed in Eqn. (17).

In order to obtain the closed form solution for $\eta$, we need to evaluate the product in Eqn. (17). For each active receiver $R_i$, $d_i$ is independent and identically distributed (i.i.d.), so again we can employ the concentric ring integral in Section III-A to calculate this product. Consider node $S_0$ surrounded by active receivers with the density of $\lambda\eta$. The entire unbounded region can be partitioned into infinite non-overlapping concentric rings centered at $S_0$. Each ring has a radius of $\rho$ and infinitesimal width of $\Delta\rho$, with $\rho$ continuously ranging from 0 to $\infty$. Now imagine each active receiver falls into a distinct ring, with the probability regulated by the Poisson distribution. Let us assign probability $P_{\text{FP}}(\lambda,\eta,\rho)$ to those rings that happen to hold an active receiver, and probability 1 to those who do not. Consider the product of the *total probability* in all the rings throughout the region. It should take the same value as the product in Eqn. (17) as $\rho$ varying continuously from 0 to $\infty$. Thus we have

$$\begin{aligned} &\prod_i P_{\text{FP}}(\lambda,\eta,d_i) \\ =\ &\prod_\rho [(2\pi\lambda\eta\rho\Delta\rho + o(\Delta\rho)) * P_{\text{FP}}(\lambda,\eta,\rho) + \\ &(1-2\pi\lambda\eta\rho\Delta\rho + o(\Delta\rho)) * 1] \\ =\ &e^{-\sum_\rho 2\pi\lambda\eta\rho\Delta\rho(1-P_{\text{FP}}(\lambda,\eta,\rho))} + o(\Delta\rho) \end{aligned} \tag{18}$$

Again we can rewrite Eqn. (18) in an integral form:

$$\prod_i P_{\text{FP}}(\lambda,\eta,d_i) = \exp[-2\pi\lambda\eta\int_0^\infty \rho(1-P_{\text{FP}}(\lambda,\eta,\rho))\,d\rho] \tag{19}$$

With Eqn. (16) and (19), Eqn. (17) finally reduces to

$$\eta = F_\Theta(0)\,\exp[-2\pi\lambda\eta\int_0^\infty \rho(1-\frac{F_\Theta(-\frac{(1-\eta)P}{\rho^\alpha})}{F_\Theta(\frac{\eta P}{\rho^\alpha})})\,d\rho] \tag{20}$$

As can be seen, from Eqn. (20) we are able to obtain the closed form solution for the Effective Transmission Rate $\eta$ as a function of node density $\lambda$. As we have pointed out, $\eta$ has a significant impact on deciding the spatial reuse through the equivalent saturation throughput $\lambda\eta$, which reflects the achievable number of simultaneous S-R pairs that can be contained in unit area. Therefore our model is shown to provide an analytical approach for the spatial reuse evaluation.

## IV. Conclusion

In this work, we develop a purely probabilistic model to evaluate the spatial reuse of generic, distributed MAC protocols. It investigates the spatial reuse in a static multi-hop network under saturated traffic conditions, where simultaneous S-R pairs are tightly packed inside a given region. We assume random traffic pattern and MAC protocols regulate the traffic via a fair scheduler. We define the spatial reuse metric in terms of Equivalent Saturation Throughput, as a function of Effective Transmission Rate. To obtain the closed form solutions, we consider the probability distribution of the aggregate interference of a node, produced by the transmission power from other active senders in its vicinity. We then use SINR constraints to establish equations to solve for the Effective Transmission Rate. The analysis shows that our framework gives tighter and more realistic bounds on the achievable saturation throughput of the network.

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