Power-Efficiencies of Multi-hop Paths for Routing in Wireless Networks

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***Abstract*— In energy constrained wireless networks, routing algorithms should base the selection of the next hop from the available nodes after taking into consideration the power efficiency of each option. With multiple routing choices available, a systematic methodology to determine the power efficiency of each path in a decentralized manner is of significant importance. This paper analyzes the power-efficiency of using multi-hop versus direct paths while taking into account the overhead power due to transceiver electronics in addition to the MAC/protocol overhead. We first obtain the threshold $l_\circ$ for the distance between the source and destination, within which direct routing is more efficient than multi-hop routing. The analysis is then extended for multi-hop routing where we obtain the optimal number of hops the route should have, and the optimal distance at which we should seek the next hop.**

I. Introduction

With decentralized routing algorithms, each node selects the next hop for a given destination from neighboring nodes based on some optimization criteria. Optimally selecting the next hop involves a number of considerations. Should the data be routed directly to the destination or though intermediate hops? If a multi-hop path is opted for, how many hops should the route have? Of all the available nodes, which should be selected for the hops in the route? The answer to these questions depends on the power-efficiency of the available options. We address these issues in this paper and present a framework for determining the optimality of the various routing choices.

This paper considers static, location-aware routing, where each node is aware of the geographical locations of its neighbors. We start by considering only the transmission power for the power consumed during routing. We show that in this case a multi-hop path would always be more power-efficient than the direct path, provided that the hops for the multi-hop path are within a certain bounded region between the source and destination nodes, which we call the *hopping region*. We then consider the additional power consumption in multi-hop routing at the intermediate nodes, due to transceiver electronics and protocol/MAC overhead, and show that multi-hop routing is preferable over direct routing only for long distances.

We also derive the threshold $l_\circ$ for the distance between the source and destination nodes, within which direct routing would be preferable over multi-hop routing and vice-versa. For multi-hop routing we derive the optimal number of hops (and optimal separation between hops) for a multi-hop route. We also quantify the tradeoffs associated with selecting the next-hop from a set of sub-optimally placed neighboring nodes.

Routing protocols for wireless ad hoc networks have been widely investigated in literature, e.g. Location-Aided Routing (LAR) [1], dynamic source routing [3], power-aware routing metrics [9] and Destination-Sequenced Distance-Vector (DSDV) routing [7]. However, these protocols do not aim for the most power-efficient route. At the same time, our analysis can be easily incorporated into these protocols. Power-efficient routing for wireless networks is also discussed in [2], [4], [10], but these assume a centralized scenario and do not account for overhead power. Energy-efficient communication in actual micro-sensor hardware is discussed in [5], [6], which provide empirical results for transmission power, channel fading etc. To the best of our knowledge, existing literature does not address the issue of evaluating the scenarios under which multi-hop paths become more preferable over direct routing, while considering the effect of overhead power.

The rest of this paper is organized as follows. In Section II we compare the power-efficiencies of direct and single-hop paths, without considering the overhead power at each node. Section III extends the analysis to the case of multi-hop paths where we determine the optimal number of hops in a path and the distance-angle tradeoff while selecting the next hop from among the set of non-optimally placed neighboring nodes. Finally Section IV presents the concluding remarks.

II. Comparison without considering overhead power

To begin with, we assume that only one intermediate node is available for routing, and Section III, we extend the analysis to the multiple intermediate node case. Note that in this paper by a hop we refer to an intermediate node in a route and not a link. Let nodes i, j and k denote the source, destination and intermediate nodes respectively and l_{ij}, l_{ik} and l_{kj} the distances between them. We consider an arbitrary position for node k satisfying $l_{ik} < l_{ij}$, $l_{kj} < l_{ij}$ and the triangle inequality, $l_{ik} + l_{kj} \geq l_{ij}$. Since we wish to optimize on power, we first find the expression for the minimum power required over each route. The constraint on the minimum transmission power is that the SINR at each receiver is at least equal to the minimum SINR required, $\mathrm{SINR}_{\min}$. The SINR at node j receiving a signal from node i has the form:

$$\text{SINR}_j = \frac{p_{ij} g_{ij}}{\eta_j + \kappa I_j}, \quad (1)$$

where p_{ij} is the transmitted power, g_{ij} is the path gain from node i to node j ($g_{ij} = \beta {l_{ij}}^{-\alpha}$), η_j is the noise power at node j, κ is a system-specific constant, and I_j is the MAI. Assuming the same amount of noise, η, and external MAI, I, at all the receivers, the minimum powers required over the direct and indirect paths, p_{D} and p_{M}, are:

$$p_{\mathrm{D}} = p_{ij} = g_{ij}^{-1}(\eta + \kappa I)\text{SINR}_{\min}, \quad (2)$$

$$p_{\mathrm{M}} = p_{ik} + p_{kj} = (g_{ik}^{-1} + g_{kj}^{-1})(\eta + \kappa I)\text{SINR}_{\min}. \quad (3)$$

Consider the ratio

$$\zeta \triangleq \frac{p_{\mathrm{M}}}{p_{\mathrm{D}}} = \frac{g_{ik}^{-1} + g_{kj}^{-1}}{g_{ij}^{-1}} = \frac{{l_{ik}}^{\alpha} + {l_{kj}}^{\alpha}}{{l_{ij}}^{\alpha}}. \quad (4)$$

$\zeta < 1$ would mean that the multi-hop path is preferable over the direct path and vice-versa. The value of ζ also depends on the node topology being considered and we now analyze the value of ζ for different values of l_{ij}, the distance between the source and destination nodes. If either $l_{ik} > l_{ij}$ or $l_{kj} > l_{ij}$, then obviously $\zeta > 1$. On the other hand, consider a topology where node k lies on the line joining nodes i and j, i.e. $l_{ik} + l_{kj} = l_{ij}$. Then $l_{ik} = \rho l_{ij}$ where $0 \leq \rho \leq 1$. So $l_{kj} = (1 - \rho) l_{ij}$. For such a topology, we have $\zeta = \rho^{\alpha} + (1 - \rho)^{\alpha}$. $\Rightarrow \zeta \leq 1$ if $\alpha \geq 1$ which is usually the case as in the two-ray ground and free space models [8].

Thus there exists a bounded region between nodes i and j, such that if node k lies in that region, a multi-hop path via node k is more power-efficient than the direct path and we call this region the *hopping region*. The shape and size of the hopping region depends on the value of α. If $\alpha < 1$, $\zeta > 1$ always, i.e. the hopping region doesn't exist or is null. If $\alpha = 1$, the hopping region is just the line joining nodes i and j, with area zero. If $1 < \alpha < \infty$, the hopping region has a finite non-zero area, with area increasing with the value of α. We now characterize the shape of the hopping region for a given α, by finding its boundary. It is more convenient here to work with polar co-ordinates (r, θ). We orient the co-ordinate axes such that node i lies at the origin, and j lies on the x-axis with co-ordinates $(l_{ij}, 0)$. The boundary of the hopping region is the set of points for which $\zeta = 1$. The boundary of the hopping region is the set of points for which $\zeta = 1$. Let k be any point on the boundary with co-ordinates (r, θ) as shown in Figure 1. The boundary is then the set of all k satisfying ${l_{ik}}^{\alpha} + {l_{kj}}^{\alpha} = {l_{ij}}^{\alpha}$. In terms of polar co-ordinates, this becomes

$$r^{\alpha} + \left(r^2 + {l_{ij}}^2 - 2 r l_{ij} \cos\theta\right)^{\alpha/2} = {l_{ij}}^{\alpha}. \quad (5)$$

When $\alpha = 2$, this curve is $r = l_{ij} \cos\theta$ and hopping region is shown in Figure 2.

The maximum off-line angle, $\theta_{\max}$, node k can attain while staying within the hopping region, equidistant from nodes i and j, is given by

$$\theta_{\max} = \cos^{-1}\left[\frac{2^{1/\alpha}}{2}\right]. \quad (6)$$

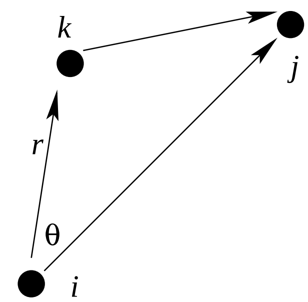

Fig. 1. Topology considered.

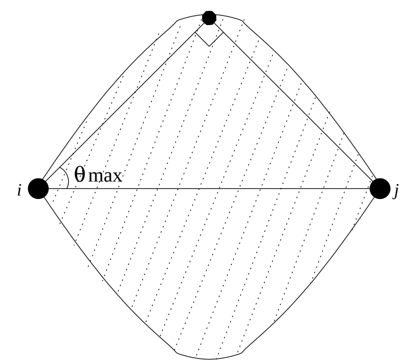

Fig. 2. Hopping region for $\alpha = 2$.

It is in some sense the "farthest" point in the hopping region. When $\alpha = 2$, $\theta_{\max} = 45°$. As expected, $\theta_{\max}$ increases with α.

The main conclusion of this analysis in this section is that the ratio ζ does not depend on the separation l_{ij}, rather it only depends on the position of node k relative to nodes i and j. $\zeta < 1$ when k lies in the hopping region between i and j. In other words, while routing between two nodes, a multi-hop path is more efficient than the direct path *irrespective of the the separation between the two nodes*, provided that we find an intermediate node in the hopping area between those two nodes.

A. Overhead cost of multi-hop routing

The analysis of the previous sub-section considered only the transmission power while comparing the direct and multi-hop paths. In addition to the transmission power, the communication involves: (1) power required by transmitter and receiver electronics and (2) protocol and MAC overhead. Obviously, the power consumed due to these overheads (which we call *overhead cost* and denote by P) would be more for multi-hop paths than for direct paths and would increase with the number of hops. We find the threshold $l_{\circ}$ such that when $l_{ij} \leq l_{\circ}$, direct routing is more efficient, and when $l_{ij} > l_{\circ}$, multi-hop is preferable.

Define $P' \triangleq (P\beta)/((\eta + \kappa I)\text{SINR}_{\min})$. Then, using Equations (2) and (3), equating the powers over the direct and multi-hop paths we get ${l_{ij}}^{\alpha} = {l_{ik}}^{\alpha} + {l_{kj}}^{\alpha} + P'$. $l_{\circ}$ is then that value of l_{ij} which solves this equation. However $l_{\circ}$ also depends on the position of k. k should lie within the hopping region (not on the boundary either), else the equation would be unsolvable.

We place k at the center of the hopping region, i.e. at the middle of the line joining nodes i and j, as this position gives the minimum power among all possible single-hop paths (power increases as k is moved away from the center). Thus if the direct path is better than this single-hop path, it would be better than *all* other single-hop paths. In this case, $l_{ik} = l_{kj} = l_\circ/2$ and we get ${l_\circ}^\alpha = 2(l_\circ/2)^\alpha + P'$ which gives

$$l_\circ = \left(\frac{P'}{1 - 2^{-(\alpha-1)}}\right)^{1/\alpha}. \quad (7)$$

Note that $0 < l_\circ < \infty$ for $\alpha > 1$. Thus, if $l_{ij} < l_\circ$, the direct path is better than all single-hop paths and if $l_{ij} = l_\circ$, the direct path and the best single-hop path both use the same power. If l_{ij} is increased over $l_\circ$, more and more single-hop paths become more efficient than the direct path. As earlier, given a $l_{ij} > l_\circ$, we have a bounded region between nodes i and j such that any single-hop path via a node in this region would be better than the direct path. We call it the *modified hopping region*. The boundary of the modified hopping region is given by all (r, θ) satisfying

$$r^\alpha + \left(r^2 + {l_{ij}}^2 - 2rl_{ij}\cos\theta\right)^{\alpha/2} + P' = {l_{ij}}^\alpha. \quad (8)$$

For $l_{ij} < l_\circ$ the modified hopping region is null, for $l_{ij} = l_\circ$ it is just a single point, and as l_{ij} is increased over $l_\circ$, it grows around the center. Unlike the hopping region, the modified hopping region does not touch nodes i and j for any l_{ij}.

III. Extension to multi-hop paths

We now consider the case where we have a number of intermediate nodes between the source and destination nodes. If $l_{ij} \le l_\circ$, in addition to single-hop paths, the direct path is better than all multi-hop paths. The proof of this is trivial and is as follows. Consider an arbitrary 2-hop path, $i \to p \to q \to j$, with both nodes p and q lying within the hopping region. Since $l_{iq} \le l_{ij}$ and $l_{ij} \le l_\circ$, $l_{iq} \le l_\circ$. So we have, ${l_{ij}}^\alpha \le {l_{iq}}^\alpha + {l_{qj}}^\alpha + P'$ and ${l_{iq}}^\alpha \le {l_{ip}}^\alpha + {l_{pq}}^\alpha + P'$. These imply ${l_{ij}}^\alpha \le {l_{ip}}^\alpha + {l_{pq}}^\alpha + {l_{qj}}^\alpha + 2P'$. This means that the direct path is preferable over the 2-hop path. Extending this argument, the direct path is preferable over an arbitrary-hop path.

Similarly if $l_{ij} > l_\circ$ a multi-hop path is preferable to the direct path. We now find the optimal number of hops in multi-hop path, and the optimal separation between them.

A. Optimal distance of next hop

We now obtain the optimal distance between hops in a route and show that as l_{ij} increases, the optimal number of hops increases and the optimal distance between hops, say $d_\circ$, changes rapidly. We have seen that if $l_{ij} > l_\circ$, the single-hop path is better than the direct path. As l_{ij} increases further, multi-hop paths become more and more power efficient. Using the same logic used to show that single-hop paths are better than direct paths, above a certain threshold for l_{ij}, the "uniform" 2-hop path becomes more power-efficient than the single-hop path. (A uniform m-hop path refers to the path with m intermediate nodes lying equally spaced and in-line with the source and destination nodes. Note that for a given number of hops, equally spaced inline nodes lead to the lowest power consumption.) Extending this argument further, there exist $l_\circ, l_2, l_3, \ldots, l_m, \ldots$, such that if $l_m < l_{ij} \le l_{m+1}$, the *uniform* m-hop path is the optimal. Equating the powers of the m and $m+1$ hop paths, l_m is obtained as the solution of

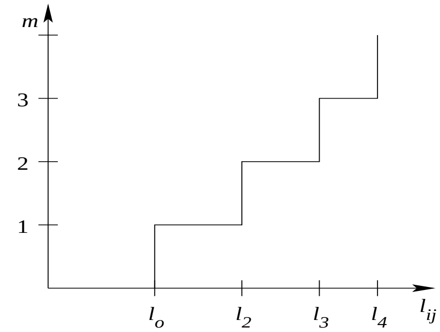

Fig. 3. Optimal number of hops m as a function of l_{ij} (approximate).

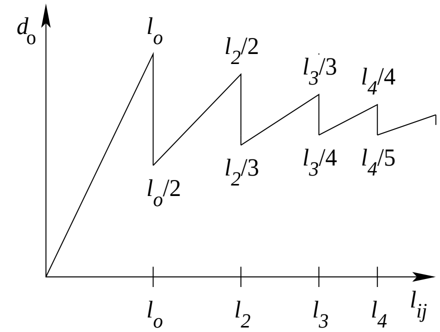

Fig. 4. Optimal distance between hops $d_\circ$ as a function of l_{ij} (approximate).

$$m\left(\frac{l}{m}\right)^\alpha = (m+1)\left(\frac{l}{m+1}\right)^\alpha + P'. \quad (9)$$

Thus given a l_{ij}, the optimal number of hops m required to route between nodes i and j, is found as the index of that l_m which satisfies $l_m < l_{ij} \le l_{m+1}$. Then, for that l_{ij} the optimal distance between two consecutive hops is $d_\circ = l_{ij}/m$.

The plots for the optimal number of hops m and $d_\circ$ as a function of l_{ij} are shown in Figures 3 and 4 respectively. Note that $d_\circ$ oscillates sharply and regularly with varying l_{ij}, but the average of the oscillations is nearly constant. The peaks of the oscillations satisfy $l_\circ > l_2/2 > l_3/3 > \ldots$, while the troughs satisfy $l_\circ/2 < l_2/3 < l_3/4 > \ldots$ (i.e. l_m satisfy $l_m/m > l_{m+1}/(m+1)$ and $l_m/(m+1) < l_{m+1}/(m+2)$ for all m). In other words, the amplitude of the oscillations ($l_m/m - l_m/(m+1)$) strictly decreases and converges to zero. Moreover, it can be shown that the optimal inter-hop distance $d_\circ = l_{ij}/m$ converges and that the limit is

$$\lim_{l_{ij}\to\infty} \frac{l_{ij}}{m} = \lim_{m\to\infty} \frac{l_m}{m} = \left(\frac{P'}{\alpha - 1}\right)^{1/\alpha}. \quad (10)$$

Thus, while seeking the next hop at any node, one can compute $d_\circ$ for the current separation l_{ij}, and then seek a node at distance $d_\circ$ away, preferably in-line. We refer to this position as the *optimal next-hop position*. If a $d_\circ$ independent of l_{ij} is

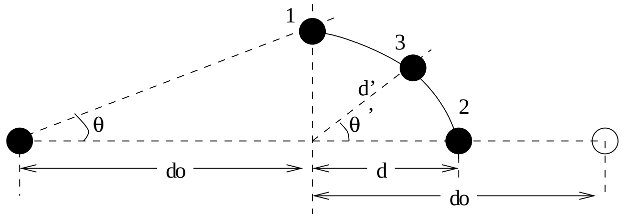

Fig. 5. Virtual destination and equivalence curve.

desired (say when l_{ij} is unknown), the best option would be to have it as $d_\circ = (P'/(\alpha - 1))^{1/\alpha}$, the limiting value of the optimal inter-hop distance.

B. Distance-angle trade-off while selecting the next hop

In a typical scenario there would not be any node lying at the optimal next-hop position and we now evaluate the tradeoffs involved in selecting the best next hop from these sub-optimally positioned nodes. Suppose we have two choices for the next hop at a node. One is an off-line node lying perpendicular to optimal next-hop position, at an off-line angle of θ, while the other is an in-line node but at a distance of $d_\circ + d$ from the current hop. This situation is depicted by nodes 1 and 2 in Fig. 5. This is a trade-off situation because while extra power is needed to transmit to node 2, it is also closer to the destination. While the optimal choice would depend on the entire network topology, in this paper we only consider the case where only the local network topology is available at the nodes, which is a realistic assumption.

We virtually move the destination node closer (at a distance of $2d_\circ$ form the current hop) to the current hop and then make a choice between the two next hops by comparing the powers consumed while routing to this *virtual* destination node. This situation is depicted in Figure 5. The powers P_1 and P_2 over the two possible paths via node 1 and 2 respectively are:

$$P_1 = \mu \left(2(d_\circ \csc\theta)^\alpha + P'\right), \quad (11)$$

$$P_2 = \mu \left((d_\circ + d)^\alpha + (d_\circ - d)^\alpha + P'\right). \quad (12)$$

So, if $P_1 < P_2$, we select node 1 as the next hop and vice-versa. Allowing $P_1 = P_2$, we can find the off-line angle θ of the node lying perpendicular to the optimal next-hop position (or center) giving the same power as the in-line node at the distance $d_\circ + d$ from the source. We call this off-line node the *corner node* for the particular θ or d used. This off-line angle is given by

$$\theta = \cos^{-1}\left[2^{1/\alpha} d_\circ \left((d_\circ + d)^\alpha + (d_\circ - d)^\alpha\right)^{-1/\alpha}\right]. \quad (13)$$

Given a θ we now find a curve around the center such that any node lying on that curve gives the same power as the corner node. Referring to Figure 5, consider an arbitrary node at distance d' and an angle of $\theta'$from the center (node 3). This node has the same power as the corner node if

$$2(d_\circ \csc\theta)^\alpha = [(d_\circ + d'\cos\theta')^2 + (d'\sin\theta')^2]^{\alpha/2} + [(d_\circ - d'\cos\theta')^2 + (d'\sin\theta')^2]^{\alpha/2}. \quad (14)$$

This equation in (d', θ') represents the *equivalence curve* characterized by the *spread angle* θ. The equivalence curve thus represents the contour along which all nodes have the same power efficiency. Note that $0 \le \theta' < 2\pi$, so the equivalence curve encloses the center.

Suppose we seek the optimal next hop within a sector of angle 2θ around the direct path (here θ is the spread angle). Consider the area bounded by the equivalence curve with spread angle θ, enclosing the center. The nodes in this area would be the most suitable candidates for the next hop. We call this area the *equivalence region* for the particular θ used. Note that the nodes in an equivalence region are not equivalent in terms of power (unlike equivalence curve). The modified hopping region discussed in Section II-A is in fact a special case of the equivalence region concept. The modified hopping region is same as the equivalence region of spread angle θ when the nodes are separated by a distance of $2d_\circ$, and when the corner node with an off-line angle θ gives the same power as the direct path between the nodes.

IV. Conclusions

In this paper we evaluated to power efficiencies of direct versus multi-hop paths for routing in location-aware wireless networks. Our analysis also accounted for the overhead power due to to transceiver electronics and MAC/protocol overhead. We obtained the threshold $l_\circ$ for distance between the source and destination, within which direct routing is preferable over than multi-hop routing. For multi-hop routing, we obtained an expression for the optimal number of hops the route should have, and the optimal distance at which we should seek the next hop.

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