Distributed Mobility Transparent Broadcasting in Vehicle to Vehicle Networks

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Abstract—In this paper we propose a distributed mobility transparent broadcast (DMTB) protocol to achieve efficient and effective broadcast in vehicle to vehicle networks. The protocol is fully distributed and highly adaptive to node mobility. Although distributed, DMTB does not suffer from the performance degradation induced by the unavailability of global information. The protocol's performance is proved to be within a constant of the optimum. Detailed analysis regarding the protocol's performance is presented. The simulation results clearly verify that the protocol maintains its performance under high node mobility.

I. INTRODUCTION

Numerous efforts are currently under progress to enhance the safety and efficiency of vehicular traffic by means of intelligent transportation systems. A number of these applications are dependent on the ability of nodes to efficiently broadcast data that they generate and also to forward important information from other vehicles. In this paper we propose a new, distributed broadcast protocol for relaying information in vehicle to vehicle networks.

Unlike other types of wireless networks, energy is usually not a constraint in vehicle to vehicle networks due to the availability of on board power. Instead, node mobility poses the greatest challenge for broadcast schemes in vehicular networks. It precludes the broadcast protocols that are highly dependent on the availability and accuracy of the network topology information at any instant. While dynamic protocols perform better than static protocols in the presence of node mobility, their broadcast efficiency is usually quite low due to the lack of global view of the network. Additionally, many existing broadcast protocols can cause *broadcast storms* due to the large amount of message relays [10]. In turn, severe *channel interference* results from the broadcast storm due to the large number of re-transmissions and the contention based channel access schemes.

The problem of interest in this paper is *how to efficiently broadcast in ad-hoc networks with high mobility*. To address this issue, we present a "distributed mobility transparent broadcast (DMTB)" protocol that is not only efficient and effective, but also highly adaptive to node mobility. By "mobility transparent" we mean that node mobility does not degrade the protocol's performance. The key contributions of

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this paper are: (a) We present a cross-layer design to achieve effective and efficient broadcast in vehicular networks; (b) The protocol's performance does not degrade as node mobility increases; (c) The proposed protocol achieves fairness by randomly rotating the set of relay nodes in different broadcast events; (d) The protocol's performance ratio (the number of relay nodes comparing to that of the optimal broadcast scheme) is proved to be less than 8; (e) Detailed analysis regarding the broadcast interference is presented; and (f) the capacity cost of the protocol is evaluated.

The rest of this paper is organized as follows. The next section presents the related work and Section III introduces the proposed protocol, along with its assumptions and groundwork. Section IV presents an analysis of the proposed protocol's performance ratio, time complexity, interference and other performance related analysis. Section V evaluates the cost of the protocol in terms of its network capacity requirements. In Section VI we present the simulation results to evaluate the proposed protocol's performance and in Section VII we present the concluding remarks.

II. RELATED WORK

In recent years design of broadcast schemes for vehicular networks has attracted considerable attention. A flooding based broadcast protocol for inter vehicle communication using short packets is proposed in [16]. Though not suitable for vehicular networks, various broadcast schemes based on improved flooding have been proposed in literature for wireless ad hoc networks. Ideally only nodes in the Minimum Connected Dominating Set (MCDS) should be elected as relay nodes, and identifying the MCDS is known to be NP-hard [5].

A probabilistic broadcast scheme for transmitting emergency warning packets is proposed in [1] to reduce the broadcast storm problem. However, how to calculate the rebroadcast probability in different scenarios has not been determined. A protocol that combines directional and intersection broadcast is proposed in [9]. The 'role-based' broadcast schemes for inter vehicle communication proposed in [2], [14] require each node to maintain the list of its neighbors at all times and send the data only to a set of these neighbors. Both these protocols suffer from the broadcast storm problem.

A hexagon-based broadcast protocol is proposed in [11]. However, the algorithm's performance is prone to deterioration due to the "hexagon skewness" problem [11] and the situation becomes worse if nodes become mobile. In [8] it is shown that when the node density is high enough so that a node can be found at any desired strategic point, the efficiency of hexagon flooding is about 68% of the optimal efficiency.

A thorough survey and comparison of a number of broadcast protocols designed for mobile ad hoc networks is presented in [18]. The compared protocols include "simple flooding", "counter-based" [10], "SBA" [12], "location-based" [10], and "AHBP and its mobile extension" [13] and it is shown that among protocols that can achieve near 100% delivery ratio, SBA has good performance in mobile scenarios with relatively small overhead. Thus, in Section VI we evaluate the proposed protocol against SBA to show its relative performance.

Although the MAC layer interference is one of the biggest issues for broadcast in ad-hoc networks, all broadcast schemes mentioned previously are designed without considering the MAC layer interference.

III. A DISTRIBUTED MOBILITY TRANSPARENT BROADCAST PROTOCOL

A. Background and Assumptions

In order to overcome the network mobility and performance degradation induced by unavailability of global information, DMTB employs an imaginary constellation as the reference to locate the relay nodes. The constellation, as shown by the dark solid lines in Fig. 1, spans the whole network and is anchored, in each broadcast event, by the source node. In the ideal case, nodes located exactly at the vertices act as the relay nodes and all others as non-relay nodes. The two theorems below provide the theoretical foundation for the constellation structure. If not stated otherwise, the words "triangle", "square" and "hexagon" are used to refer to the corresponding equilateral polygons.

Theorem 3.1: If overlapping and gaps are not allowed and only one type of polygon is used, a plane can only be tiled by triangle, square or hexagon. This is called the Plane Tiling Theorem.

Proof of this theorem is presented in the Appendix A.

Theorem 3.2: Overlapping with hexagon tiling is the least among the three polygon tiling methods (hexagon tiling, triangle tiling or square tiling).

This can be proved by using a method similar to that used in [8].

From theorems above it can be seen that hexagon tiling has the highest broadcast efficiency, as also indicated in [8]. In Section IV we analytically address the broadcast efficiency issue.

The ideal scenario described above is impractical to realize since in most vehicular network scenarios, it is difficult or even impossible to find a node that is located exactly at the required position. Also the construction and maintenance of the constellation may introduce significant overhead. These observations motivate our design of an "imaginary constellation" based, fully distributed broadcast protocol and we describe it in detail in the following sections. The vertices shown in Fig. 1 will be referred to as the *benchmark positions*, and the distance between two neighboring vertices as the *cell radius*. The constellation in Fig. 1 has another advantage that each point in the network is covered by the transmissions from more that one relay node. This provides certain resilience against message losses due to link failure or interference.

DMTB is based on IEEE 802.11's MAC layer broadcast function in ad-hoc mode, with only physical carrier sensing
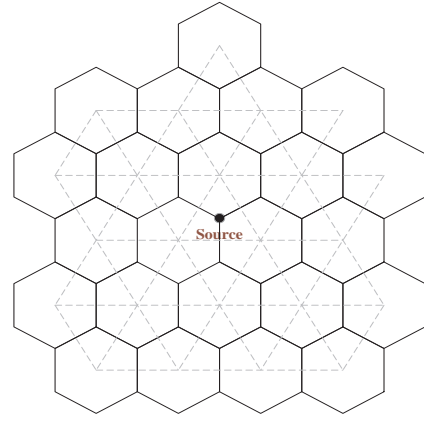

Fig. 1. DMTB's broadcast reference constellation

a) The broadcast message generated by the source node s

b) The broadcast message that node i receives from node j

c) The broadcast message sent out by node i

Fig. 2. Message formats in the proposed broadcast protocol.

function (no RTS/CTS or ACK as used by the virtual carrier sensing). We assume that nodes are aware of their geographic location. Significantly, no neighborhood information is required.

B. Protocol Description

DMTB solves two problems for the constellation based broadcast protocol: *How to construct and maintain the constellation with little overhead* and *how to find a relay node for each benchmark position*. The first problem is addressed collaboratively by the source node and all other nodes, while the second problem is addressed locally in the neighborhood of each benchmark position.

1) The Imaginary Constellation and the Benchmark Position Association Function: The imaginary constellation can be specified by its origin, *cell radius* r and orientation θ. Each time a source node broadcasts a message, it formulates the broadcast message in the format shown in Fig. 2(a). The message supplies important information to anchor the reference constellation: the constellation's origin (x_s, y_s), cell radius r and orientation θ (randomly generated within the range of $[0, \frac{2}{3}\pi]$). The $MsgTY$ and $seqnum$ fields denote the message type and sequence number, respectively. This information is enough to fix the reference constellation that spans the whole network, which means all benchmark positions are implicitly fixed. The random choice for θ ensures that even for different broadcasts from the same source, the chosen relay nodes are
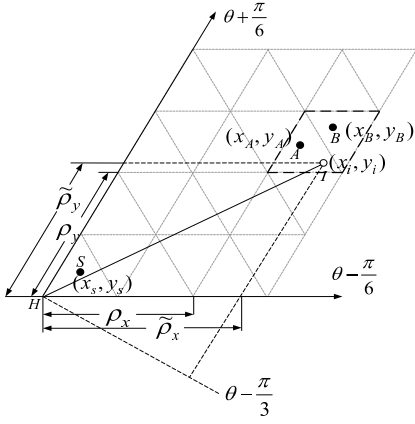
Fig. 3. Finding a node's benchmark position.

different, thereby distributing the broadcast load and achieving fairness.

Upon reception of a broadcast message for the first time (duplicate messages are discarded), a node identifies its benchmark position p and prepares the broadcast message according to p's information. Fig. 1 gives a constellation example with all vertices as the benchmark positions. The gray dashed lines form a Voronoi tessellation and each Voronoi cell contains one of the benchmark positions at the cell's center. The property of Voronoi tessellation guarantees that an arbitrary node in a Voronoi cell is closer to the benchmark position that is located in the same Voronoi cell than any other benchmark position. All nodes within a Voronoi cell will mark themselves as the relay candidates for the benchmark position in the same Voronoi cell. The details of finding the benchmark position is shown in Fig. 3 where suppose a broadcast message is generated from the source node, with the constellation origin located at (x_s, y_s). The broadcast message is shown in Fig. 2(a). Node i first receives this broadcast message from its neighbor node j, with the message content shown in Fig. 2(b). Given node i's location (x_i, y_i), the received broadcast message will trigger node i to calculate its nearest benchmark position (x_i^p, y_i^p), which equals (x_A, y_A) in Fig. 3. Please refer to Section III-B3 for details on calculating (x_i^p, y_i^p) based on the source node's position, the imaginary constellation's *cell radius* r and orientation θ.

Once the nearest benchmark position (x_i^p, y_i^p) is located, node i first checks whether the message sequence number is outdated and if (x_i^p, y_i^p) is identical to x_j^p, y_j^p, which is the benchmark position indicated in the broadcast message i receives from node j. Only when the two conditions are not true will node i mark itself as a relay node candidate and reformulate the broadcast message as indicated in Fig. 2(c).

When a relay node reformulates the broadcast message, the associated benchmark position, instead of its own location, is encapsulated and relayed to its neighbors. Thus each node actually takes the imaginary constellation as its reference, which effectively avoids problem of skewness propagation (skewness of the actual constellation formed by relay nodes compared to the imaginary constellation) incurred due to limited node density. When the node density is fairly low, in order to counteract the side effect introduced by the relatively big gap between the relay nodes' actual position and their corresponding benchmark position, the cell radius r can be set smaller than the node's normal transmission range. The protocol automatically terminates when the broadcast message reaches the boundary of the network area since the benchmark positions are out of all nodes' transmission range.

2) Relay Node Election: The primary purpose of the "relay node election" function is to elect a relay node from all candidates that are identified by the previous function component. The elected relay node forwards the message to all nodes in its transmission range, while all other candidates simply drop the message. Once it identifies itself as a relay node candidate, node i starts a deferring timer with the initial value T_i as:

$$T_i = \mathcal{F}(d_i) \qquad (1)$$

where $\mathcal{F}()$ can be any increasing function and d_i is the distance from node i to its associated benchmark position. Function $\mathcal{F}()$ should be chosen such that the maximum allowable value of T_i is much smaller than the time associated with network dynamics in order to minimize the likelihood of a node crossing Voronoi cell boundaries while the timer decrements. Node i sends out the broadcast message only when following conditions are true:

1) i does not hear any other candidates relaying the same message before its timer expires;
2) As can be seen in Fig. 1, each relay node has three neighbor nodes that are also relay nodes. i relays the message only when it does not hear all of its 3 neighbor relay nodes relay the same message for their corresponding benchmark positions.

Rather than explicitly designating any node to relay the message, DMTB counteracts the topology changes by letting the most suitable nodes elect themselves to relay the message. This effectively increases the protocol's robustness and resilience against node mobility or failure.

3) Benchmark Position of an Arbitrary node during a broadcast: In the example shown in Fig. 3, to calculate node i's corresponding benchmark position (x_i^p, y_i^p), we need to find out the *Voronoi* cell in which (x_i, y_i) resides in. We see that Voronoi cells are equilateral triangles with side length of $\sqrt{3}r$, where r is the radius of the hexagon. For any node inside a cell, its benchmark position should be the center of the triangle. We place our Voronoi tessellation in affine coordinates with oblique axes in the orientation of $\theta - \frac{\pi}{6}$ and $\theta + \frac{\pi}{6}$, as shown in Fig. 3. Recall that s is the source node position. Let H be the bottom-left vertex of the Voronoi cell that holds S. θ is the orientation of $\overrightarrow{HS}$, which is also the hexagon orientation embedded in the previous broadcast message.

As shown in the Fig. 3, the coordinate origin and node i's position are denoted by H and I respectively. Let $< \alpha, \beta >$ denote vector $\overrightarrow{HI}$, where α and β are the orthogonal projection of $\overrightarrow{HI}$ on the x and y axes, respectively:

$$\begin{aligned} \alpha &= x_i - x_s + r\cos\theta \\ \beta &= y_i - y_s + r\sin\theta \end{aligned}$$

Now we are interested in the oblique projection of vector $\overrightarrow{HI}$. In Fig. 3, it is seen that its oblique projection on the axis $\theta - \frac{\pi}{6}$

is equal to its orthogonal projection on $\theta - \frac{\pi}{3}$ divided by $\cos \frac{\pi}{6}$. Thus we can calculate this projection using the inner product:

$$\widetilde{\rho_x} = \frac{1}{\cos \frac{\pi}{6}} \left(\alpha \cos \left(\theta - \frac{\pi}{3} \right) + \beta \sin \left(\theta - \frac{\pi}{3} \right) \right)$$

Likewise, the oblique projection of $\overrightarrow{HI}$ on the axis $\theta + \frac{\pi}{6}$ is equal to its orthogonal projection on $\theta + \frac{\pi}{3}$ divided by $\cos \frac{\pi}{6}$:

$$\widetilde{\rho_y} = \frac{1}{\cos \frac{\pi}{6}} \left(\alpha \cos \left(\theta + \frac{\pi}{3} \right) + \beta \sin \left(\theta + \frac{\pi}{3} \right) \right)$$

Using the affine coordinates in Fig. 3, we can confine a node within a parallelogram composed of two adjacent Voronoi cells. For example, I is in a parallelogram which contains two cells, centered at A and B, respectively. To obtain the coordinates of A and B, we need to locate this parallelogram by its oblique projection:

$$\rho_x = \sqrt{3}r \cdot \left| \frac{2\alpha}{3r} \sin \left(\theta + \frac{\pi}{6} \right) - \frac{2\beta}{3r} \cos \left(\theta + \frac{\pi}{6} \right) \right|$$

$$\rho_y = \sqrt{3}r \cdot \left| -\frac{2\alpha}{3r} \sin \left(\theta - \frac{\pi}{6} \right) + \frac{2\beta}{3r} \cos \left(\theta - \frac{\pi}{6} \right) \right|$$

Recall that $\sqrt{3}r$ is the side length of a Voronoi cell. Therefore the coordinates of A and B are:

$$x_A = x_s + \rho_x \cos \left(\theta - \frac{\pi}{6} \right) + \rho_y \cos \left(\theta + \frac{\pi}{6} \right)$$

$$y_A = y_s + \rho_x \sin \left(\theta - \frac{\pi}{6} \right) + \rho_y \sin \left(\theta + \frac{\pi}{6} \right)$$

$$x_B = x_A + r \cos \theta$$

$$y_B = y_A + r \sin \theta$$

It is seen that A and B are two candidates of benchmark position for I. The final benchmark position is the one that is closer to I. Define:

$$\eta = [(x_i - x_A)^2 + (y_i - y_A)^2] - [(x_i - x_B)^2 + (y_i - y_B)^2]$$

then:

$$(x_i^p, y_i^p) = \begin{cases} (x_A, y_A) & \text{if } \eta < 0 \\ (x_B, y_B) & \text{if } \eta > 0 \end{cases}$$

IV. PERFORMANCE AND INTERFERENCE ANALYSIS

In this section we analyze DMTB's message complexity, time complexity, the broadcast interference and the *cell radius* for different node densities. Various other aspects for the protocol's performance are also evaluated. Throughput this section and subsequently in the paper we assume that the network is connected.

A. Message Complexity

Definition The broadcast algorithm's *message complexity* is defined as the number of re-broadcast occurrences during each broadcast event with respect to the total number of nodes present in the network.

According to the definition above, the broadcast algorithm's message complexity is actually the percentage of nodes that act as relay nodes. Next we compare DMTB's message complexity with that of the optimal algorithm: MCDS (Minimum Connected Dominating Set).
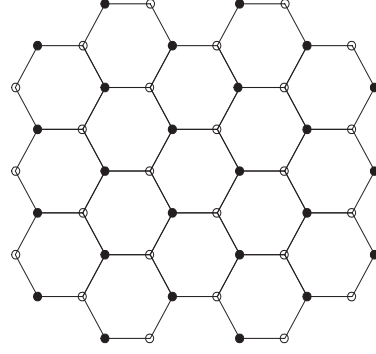

Fig. 4. DMTB's broadcast reference constellation

Definition Denote a graph as $G = (V, E)$. An *independent set* of a graph G is a subset of the vertices such that no two vertices in the subset represent an edge of G. Let *opt* denote the size of any MCDS.

The following lemma is proved in [17].

Lemma 4.1: The size of any independent set in a unit-disk graph $G = (V, E)$ is at most $4 \cdot opt + 1$.

Since *opt* represents the lower bound for the size of the relay node set, we evaluate DMTB's message complexity through its performance ratio, defined as the ratio of DMTB's message complexity to that of any MCDS algorithm.

Theorem 4.2: The performance ratio of DMTB is within 8 of the global optimum.

Proof: As described in previous sections, relay nodes form a constellation as shown in Fig. 4. Based on the definition mentioned above, all circled nodes, which is exactly half the amount of all nodes, form an independent set. Thus, the number of solid nodes (or hollow nodes) is at most $4 \cdot opt + 1$. Similarly the number of hollow nodes is also at most $4 \cdot opt + 1$. This proves that the number of relay nodes selected by our protocol is at most $8 \cdot opt + 2$. ■

Note that the performance ratio is independent of the relative size of the hexagons with respect to the transmission radius of the relay nodes. However, the overall number of transmissions increases if the size of the hexagons is kept smaller that the transmission range.

B. Time Complexity

Definition The broadcast algorithms' *time complexity* is defined as the time delay (T_d) for a node that is d distance away from the source to receive the broadcast message.

We assume that the hop count h between node u and node v is roughly estimated as the ratio of their distance from each other to the transmission range R: $h = \frac{d}{R}$. Suppose there are k intermediate nodes between u and v, and denote the message forwarding delay at each hop as $T_i, i \in 1, 2, 3, \cdots, k$, then we have: $T_d = \sum_{i=1}^{k} T_i$.

Node mobility poses a great challenge in determining the geographical distribution of the nodes at an arbitrary instant. Here we present the analysis for scenarios where the nodes obey Poisson distribution through the network and the network
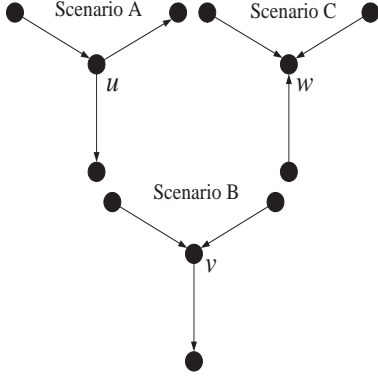
Fig. 5. Three interference scenarios

is connected. In such scenarios, the delay at each intermediate node has the same distribution. The major delay at each intermediate hop is introduced by DMTB's relay node election procedure, as indicated by Eqn. (1). To evaluate T_d, we first present the expectation of T_i.

Lemma 4.3: In a Poisson node cloud with density λ_0, the distance from any node to its nearest neighbor, denoted by r_0, follows the Rayleigh distribution with mean $\frac{1}{2\sqrt{\lambda_0}}$:

$$f_{R_0}(r_0; \lambda_0) = 2\pi \lambda_0 r_0 e^{-\pi \lambda_0 r_0^2} \qquad (2)$$

Proof: Let R_0 denote a random variable representing the distance to the nearest neighbor. When R_0 is greater than some fixed value r_0, we have no node occurrence within the disk of radius r_0. For a Poisson process, this probability is $\mathbb{P}\{R_0 > r_0\} = e^{-\lambda_0(\pi r_0^2)}$. Hence we can obtain the CDF (Cumulative Distribution Function) for r_0 as

$$F_{R_0}(r_0; \lambda_0) = \mathbb{P}\{R_0 \leqslant r_0\} = 1 - e^{-\pi \lambda_0 r_0^2} \qquad (3)$$

By computing its derivative we obtain the PDF $f_{R_0}(r_0; \lambda_0)$ as in Eqn. (2), which represents a Rayleigh distribution with mean $\frac{1}{2\sqrt{\lambda_0}}$. ∎

Based on Eqn. (1), we have:

$$\mathbb{E}(T_d) = \mathbb{E}(\sum_{i=1}^{k} T_i) = \mathbb{E}(\sum_{i=1}^{k} \mathcal{F}(d_i)) \qquad (4)$$

Based on Lemma 4.3, if we make $\mathcal{F}()$ a linear function, then the equation above can be further presented as:

$$\mathbb{E}(T_d) = \mathbb{E}(\sum_{i=1}^{k} \mathcal{F}(d_i)) = \sum_{i=1}^{k} \mathcal{F}(\mathbb{E}(d_i))$$
$$= \sum_{i=1}^{k} \mathcal{F}(\frac{1}{2\sqrt{\lambda_0}}) = k \cdot \mathcal{F}(\frac{1}{2\sqrt{\lambda_0}}) \qquad (5)$$

C. MAC Layer Interference

To theoretically analyze the interference during one DMTB broadcast, we observe the interference experienced by each relay node. In the ideal scenario when a node can be found exactly at each of the benchmark positions, we have following theorem:

Theorem 4.4: During a single DMTB broadcast, at least 50%-75% of the relay nodes will not experience a collision when receiving the broadcast message.

Proof: In each broadcast event, there exists a conservation law: the number of messages received at all relay nodes equals the number of messages sent out from all relay nodes. This is because each node relays the message at most once. In our broadcast constellation, each relay node has three neighbors that are also relay nodes. Fig. 5 shows three message sending/receiving scenarios:

1) Node u receives the message from 1 neighbor and relays it to 2 other neighbors;
2) Node v receives the message from 2 neighbors and relays it to 1 neighbor;
3) Node w receives the message from 3 neighbors. It does not relay the message anymore.

Denote the fraction of the nodes that belong to each of the three types of nodes by $N(u), N(v)$ and $N(w)$ respectively. We have:

$$N(u) + 2N(v) + 3N(w) = 2N(u) + N(v)$$
$$N(u) + N(v) + N(w) = 1 \qquad (6)$$

From equations above we can get: $0.5 \leq N(u) \leq 0.75$, which means that in the ideal scenario, at least 50%-75% of relay nodes receive the broadcast message exactly once and thus experience no collision. ∎

For the nodes that do receive multiple copies of the broadcast message from more than one neighbors (it is possible to receive another copy of the same message when nodes are backing off during the self-election procedure), interference only happens when these neighbors receive the message at exactly the same instant. In our protocol the deferring timer (shown in Eqn. 1) contributes to the differentiation of message delivery time to the same node through different paths. Thus the probability of occurrence of interference events is greatly decreased.

D. Scalability of DMTB

Although our protocol is based on building an imaginary global constellation, it does not incur much constellation maintenance overhead. The overhead involved is the constellation information carried in the broadcast message. This information is embedded in the broadcast message and does not affect DMTB's scalability. Scalability of DMTB concerns three parameters: the network size, node density and the number of broadcast sources. Regarding DMTB's scalability with respect to the network size and node density, we have the following theorem:

Theorem 4.5: The expected number of relay nodes employed in a single DMTB broadcast in a network of area $\mathcal{A}$ and *cell radius* r is estimated as $\frac{4 \cdot \mathcal{A}}{3\sqrt{3} \cdot r^2}$. Denoting the node density with λ and the theoretical number of relay nodes by $\mathcal{N}$, we have:

$$\lim_{\lambda \to \infty} \mathcal{N} \leq \frac{4 \cdot \mathcal{A}}{3\sqrt{3} \cdot 250^2}$$

Proof: In a broadcast constellation with *cell radius* r, the hexagon unit's area is $A_h = \frac{3\sqrt{3}r^2}{2}$. Then the number of

hexagons needed to tile a network of area $\mathcal{A}$ is $\frac{2A}{3\sqrt{3}r^2}$. As shown in Fig. 4, two extra relay nodes are needed to expand each hexagon. Thus, the number of relay nodes needed by DMTB is estimated as $\frac{4\cdot\mathcal{A}}{3\sqrt{3}\cdot r^2}$. When the node density exceeds a certain threshold, the constellation *cell radius* can be set as equal to the transmission range, 250m. Thus the theoretical number of relay nodes is bounded by $\frac{4\cdot\mathcal{A}}{3\sqrt{3}\cdot 250^2}$ ∎

Another scalability issue concerns multiple broadcast sources. When these sources broadcast concurrently, in the worst case scenario, a node has to maintain $m \geq 1$ (the number of concurrent broadcast) timers for the relay node self-election, as specified in Section III-B2. But these timers are temporary and can be discarded as soon as the broadcast ceases in the neighborhood.

E. Other Parameters

1) **Reliability:** Our protocol inherently provides reliability and resilience against node failures. If we denote a single node's failure rate by p_f, then the probability that relay fails at an arbitrary benchmark position is p_f^n, where n is the average number of nodes in one Voronoi cell: $n = \lambda \cdot \frac{3\sqrt{(3)}}{4}r^2$. From Fig. 1 it can be seen that a non-relay node might receive the broadcast message from multiple relay nodes.

2) **Network Dynamics:** The proposed protocol adapts to network dynamics easily since the broadcast does not depend on the presence of any node at any specific position. Nodes can move or switch to sleep mode. The absence of nodes in any Voronoi cell results in no broadcast message being sent in that cell. However, this does not mean that the broadcast is terminated since the broadcasts from nearby cells continue to propagate.

V. CAPACITY COST

In [6] the notion of network capacity for wireless networks was introduced and scaling laws for random and arbitrary networks was derived. For optimally placed nodes, it was shown that the *transport capacity* of the network where each node is capable of transmitting W bits/sec, is $\Theta(W\sqrt{An})$ bit-meters/sec where A is the area of the network and n is the number of nodes in the networks. If a particular broadcast scheme is inefficient, it uses a large fraction of the network capacity, thereby starving other communications in the network. *Thus the cost metric we use in this paper is the network transport capacity that is used by the broadcast mechanism.*

To evaluate the cost of a broadcast protocol, consider a uniform distribution of vehicles or nodes in the plane $\mathbb{R}^2$. For a point P in $\mathbb{R}^2$ and $r \in \mathbb{R}^+$, we represent by $D(P, r)$ the closed disk centered at P with radius r. Consider a subset A of the real plane $\mathbb{R}^2$. We define a cover of A as the set of disks $\mathcal{R} = \{D(P_i, r_i),\ P_i \in A,\ r_i \in \mathbb{R}^+,\ i \in I\}$, indexed by a countable set I such that: (a) the union of the disks of $\mathcal{R}$ contains A and (b) any compact of the plane only meets a finite number of disks of $\mathcal{R}$.

From the perspective of the broadcast protocol, the region A corresponds to the area to be covered and the points P_i correspond to the positions of the nodes chosen to forward the packet. Also, the radius r_i depends on the transmission power used by the nodes. The cost of any particular transmission depends on the transmission power or equivalently the range of the transmission (since other nodes in this range are not allowed to transmit during this period). We assume omni-directional antennas with a circular transmission pattern. We denote by ζ the unit capacity cost of a transmission in (bit-m/sec)$/m^2$ and thus the cost of transmitting a bit to a distance r_i is given by

$$C_i = \zeta\pi r_i^2 \qquad \text{bit-}m/\text{sec} \qquad (7)$$

Then the absolute cost of a cover (the cover may consist of partially overlapping disks) is

$$C_{abs} = \sum_{i \in I} C_i \qquad (8)$$

and the relative cost of the cover, considering that the plane $\mathbb{R}^2$ has its origin at O, is given by [4]

$$C_{rel} = \limsup_{t \to \infty} \frac{\sum_{i \in I, P_i \in D(O,t)} C_i}{\text{Area}(A \cap D(O,t))} \qquad (9)$$

We define a cover $\mathcal{R}$ indexed by I to be periodical if there is a finite set $J \subset I$ and two vectors u and v of $\mathbb{R}^2$ such that

$$\mathcal{R} = \bigcup_{(m,n) \in \mathbb{Z}^2} \{D(P_j + mu + nv,\ r_j),\ j \in J\} \qquad (10)$$

For periodic covers the expression for the relative cost may be further simplified. Consider a compact region $B \subset \mathbb{R}^2$ such that $\mathbb{R}^2 = \cup_{(m,n) \in \mathbb{Z}^2}(B + mu + nv)$ and such that the interiors of B and $B + mu + nv$ are disjoint for all $(m, n) \in \mathbb{Z}^2$. B, for example, could be a hexagon. The relative cost of the cover $\mathcal{R}$ is then given by [4]

$$C_{rel} = \frac{\sum_{j \in J} \pi r_j^2}{\text{Area}(B)}\zeta \qquad (11)$$

We now derive some properties related to the capacity cost of a broadcast protocol. The capacity cost of any broadcast protocol follows:

Proposition 5.1: For any $\epsilon > 0$, there exists a connected cover (i.e. the points P_i are in range of each other) whose relative cost is between ζ and $\zeta + \epsilon$.

To prove this proposition we use the results in the following lemmas.

Lemma 5.2: Consider a region A corresponding to the hexagon of side η centered at O. There exists a sequence of disks D_i contained in A whose interiors are pairwise disjoint and $\lim_{n\to\infty} \bigcup_{i=1}^{n} \text{Area}(D_i) = \text{Area}(A)$.

Proof: Consider the disk $D_1 = D\left(0, \frac{\sqrt{3}}{2}\eta\right)$. This is the unique disk that is tangent to the six sides of the hexagon. In every corner of the hexagon, we now have an uncovered region bounded by three arcs, one from the boundary of D_1 and the other two from the two sides of the the hexagon, which can be considered to be arcs of circles of length infinity. We consider D_2 to be the circle that is tangent to the inner circle D_1 and also to the two sides of the hexagon (akin to Soddy circles [15]). This partly fills the corner but creates three new, smaller, uncovered regions. We can now repeat this process

by partly filling each corner with a smaller Soddy circle. It is well known that the union of these circles will have the same area as the region they are covering, i.e. A [7]. Also, by construction, the circles are disjoint. ∎

Lemma 5.3: Let B be a compact region of the real plane, $B \subset \mathbb{R}^2$. For each $\eta > 0$, there exists a cover $\mathcal{R}(\eta)$ of B such that

$$\text{cost}(\mathcal{R}(\eta)) \leq \alpha\zeta\text{Area}(B) + e(\eta) \tag{12}$$

where $\alpha = \frac{2\pi}{3\sqrt{3}}$ is the density of the simple hexagonal cover and $\lim_{\eta \to 0} e(\eta) = 0$.

Proof: This lemma is proved in Lemma 5 of [4]. ∎

Lemma 5.4: The relative cost of broadcasting on a hexagon, for a given $\epsilon > 0$ is at most $\zeta + \epsilon$.

Proof: Consider the sequence of disks D_n as in Lemma 5.2 covering a hexagon of side η. We denote by a_n the area of the uncovered region of the hexagon

$$a_n = \frac{3\sqrt{3}}{2}\eta^2 - \text{Area}(D_1 \cup D_2 \cup \cdots \cup D_n) \tag{13}$$

We now pick the value of n, $n \in \mathbb{Z}$ such that $a_n \leq \frac{\epsilon}{2\zeta\alpha}\left(\frac{3\sqrt{3}}{2}\eta^2 - 1\right)$. We denote by β the region $\beta = A - \cup_{i=1}^{n}D_i$ and choose a cover of $\mathcal{R}(\eta')$ of β as in Lemma 5.3 such that $\text{cost}(\mathcal{R}(\eta')) \leq \alpha\zeta\text{Area}(\beta) + \frac{\epsilon}{2}$. Consider the cover $\mathcal{P}$ of the hexagon given by $\mathcal{P} = \mathcal{R}(\eta') \cup (\cup_{i=1}^{n}D_i)$. The cost of this cover is

$$\begin{aligned}
\text{cost}(\mathcal{P}) &= \text{cost}(\mathcal{R}(\eta')) + \text{cost}(\cup_{i=1}^{n}D_i) \\
&\leq \left[\alpha\zeta\text{Area}(\beta) + \frac{\epsilon}{2}\right] + \zeta\text{Area}(\cup_{i=1}^{n}D_i) \\
&\leq \alpha\zeta a_n + \frac{\epsilon}{2} + \zeta\frac{3\sqrt{3}}{2}\eta^2 \\
&= \alpha\zeta\left[\frac{\epsilon}{2\alpha\zeta}\left(\frac{2\sqrt{3}}{2}\zeta^2 - 1\right)\right] + \frac{\epsilon}{2} + \frac{3\sqrt{3}}{2}\zeta\eta^2 \\
&= \frac{3\sqrt{3}}{2}\eta^2\left(\zeta + \frac{\epsilon}{2}\right) \tag{14}
\end{aligned}$$

The relative cost of the cover $\mathcal{P}$ is obtained by dividing the cost above by the area of the hexagon being covered:

$$\frac{\text{cost}(\mathcal{P})}{\frac{3\sqrt{3}}{2}\eta^2} \leq \zeta + \frac{\epsilon}{2} \tag{15}$$

∎

Using the results above we now prove the result in Proposition 5.1.

Proof: The unit capacity cost associated with transmitting a packet is ζ (bit-m/sec)/m^2. Thus it is obvious that the relative cost of any cover is at least ζ.

Now for any $\zeta > 0$ consider a plane tiling of the region A by hexagons of size η. Let S be the set of such hexagons B satisfying $B \cup A \neq \phi$. Now we cover each hexagon of S by a cover of relative cost less than $\zeta + \frac{\epsilon}{2}$, as done in Lemma 5.4. The union of these covers for each of the hexagons is a cover, say $\mathcal{P}$, of A and contains the points in the area

$$A(\mathcal{P}) = \{x : x \in \mathbb{R}^2, \text{ dist}(x, A) \leq 2\eta\} \tag{16}$$

Since the cost of each hexagon is less that $\zeta + \frac{\epsilon}{2}$, the total cost of the cover is less than $\left(\zeta + \frac{\epsilon}{2}\right)\text{Area}(\mathcal{P})$. Now,
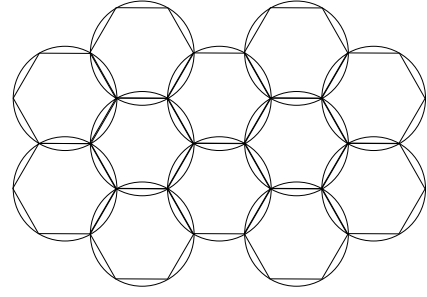

Fig. 6.   Simple hexagonal cover

$\lim_{\eta \to 0}\text{Area}(\mathcal{P}) = \text{Area}(A)$. Then we can choose an η such that

$$\text{Area}(\mathcal{P}) \leq \text{Area}(A)\left(1 + \frac{\epsilon}{2\left(\zeta + \frac{\epsilon}{2}\right)}\right) \tag{17}$$

With this choice of η

$$\begin{aligned}
\text{cost}(\mathcal{P}) &= \left(\zeta + \frac{\epsilon}{2}\right)\text{Area}(\mathcal{P}) \\
&\leq \left(\zeta + \frac{\epsilon}{2}\right)\text{Area}(A)\left(1 + \frac{\epsilon}{2\left(\zeta + \frac{\epsilon}{2}\right)}\right) \\
&= (\zeta + \epsilon)\text{Area}(A) \tag{18}
\end{aligned}$$

The relative cost of the cover is thus

$$\frac{(\zeta + \epsilon)\text{Area}(A)}{\text{Area}(A)} = \zeta + \epsilon \tag{19}$$

∎

A. Capacity cost of the Proposed Scheme

The proposed scheme is a hexagonal constellation based broadcast protocol. Fig. 6 shows the best possible case for a hexagonal cover of a region in $\mathbb{R}^2$. Assuming that the radius of the circles and thus each side of the hexagons is η, the relative cost of this cover is

$$C_{rel} = \frac{\pi\eta^2}{\frac{3\sqrt{3}}{2}\eta^2}\zeta = 1.209\zeta \tag{20}$$

However this cover is not connected. For the broadcast case, we keep the same radius and side lengths but move the centers of the circles to the hexagon vertices. The cost of this cover is twice that of the previous cover:

$$C_{rel} = 2\frac{\pi\eta^2}{\frac{3\sqrt{3}}{2}\eta^2}\zeta = 2.418\zeta \tag{21}$$

Thus the capacity cost of our hexagonal broadcast scheme is at least 2.418 times the minimum possible.

VI. SIMULATION RESULTS

In this section we present the simulation results. The proposed broadcast protocol was implemented in the *ns-2* simulator. IEEE 802.11 is assumed as the MAC layer protocol. If not stated otherwise, nodes are initially uniformly deployed in the concerned areas. The transmission range of each node is 250m, and the broadcast message payload is 100 bytes. The

transmission rate of each node is 1Mbps. We use the following performance indicators for our evaluation:

- **Broadcast Effectiveness**: reflected by the broadcast message's delivery rate, which is defined as the percentage of nodes that successfully receive the broadcast message;
- **Broadcast Efficiency**: reflected by the percentage of nodes that relay the broadcast message. Since nodes will relay the message at most once, the number of relay nodes equals the number of message retransmissions.
- **Collision Rate**: the average number of message collisions during each broadcast;
- **End-to-End Delay**: defined as the time it takes for all nodes to cease receiving/sending the broadcast message;
- **Fairness**: reflected by the number of times a given node is chosen for broadcasting. We denote the number of times a node is selected for relaying at time t by the vector $e(t)$. We observe $e(t)$'s coefficient of variance (CV), which is the ratio of $e(t)$'s standard deviation to its mean, with $CV = 0$ being the fairest scenario.

The mobility model used in the simulations is the "Random Direction Model". In this model, a node travels in a pre-picked random direction and a random speed until it reaches the area boundary, where it chooses a different direction and speed to continue moving. Each mobility pattern file is generated with a given average node speed V_{avg}, and each node's speed is randomly chosen between 0 and $2V_{avg}$.

In this set of simulations, the size of ad-hoc network area is $5000 \times 5000 m^2$. We examined the protocol performance for different node densities (sparse and dense) and the simulations were carried for 200 broadcast events. Each broadcast event was generated at a random source and were fed into the network at random instants of time. The results presented are the average over these 200 events. Our results are compared with the results of two protocols proposed in literature: Mobility Management (MM) [19] and Scalable Broadcast Algorithm (SBA) [12]. These two protocols are chosen for comparison since they outperform most of other protocols in mobile environments [18].

1) Broadcast Effectiveness/Efficiency vs. Node Mobility: We first verify the proposed protocol's effectiveness/efficiency in Fig. 7 and Fig. 8. Fig. 7 presents DMTB, MM and SBA's broadcast message delivery rate under different node speeds. Fig. 8 shows the percentage of nodes that are employed by the three protocols as a function of the node speed. While SBA has high delivery rates, its percentage of forwarding nodes is much higher than that of DMTB, especially when nodes move fast. On the other hand, while MM employs only a small fraction of nodes for forwarding, its delivery rates are much smaller. It is worth noting that the performance of the proposed algorithm does not degrade as nodes move faster.

2) Collision vs. Node Mobility: In Fig. 9 we plot the number of collision occurrences during each broadcast as a function of the nodes' average speed. As can be seen from the figure, the occurrence rate stays fairly low for DMTB. Collision occurrences of MM and SBA are also observed, and we note that unlike DTMB, their performance degrades as the node density increases. For SBA, now more neighbors compete for relaying the broadcast messages which unnecessarily

leads to high collisions.

3) Broadcast End-to-end Delay vs. Node Mobility: The end-to-end delays in networks with different node speeds and node densities are presented in Fig. 10. In these simulations, the deferring timer function $\mathcal{F}(d_i)$ at each node randomly picks a value between $[0, 10\mu s \cdot d_i]$ ($10\mu s$ is picked with reference to the typical slot time of IEEE 802.11), where d_i is the distance from the relay node i to its corresponding benchmark position. The end-to-end delay for DMTB stays around 10ms even when the node density is relatively low and is much lower than both MM and SBA.

4) Fairness vs. Node Mobility: In Fig. 11 we show the CV of the capacity consumed at each node for broadcasting. From Fig. 11 we can see that the system CV is fairly small for different node densities, and more importantly, it stays roughly the same as the node average speed changes from 20km/hr to 160km/hr. We also note that DMTB's fairness is much better than MM and SBA.

A. Protocol Robustness

To verify the protocol's robustness against node failures or its applicability in network areas of irregular shape, we simulate broadcasting in a $2500 \times 2500 m^2$ area shown in Fig. 12. The node density is 1.5×10^{-4}, and static nodes are uniformly distributed in the area except the subarea denoted by "-", where no nodes are present. With the source node randomly picked in each broadcast, 100 broadcast messages are sent throughout the simulation. The delivery rate is observed to be 100%. With non-relay nodes denoted by the light color "·" and relay nodes by "*", Fig. 12 shows a snapshot of the distribution of relay nodes for one broadcast event.

B. Protocol Scalability

In this section we evaluate the scalability of DMTB as number of nodes in the network and the network size changes. The results in this section also compare the protocol's performance with that of the "AHBP (Ad Hoc Broadcast Protocol)" algorithm previously proposed in literature [13]. The reason for choosing AHBP for these results is that the authors of [3] have shown that the scalability properties of AHBP are superior to other protocols.

To observe the effect of node density, we simulate a $2500 \times 2500 m^2$ area and vary the number of nodes from 600 to 1200. 100 broadcast messages are sent out by arbitrarily picked source nodes. Fig. 13 shows DMTB and AHBP's average delivery rate for different node densities and different constellation *cell radius* (r). We note that while DMTB's delivery rate stays near 100% even for a fairly low density, AHBP performs well only with higher node densities. AHBP's broadcast delivery rate increases directly rely on more relay nodes, while the number of relay nodes employed by DMTB stays roughly the same as the node density increases, as shown in Fig. 14.

We now observe the effect of the network size on various aspects of DMTB's performance. In these results, network area is a square and its side length is varied from $500m$ to $2500m$ with 500m as the step size. For each area size,
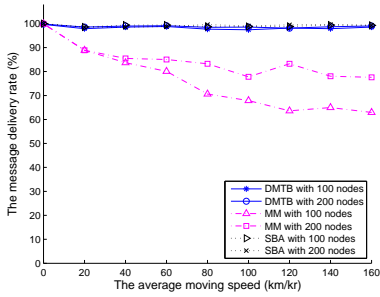
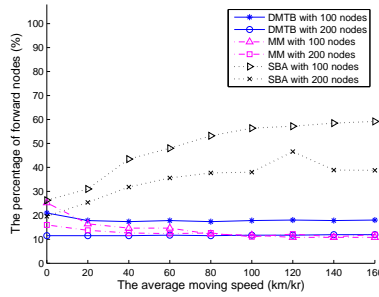Fig. 7. The delivery rate with different node average speeds.

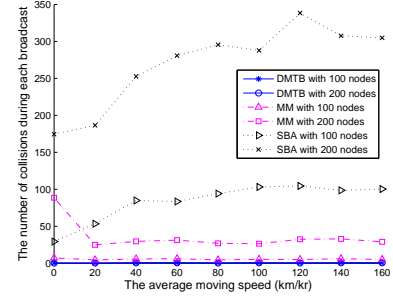Fig. 8. The amount of forwarding nodes with different node average speeds.

Fig. 9. The average number of collisions per broadcast with different node average speeds.
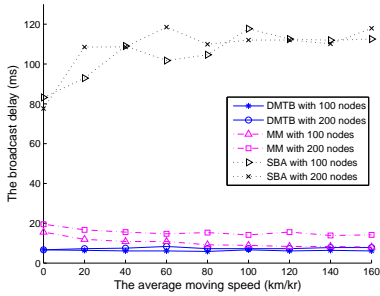

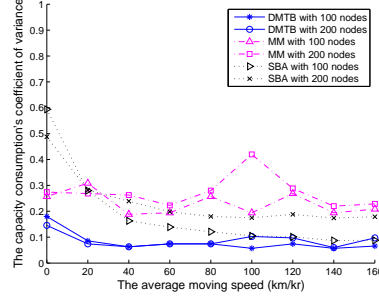Fig. 10. The average broadcast delay with different node average speeds.

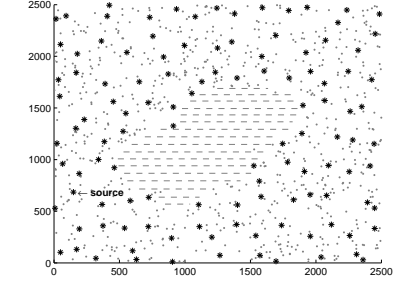Fig. 11. The capacity consumption's coefficient of variance with different node average speeds.

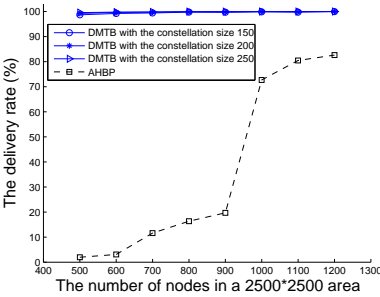Fig. 12. Broadcasting in a network area of irregular shape.

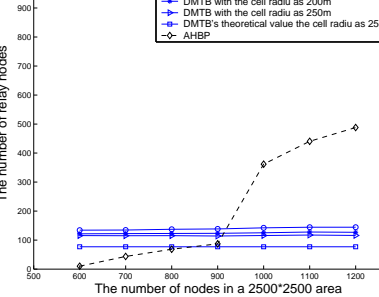Fig. 13. The delivery rate to different node densities and different constellation *cell radii r*.

Fig. 14. The percentage of relay nodes to different node densities and constellation *cell radii r*.
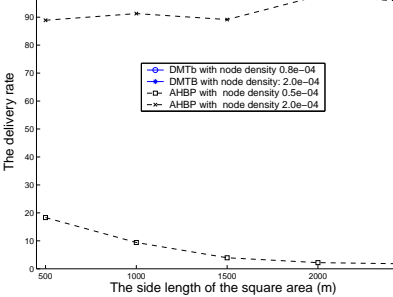

Fig. 15. The delivery rate with different network area size and node densities.

results for high and low node densities are presented. 100 messages are generated from randomly picked source nodes and broadcast to the whole network. The imaginary *cell radius* is kept as 250m throughout the simulations. In Figs. 15, 16, 17 and 18 we compare the delivery rate, percentage of relay nodes, average number of collision occurrences and end-to-end delays, respectively, of DMTB and AHBP. For all metrics, DMTB outperforms AHBP and more importantly, its performance scales gracefully with both the network size as well as the node density.

VII. CONCLUSION

In this paper we presented a cross-layer designed, constellation based broadcast protocol for vehicular networks. The protocol's performance is bounded to be within a constant of the optimum. The procedure for choosing the relay node for each benchmark position is probabilistic so that the protocol does not rely on any specific node to relay the message. Thus, node mobility is accommodated and protocol resilience is enhanced. Both protocol efficiency and fairness are taken into consideration. Analytic and simulation results are presented to address and verify DMTB's performance.

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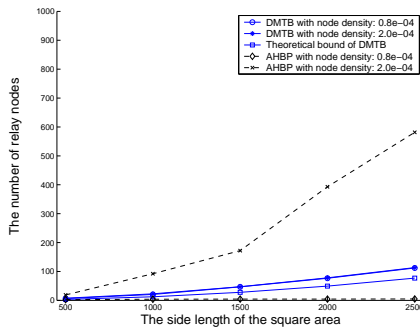
Fig. 16. The percentage of relay nodes with different area sizes and node densities.
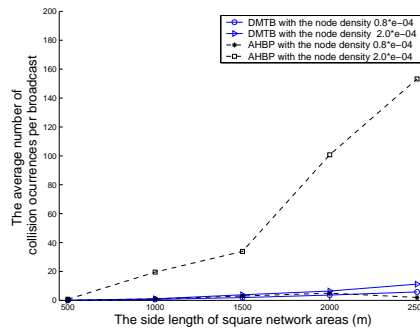

Fig. 17. The average number of collisions per broadcast with different network sizes.
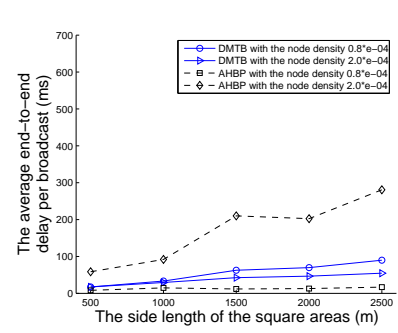

Fig. 18. End-to-end broadcast delay to network areas of different sizes and node densities.

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APPENDIX A
PROOF OF THE PLANE TILING THEOREM

Proof: Let m denote the number of vertices of a m-polygon and n the number of m-polygons needed to tile 2π degrees. We have the following equation:

$$\frac{(m-2)n\pi}{m} = 2\pi$$
$$(m-2)(n-2) = 4$$

Since both m and n are integers, the only solution for two integers' product to be 4 is $(1,4), (2,2)$ and $(4,1)$. Thus the three solutions of m is $3, 4, 6$, which is triangle, square or hexagon. ∎

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