# A Multistate Markov Model for Dimensioning Solar Powered Cellular Base Stations

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***Abstract*—The dimensioning of photovoltaic (PV) panel and battery sizes is one of the major issues regarding the design of solar powered cellular base stations (BSs). This letter proposes a multistate Markov model for the hourly harvested solar energy to determine the cost optimal PV panel and battery dimensions for a given tolerable outage probability at a cellular BS.**

***Index Terms*—Green communications, solar energy.**

## I. Introduction

SOLAR POWERED, offgrid cellular base stations (BSs) provide a communication infrastructure in places without reliable grid power. This letter presents a Markov model for hourly solar energy and applies it to dimensioning offgrid cellular BSs. Existing Markov models for solar energy lack the day-level weather correlations that are critical for dimensioning high-reliability systems [1], [2]. Thus, we propose a model that combines hourly and daily transitions in the weather conditions.

## II. Background Details

This letter considers a long-term valuation (LTE) cellular BS whose power consumption at time $t$ is given by [3]

$$P_{\mathrm{BS}}(t) = N_{\mathrm{trx}}(P_0 + \Delta_p P_{\max} K), \qquad 0 \le K \le 1 \tag{1}$$

where $N_{\mathrm{trx}}$ is the number of transceivers, $P_0$ is the power consumption at no load (zero traffic), $\Delta_p$ is a BS specific constant, $P_{\max}$ is the output of the power amplifier at the maximum traffic, and $K$ is the normalized traffic at the given time.

To model the traffic, Poisson distributed call arrivals with time-of-day dependent rates, and exponentially distributed call durations with mean 2 min are used [4]. $K$ is obtained by normalizing the instantaneous traffic by the maximum number of calls that the BS can support at any time. We assume that lead acid batteries are used. The battery lifetime is calculated by counting the charge/discharge cycles for each range of depth of discharge (DoD) for a year and is given by [5]

$$L_{\mathrm{Bat}} = 1 \Bigg/ \left( \sum_{i=1}^{N} \frac{Z_i}{\mathrm{CTF}_i} \right) \tag{2}$$

where $Z_i$ is the number of cycles with DoD in region $i$, and $\mathrm{CTF}_i$ is the cycles to failure corresponding to region $i$. Given $n_{\mathrm{PV}}$ photovoltaic (PV) panels each with dc rating $E_{\mathrm{panel}}$, and $n_b$ batteries, each with capacity $E_{\mathrm{bat}}$, the overall PV panel dc rating is $\mathrm{PV}_w = n_{PV} E_{\mathrm{panel}}$, and the battery bank capacity is $B_{\mathrm{cap}} = n_b E_{\mathrm{bat}}$. This letter uses solar irradiance data made available by National Renewable Energy Laboratory (NREL), USA [6].

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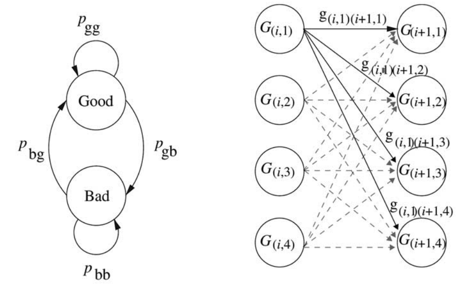


(a) (b)

Fig. 1. (a) Transition between good and bad days. (b) Hourly transition in a good day. For clarity, only the transitions from state $G_{(i,1)}$ are marked.

## III. Model Description

To develop the solar energy model, for any site, solar irradiance data of 10 years are fed into NREL's System Advisor Model tool [6] to calculate the hourly energy generated by a PV panel with 1-kW dc rating. This data is then parsed on a monthly basis. The solar energy output for each day in a given month is computed and the days are sorted based on this energy. $\beta\%$ of the days with the lowest energy are termed "bad," and the rest, "good" days. The probability of transition from one day type to another is calculated from the data. This is modeled as a Markov process [Fig. 1(a)] with transition matrix

$$\mathbf{T} = \begin{bmatrix} p_{\mathrm{gg}} & p_{\mathrm{gb}} \\ p_{\mathrm{bg}} & p_{\mathrm{bb}} \end{bmatrix} \tag{3}$$

where $p_{\mathrm{gg}}$ ($p_{\mathrm{bb}}$, respectively) is the transition probability from good to good (bad to bad), and $p_{\mathrm{gb}} = 1 - p_{\mathrm{gg}}$ ($p_{\mathrm{bg}} = 1 - p_{\mathrm{bb}}$, respectively) is the transition probability from good to bad (bad to good) day.

Within a day, the harvested solar energy varies with time. We model these variations on a hourly basis as a Markov process. For each day type (good/bad), the minimum and maximum PV panel output for each hour of the day are calculated. The region between the minimum and maximum values is divided uniformly into four regions, as shown in Fig. 2. Each of these regions, along with the day type, represents a "state" of the harvested solar energy. The state at time $t$ is denoted by

$$S_t : S_t \in \{G_{(x,y)}, B_{(x,y)}\}, \quad x \in \{1, 2, \ldots, 24\}, \quad y \in \{1, 2, 3, 4\}$$

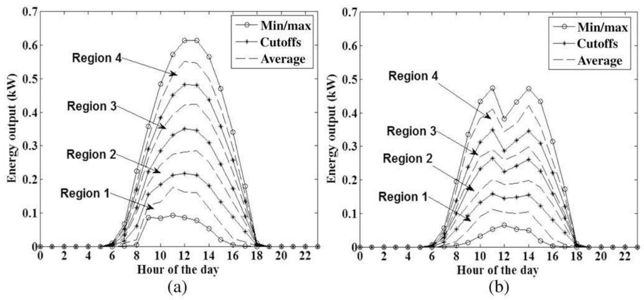


Fig. 2. (a) States for a good day. (b) States for a bad day.

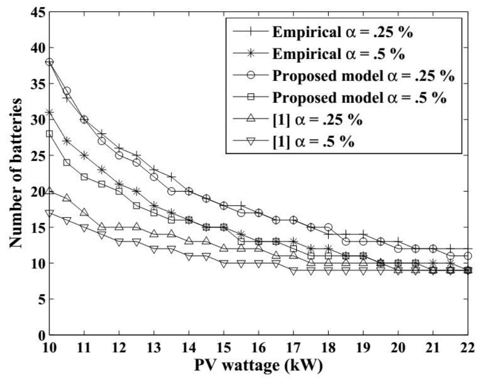


Fig. 3. PV wattage versus the number of batteries required for various outage probabilities for Kolkata.

where $G$ and $B$ indicate good and bad days, respectively, $x$ is the hour of the day, and $y$ is the region for the solar energy. The irradiance data are used to calculate the average hourly solar radiation $E_{S_t}$ for each state and the state transition probabilities. For good days, the state transition matrix is given by

$$\mathbf{G} = \begin{bmatrix} g_{(1,1)(1,1)} & \cdots & g_{(1,1)(24,4)} \\ \vdots & \ddots & \vdots \\ g_{(24,4)(1,1)} & \cdots & g_{(24,4)(24,4)} \end{bmatrix} \tag{4}$$

where $g_{(i,j)(k,l)}$ is the probability of transition from region $j$ of hour $i$ to region $l$ in hour $k$ on a good day. Note that from a state in a given hour, a transition can only be made to one of the states in the next hour, as shown in Fig. 1(b). The transition matrix for bad days $\mathbf{B}$ is similarly defined.

Consider a solar powered BS with a PV panel dc rating $\text{PV}_w$ and battery capacity $B_{\text{cap}}$. The multistate Markov model proposed above gives the solar energy generated by a PV panel with dc rating of 1 kW. For a panel with rating $\text{PV}_w$, the energy generated at time $t$ (in hours) in state $S_t$ is given by

$$E(t) = \text{PV}_w E_{S_t}. \tag{5}$$

To avoid deep discharges which adversely affect the battery life, we disconnect the battery from the system when the overall charge level goes below 70% DoD. Let $B(t)$ denote the battery power at time $t$. Using the notation $B'(t) = B(t-1) + E(t) - P_{\text{BS}}(t)$, $B(t)$ is then given by

$$B(t) = \begin{cases} B_{\text{cap}}, & B'(t) \geq B_{\text{cap}} \\ B(t-1) + E(t) - P_{\text{BS}}(t), & 0.3B_{\text{cap}} < B'(t) < B_{\text{cap}} \\ 0.3B_{\text{cap}}, & B'(t) \leq 0.3B_{\text{cap}} \end{cases} \tag{6}$$

with $B(0) = B_{\text{cap}}$. The hours when the battery level is either less than or equal to $0.3B_{\text{cap}}$ correspond to outage events. The outage probability is denoted by $O$ and is given by

$$O = H_{\text{outage}}/H \tag{7}$$

where $H_{\text{outage}}$ is the number of outage hours and $H$ is the total hours of operation. The optimal PV panel and battery dimensioning problem is to determine the least cost configuration in order to satisfy a limit on the outage probability. The cost optimization problem can be expressed as

$$\begin{aligned} \text{Minimize}: \quad & N_{\text{Bat}} C_B + \text{PV}_w C_{\text{PV}} \\ \text{Subject to}: \quad & O < \alpha \end{aligned}$$

where $C_B$ is the unit battery cost, $C_{\text{PV}}$ is the PV panel cost per kW, and $\alpha$ is the operator's limit on the outage probability. The number of batteries required $N_{\text{Bat}}$ is given by

$$N_{\text{Bat}} = n_b (T_{\text{run}}/L_{\text{Bat}}) \tag{8}$$

where $T_{\text{run}}$ is the desired operational system lifetime. The optimization problem can be solved using standard techniques.

TABLE I
OPTIMAL CONFIGURATION FOR VARIOUS OUTAGE PROBABILITIES

| Location | Empirical PV | Empirical $n_b$ | [1] PV | [1] $n_b$ | Proposed model PV | Proposed model $n_b$ |
|---|---|---|---|---|---|---|
| $\alpha = .25\%$ | | | | | | |
| Kolkata | 14 | 20 | 10.5 | 19 | 13.5 | 20 |
| Miami | 11 | 20 | 10 | 15 | 11 | 20 |
| $\alpha = .5\%$ | | | | | | |
| Kolkata | 12 | 21 | 10 | 17 | 11 | 22 |
| Miami | 10.5 | 18 | 9.5 | 14 | 10.5 | 18 |

## IV. RESULTS

To validate our model, we consider two cities: 1) Kolkata (India) and 2) Miami (USA). We consider an LTE BS with 10-MHz bandwidth and 2×2 multiinput multioutput configuration. The BS is assumed to have three sectors, each with two transceivers ($N_{\text{TRX}} = 6$). We assume that 12 V, 205 Ah flooded lead acid batteries are used in the BS. The results use $T_{\text{run}} = 20$ years and based on market statistics, we use $C_B$ of US$ 280 and $C_{\text{PV}}$ of US$ 1000. Our solar energy model uses $\beta = 15\%$.

Fig. 3 shows the number of batteries required for a given PV panel size for Kolkata as obtained from our model, empirical data, and the model proposed in [1]. The optimal cost configuration predicted by the three methods for the two locations are shown in Table I. Our model has a close match to the empirical results and greater accuracy compared to [1].

## V. Conclusion

This letter proposed a multistate Markov model for characterizing the hourly solar irradiation. The model was used for dimensioning solar powered cellular BSs in terms of the cost optimal PV panel and battery bank size.

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