Comparison of Broadcasting Schemes for Infrastructure to Vehicular Communications

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***Abstract*—A large set of potential applications being designed for Intelligent Transportation Systems (ITS) depend on the broadcasting of information and control packets by roadside infrastructure points to vehicles in their vicinity. This paper considers the *broadcast capacity* of broadcast schemes and evaluates and compares the broadcast capacity of strategies based on time-splitting, frequency-splitting and superposition coding. Frequency-splitting is shown to always dominate time-splitting and the conditions under which superposition coding dominates the other two are derived. For these regimes, it is shown that the broadcast capacities associated with superposition coding are optimal. A proportionally fair algorithm for scheduling broadcast packets is then proposed and its performance compared against other schedulers.**

***Index Terms*—Broadcasting, multiplexing, scheduling**

I. Introduction

The development of Intelligent Transportation Systems promises to revolutionize vehicular technology in terms of safety, efficiency, entertainment and environmental impact. Many ITS systems depend on the timely transfer of data between vehicles and roadside infrastructure points. This paper addresses the problem of developing a strategy for broadcasting in infrastructure to vehicle communications. The infrastructure based broadcasts considered in this paper are for the scenarios where vehicles are given information based on their location. Such broadcasts in vehicular networks may be used by numerous applications, the most important of them being safety and traffic information related [13], [17], [21]. For example, lane closure information at toll plazas, construction related lane closures and delays, information about unsafe road conditions (e.g. icy bridges), changes in speed limits due to weather conditions or construction etc. may all be broadcast by roadside infrastructure points. Also, while many forms of emergency messages originate at the vehicles, a subsequent broadcast by an infrastructure point may be a more efficient means of distributing such messages. Broadcasts by infrastructure points may also be used to distribute non-critical information such as expected travel times, available facilities in a rest area, information on parking availability etc. There are also commercial applications that may benefit from infrastructure point based broadcasts such as advertisements by establishments in a neighborhood.

In typical ITS system applications listed above such as traffic advisories, road condition information, local maps and restaurants, etc. a roadside infrastructure point may want to deliver different information to vehicles in different geographical locations. There is an inherent tradeoff between the rate of transmissions and the distance till which they may be correctly decoded, as characterized by the well known Shannon's channel coding theorem [19]. Thus a key decision that the scheduler associated with a broadcasting station has to make is: *which region to broadcast to and at what rate, in order to maximize the throughput while maintaining fairness?* This is the problem addressed in this paper. Note that this paper is interested only in the one hop transmissions from an infrastructure point (that are assumed to cover their intended region) and not multi-hop transmissions to expand the scope of the broadcast.

To address this problem, this paper uses the notion of *broadcast capacity*, analogous to the notion of transport capacity developed in [7], to determine the utility associated with a broadcast packet. In [7], the transport capacity of a transmission is defined as the product of the rate and the distance it traverses. In our case, we modify this definition by giving multiple credit for broadcast packets, corresponding to the number of receivers of the packet, as done in [18]. This paper compares the broadcast capacity of various transmission strategies and evaluates the conditions under which one strategy dominates the other. Finally, a proportionally fair scheduling policy is proposed that strives to maximize the broadcast utility while maintaining fairness.

For a given bandwidth and transmission power, a typical broadcast strategy would be to either use the entire available bandwidth to transmit to different regions over different periods of time (time-splitting) or to split the available bandwidth into non-overlapping bands and transmit to different regions simultaneously over different bands (frequency-splitting). Existing literature focusing on the communications from infrastructure points to vehicles are primarily based on variations of the two schemes above. In [9] the author proposes variations of centralized and decentralized time-splitting mechanisms. Technologies based on both time and frequency-splitting for infrastructure to vehicular communications are considered in [10], [12], [15]. The upcoming IEEE 802.11p WAVE standards for infrastructure to vehicular communications and the US Department of Transport's Dedicated Short Range Communications System (DSRC) is also based on time-splitting [6] while a multi-channel variant is considered in [14]. Much of the existing literature on broadcast protocols for vehicular networks are for the multi-hop case with the focus on reducing the number of re-broadcasts and are aimed primarily at ad hoc networks and vehicle-vehicle communications [1], [3], [11],

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[20]. These papers do not address the problem considered in this paper. Finally, while code division multiplexing may also be used for vehicular communications [4], [16], the dynamic reallocation of codes as vehicles move to different parts of the network introduces additional overhead and is thus not considered in this paper.

This paper first compares the effectiveness of schemes based on frequency and time-splitting with a third mechanism that uses superposition coding [5]. This paper shows that while the broadcast capacity of frequency-splitting always dominates that of time-splitting, under certain conditions, superposition coding may dominate both time and frequency-splitting. Unlike [2] where it is shown that the rates achievable by superposition coding *always* dominate that of time and frequency-splitting, we show that this result does not hold in all cases when *broadcast capacity* is considered as the metric. The paper then proposes a broadcast scheduling algorithm that maintains proportional fairness. Simulation results are used to evaluate the performance of the proposed scheduler.

The rest of the paper is organized as follows. Section II compares the broadcast utilities of different transmission strategies and derives the conditions under which superposition coding dominates other strategies. Section III proves the optimality of the utilities associated with superposition coding and Section IV presents the proportionally fair scheduling algorithm. Finally, Section V presents the simulation results while Section VI presents the concluding remarks.

II. Communication Strategies

This section considers three communication strategies and evaluates them in terms of their broadcast capacity: time-splitting, frequency-splitting and superposition coding. While the Shannon capacity of these schemes is well known [2], this paper considers the slightly different notion of broadcast capacity. While superposition coding always dominates the other two transmission schemes in terms of the Shannon capacity [2], we show that this is not necessarily the case when considering the broadcast capacity.

A. Transport Capacity

This section formally defines the notion of the broadcast capacity of a broadcast packet based on [7], [18]. Consider a communication system with a transmission power of P watts where the transmitter and receiver are separated by a distance of d meters. The channel bandwidth is assumed to be W Hz and the communication channel is subject to additive white Gaussian noise with power spectral density of N_o watts/Hz. A transmitted signal is assumed to decay according to $d^{-\alpha}$ with distance, where α is the channel attenuation constant and assumed to be $\alpha > 2$. All antenna and system parameters are assumed to be 1. Shannon's theorem on channel capacity then states that considering all coding schemes, the largest rate C at which the transmitter may send messages with arbitrarily low bit error rates to the receiver is given by [19]

$$C = W \log_2 \left(1 + \frac{P d^{-\alpha}}{W N_o} \right) \tag{1}$$

In this paper, the terms "error free communication" and "communication with arbitrarily low bit error rates" are used interchangeably, as is the practice in literature.

The distance till which a given rate may be supported with arbitrarily low error rates depends on the transmit power and the ambient noise. Since a broadcast packet conveys information targeted to all nodes that receive it, the broadcast capacity associated with the transmission is a function of the transmission rate, the distance it traverses, as well as the number of nodes that receive the packet. In this paper we are only interested in one hop broadcast data and not multi-hop traffic. Thus the distance a message propagates due to a broadcast transmission is not important in our context. Consequently, we interpret the broadcast capacity as a quantity that is proportional to the product of the data rate and the number of receivers of the broadcast data. If the region around the transmitter where vehicles may successfully receive the broadcast packet can be described by a circular region with radius d_{max}, the broadcast capacity of the transmission is defined as [18]

$$U = C(d_{max})^{\gamma} \tag{2}$$

where γ is a parameter that can be selected to represent the relationship between the transmission range and the number of vehicles. The parameter is bounded by $1 \le \gamma < 2$ since the road lengths and parking areas grow at least linearly with the radius but not faster than $(d_{max})^2$. In the context of vehicular networks, the broadcast capacity as defined above gives an indication of the effectiveness of each transmission in terms of the number of vehicles it serves and the rate at which it transmits data.

Commercial wireless communication systems are built to transmit at one or more predefined rates. For example, devices complying to the IEEE 802.11a standards may transmit at 6, 9, 12, 18, 24, 36, 48 and 54 Mbps and employ different modulation and coding schemes for different rates. Given that a transmitter needs to convey information to receivers at geographically diverse locations, different modulation and coding schemes may be used to select the rate and split the available power and bandwidth. In this paper, we assume that two modulation and coding schemes are available to the infrastructure point transmitter corresponding to rates of $C1$ and $C2$ (in contrast, existing work such as [18] assume that a node may transmit at any arbitrary rate, leading to different results). Without loss of generality, we assume that $C1 < C2$. While we expect our results to generalize for larger number of modulation and coding schemes, extension of our results for these cases remains an open problem. The broadcast capacity of the three communication strategies are evaluated next.

B. Time Splitting

In the time-splitting strategy, the transmitter alternates between transmitting at rates of $C1$ and $C2$ and uses the entire spectrum and power for each rate. We denote by τ the fraction of time that the transmitter devotes to rate $C1$ while the remaining $1 - \tau$ is spent transmitting at rate $C2$. Let d_1 and d_2 denote the maximum distance till which error free transmissions may be received at rates of $C1$ and $C2$, respectively. From Eqn. (1) we then have

$$d_i = \left[\frac{P}{WN_o(2^{Ci/W}-1)}\right]^{\frac{1}{\alpha}} \tag{3}$$

where d_i and Ci, $i = 1, 2$ correspond to the two modulation and coding schemes. Since a transmission rate of $C1$ is achieved for a fraction τ of the time while $C2$ is achieved for $1-\tau$, the broadcast capacities that can be achieved by time-splitting is given by

$$U_1^{TS} = \tau C1 \left[\frac{P}{WN_o(2^{C1/W}-1)}\right]^{\frac{\gamma}{\alpha}} \tag{4}$$

$$U_2^{TS} = (1-\tau) C2 \left[\frac{P}{WN_o(2^{C2/W}-1)}\right]^{\frac{\gamma}{\alpha}} \tag{5}$$

and τ may be varied in the interval $[0, 1]$ to obtain the entire range.

C. Frequency Splitting

With frequency-splitting, transmissions at both rates may be carried out simultaneously by splitting the available bandwidth into two non-overlapping bands. Additionally, the available power may also be split between the two transmissions to control the distance till which error free communications may be made at either rate. Denote by δ and $1-\delta$, $0 \le \delta \le 1$, the fraction of the bandwidth allocated to transmissions at rate $C1$ and $C2$, respectively. Also, let ϵ and $1-\epsilon$ denote the fraction of available power devoted to transmissions at rate $C1$ and $C2$, respectively. For a receiver at a distance d from the sender transmitting with power ϵP and bandwidth δW, the maximum achievable error free communication rate is

$$C = \delta W \log_2\left(1 + \frac{\epsilon P d^{-\alpha}}{\delta W N_o}\right) \tag{6}$$

The maximum distances till which transmissions at arbitrarily low error rates can then be received for the two modulation and coding schemes is then given by

$$d_i = \left[\frac{\epsilon_i P}{\delta_i W N_o(2^{Ci/\delta_i W}-1)}\right]^{\frac{1}{\alpha}} \tag{7}$$

where $i \in \{1, 2\}$ corresponding to transmissions at rate $C1$ and $C2$ respectively and $\epsilon_1 = \epsilon$, $\epsilon_2 = 1-\epsilon$, $\delta_1 = \delta$ and $\delta_2 = 1-\delta$. The corresponding broadcast capacities are then

$$U_1^{FS} = C1 \left[\frac{\epsilon P}{\delta W N_o(2^{C1/\delta W}-1)}\right]^{\frac{\gamma}{\alpha}} \tag{8}$$

$$U_2^{FS} = C2 \left[\frac{(1-\epsilon)P}{(1-\delta)WN_o(2^{C2/(1-\delta)W}-1)}\right]^{\frac{\gamma}{\alpha}} \tag{9}$$

The parameters δ and ϵ can be varied independently over the range $[0, 1]$ to obtain the entire range of achievable broadcast capacity points.

D. Superposition Coding

Superposition coding schemes have been proposed in [5] wherein the transmitter uses the entire bandwidth independently for transmitting simultaneously to multiple receivers, using two or more modulation and coding schemes. Consider the operation of a superposition scheme with two simultaneous transmissions. With superposition coding, in addition to sending a message to a primary receiver, the transmitter superimposes an additional message destined to a secondary receiver on top of the message destined for the primary receiver. The available transmission power is split between these two transmissions. The transmitter then modulates and encodes the two packets separately at the desired rates and the modulated symbols are scaled according to the desired power split. The two signals are then summed to obtain the signal to be transmitted.

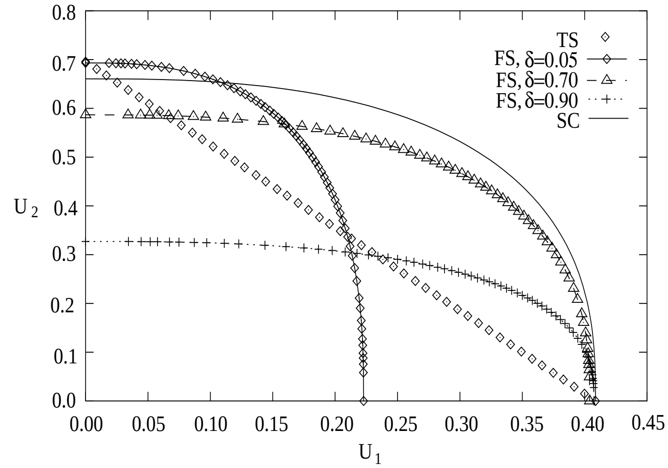

Fig. 1. Comparison of broadcast capacities of time-splitting (TS), frequency-splitting (FS) for three values of δ and superposition coding (SC). Parameters used: $C1 = 0.2$, $C2 = 0.5$, $\alpha = 4$, $\gamma = 1.5$, $W = 1$, $N_o = 1$ and $P = 1$. Note: U_1 is the primary receiver (with rate $C1$) and U_2 is the secondary receiver (with rate $C2$).

Different rules are used by the two receivers to decode their packets. The primary receiver decodes its packet while treating the superimposed signal as interference. The secondary receiver first decodes the primary packet, then re-encodes the packet, and then subtracts it from the original received signal. It then decodes the remaining signal to obtain the secondary transmission (also called successive interference cancellation). If the distances from the transmitter to the primary and secondary receivers are d_1 and d_2 respectively and a fraction β of the power is spent on the primary transmission, the achievable rates to the two receivers is given by

$$C_1 = W \log_2\left(1 + \frac{\beta P d_1^{-\alpha}}{(1-\beta)Pd_1^{-\alpha} + WN_o}\right) \tag{10}$$

$$C_2 = W \log_2\left(1 + \frac{(1-\beta)Pd_2^{-\alpha}}{WN_o}\right) \tag{11}$$

To ensure that the secondary receiver is able to decode the primary transmission whenever the primary receiver is able to, and also to ensure that the remaining signal after the subtraction has a sufficiently high signal to noise ratio, the channel quality to the secondary receiver should be better than that of the primary receiver.

This paper considers the applicability of superposition coding schemes for infrastructure based broadcasting. If the ambient noise in the channel is governed by the same white Gaussian noise process, regions closer to the infrastructure point would have a statistically better channel quality than those farther away. Thus vehicles in a nearer region can operate as the secondary receivers while those in a region further away may serve as primary receivers. Given that the infrastructure point can only transmit at rates $C1$ and $C2$, substituting $C_1 = C1$ and $C_2 = C2$ in Eqns. (10) and (11) and solving for d_1 and d_2, the maximum distances till which superposition coding may be successfully employed are given by

$$d_1 = \left[\frac{P-(1-\beta)P2^{C1/W}}{WN_o(2^{C1/W}-1)}\right]^{\frac{1}{\alpha}} \quad (12)$$

$$d_2 = \left[\frac{(1-\beta)P}{WN_o(2^{C2/W}-1)}\right]^{\frac{1}{\alpha}} \quad (13)$$

Note that Eqn. (12) implies that the power allocated to the secondary transmissions must be kept sufficiently small in order to make superposition coding feasible. Specifically, the fraction of power allocated to secondary transmissions should satisfy $(1-\beta) < \frac{1}{2^{C1/W}}$ to ensure $d_1 > 0$. When this condition is satisfied, the broadcast capacities associated with the superposition coding based broadcast are then given by

$$U_1^{SC} = C1\left[\frac{P-(1-\beta)P2^{C1/W}}{WN_o(2^{C1/W}-1)}\right]^{\frac{\gamma}{\alpha}} \quad (14)$$

$$U_2^{SC} = C2\left[\frac{(1-\beta)P}{WN_o(2^{C2/W}-1)}\right]^{\frac{\gamma}{\alpha}} \quad (15)$$

When $(1-\beta) \geq \frac{1}{2^{C1/W}}$, only one set of transmissions is feasible and the achievable broadcast capacity points (U_1^{SC}, U_2^{SC}) belong to the set $(0, U_2^{SC})$. The parameter β can be varied over the range $[0, 1]$ to obtain the entire range of achievable broadcast capacity points.

Figure 1 compares the set of broadcast capacity points achievable using superposition coding against those achieved by time and frequency-splitting. For each case, the curves are generated by varying the fraction of power allocated to transmissions at rate $C1$ in the range $[0, 1]$. While superposition coding broadcast capacities dominate that of time and frequency-splitting over a part of the figure, the relationship is reversed in the rest[1]. The following subsections study the relative performance of the three schemes.

Finally, we note that the rates $C1$ and $C2$ for the primary and the secondary transmissions may be interchanged without loss of generality. The results in this section as well as the subsequent ones still hold and we only need to interchange $C1$ and $C2$ in the equations.

E. Dominance of Frequency over Time-Splitting

Figure 1 shows that for a given δ, there exists a region where the broadcast capacities achieved by frequency-splitting dominates that of time-splitting. This section analytically shows that the performance of a broadcast scheme using frequency-splitting dominates that of a scheme based on time-splitting.

[1] If all the available power is allocated to one transmission, then both the X and Y-axis intervepts would be the same for the superposition coding and time-splitting curves.

Result 1: For any broadcast capacity point (U_1^{TS}, U_2^{TS}) achievable using time-splitting, there exists an broadcast capacity point (U_1^{FS}, U_2^{FS}) that dominates it in the sense

$$U_1^{FS} \geq U_1^{TS} \quad (16)$$

$$U_2^{FS} \geq U_2^{TS} \quad (17)$$

where $U_1^{TS}, U_2^{TS}, U_1^{FS}$ and U_2^{FS} are given by Eqns. (4), (5), (8) and (9) respectively. The inequalities are strictly satisfied in all cases except when $\tau = 0$ or $\tau = 1$ when the expressions hold with an equality.

Proof: First consider the case when $0 < \tau < 1$. The proof for this part proceeds in two steps. The first step shows that for a given δ, the curve corresponding to the set of broadcast capacity points achieved by varying ϵ is concave downward. Then the concavity result is combined with a result showing that for any $0 < \tau < 1$ there exists a $0 < \delta < 1$ and $0 < \epsilon < 1$ such that the frequency-splitting broadcast capacity curve defined by the given δ, as a function of ϵ, intersects with the line defining the broadcast capacities achieved by time-splitting.

Consider the parametric curve defined by the broadcast capacity points (U_1^{FS}, U_2^{FS}) associated with frequency-splitting for a given δ as ϵ varies in the range $(0, 1)$. To show that this curve is concave (downward), it suffices to show that $\frac{d^2y}{dx^2} < 0$ with $x = U_1^{FS}$ and $y = U_2^{FS}$. Now,

$$\frac{dx}{d\epsilon} = C1\left[\frac{P}{\delta WN_o(2^{C1/\delta W}-1)}\right]^{\frac{\gamma}{\alpha}} \frac{\gamma}{\alpha}(\epsilon)^{\frac{\gamma}{\alpha}-1} > 0 \quad (18)$$

$$\frac{dy}{d\epsilon} = -C2\left[\frac{P}{(1-\delta)WN_o(2^{C2/(1-\delta)W}-1)}\right]^{\frac{\gamma}{\alpha}} \frac{\gamma}{\alpha}(1-\epsilon)^{\frac{\gamma}{\alpha}-1} \quad (19)$$

Combining the two equations above, we have

$$\frac{dy}{dx} = -\frac{C2}{C1}\left[\frac{\delta(2^{C1/\delta W}-1)}{(1-\delta)(2^{C2/(1-\delta)W}-1)}\right]^{\frac{\gamma}{\alpha}} \left[\frac{1-\epsilon}{\epsilon}\right]^{\frac{\gamma}{\alpha}-1} \quad (20)$$

$$\frac{d}{d\epsilon}\frac{dy}{dx} = \frac{C2}{C1}\left[\frac{\delta(2^{C1/\delta W}-1)}{(1-\delta)(2^{C2/(1-\delta)W}-1)}\right]^{\frac{\gamma}{\alpha}} \left[\frac{\gamma-\alpha}{\alpha\epsilon^2}\right]\left[\frac{1-\epsilon}{\epsilon}\right]^{\frac{\gamma}{\alpha}-2} < 0 \quad (21)$$

where Eqn. (21) uses the fact that $\gamma - \alpha < 0$ while all other quantities are positive. Using the results of Eqns. (21) and (18) we then have

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{d\epsilon}\frac{dy}{dx}}{\frac{dx}{d\epsilon}} < 0 \quad (22)$$

Since the second derivative is negative, the curve corresponding to the frequency-splitting broadcast capacities for any δ is concave.

Next, consider the intersection point of the time-splitting broadcast capacity line and a frequency-splitting broadcast capacity curve. For a given τ, the broadcast capacities achieved by time-splitting are given by Eqns. (4) and (5). The values of δ and ϵ where the frequency-splitting broadcast capacities are equal to the time-splitting broadcast capacities specified by the given τ are obtained by equating $U_1^{TS} = U_1^{FS}$ and $U_2^{TS} = U_2^{FS}$ and solving simultaneously for ϵ and δ. Equating

$U_1^{TS} = U_1^{FS}$ gives

$$\tau C1\left[\frac{P}{WN_o(2^{C1/W}-1)}\right]^{\frac{\gamma}{\alpha}} = C1\left[\frac{\epsilon P}{\delta W N_o(2^{C1/\delta W}-1)}\right]^{\frac{\gamma}{\alpha}}$$

$$\Rightarrow\ \epsilon = \tau^{\frac{\alpha}{\gamma}}\left[\frac{\delta(2^{C1/\delta W}-1)}{2^{C1/W}-1}\right] \quad (23)$$

Similarly, equating $U_2^{TS} = U_2^{FS}$ gives

$$(1-\tau)C2\left[\frac{P}{WN_o(2^{\frac{C2}{W}}-1)}\right]^{\frac{\gamma}{\alpha}} = C2\left[\frac{(1-\epsilon)P}{(1-\delta)WN_o(2^{\frac{C2}{(1-\delta)W}}-1)}\right]^{\frac{\gamma}{\alpha}}$$

$$\Rightarrow\ 1-\epsilon = (1-\tau)^{\frac{\alpha}{\gamma}}\left[\frac{(1-\delta)(2^{C2/(1-\delta)W}-1)}{2^{C2/W}-1}\right] \quad (24)$$

Substituting ϵ from Eqn. (23) in Eqn. (24), δ can be obtained by solving $f(\delta) = 0$ for δ where

$$\begin{aligned} f(\delta) &= (1-\tau)^{\frac{\alpha}{\gamma}}\left[\frac{(1-\delta)(2^{C2/(1-\delta)W}-1)}{2^{C2/W}-1}\right] \\ &\quad - \left[1-\tau^{\frac{\alpha}{\gamma}}\left(\frac{\delta(2^{C1/\delta W}-1)}{2^{C1/W}-1}\right)\right] \quad (25) \end{aligned}$$

Note that a solution to the above equation exists for $0 < \delta < 1$. This can be seen by observing that

$$\begin{aligned} f(0) &= +\infty & (26) \\ f(1) &= -\left[1-\tau^{\frac{\alpha}{\gamma}}\right] < 0 & (27) \end{aligned}$$

Thus $f(1)$ and $f(0)$ are of opposite signs and a solution must exist in the range $(0, 1)$. The value of ϵ can then be obtained by substituting δ in Eqn. (23). Now, Eqn. (24) implies

$$\begin{aligned} (1-\tau)^{\frac{\alpha}{\gamma}} &= (1-\epsilon)\frac{2^{C2/W}-1}{(1-\delta)(2^{C2/(1-\delta)W}-1)} \\ &< 1-\tau^{\frac{\alpha}{\gamma}}\left(\frac{\delta(2^{C1/\delta W}-1)}{2^{C1/W}-1}\right) & (28) \\ \Rightarrow \tau^{\frac{\alpha}{\gamma}} &< \left[1-(1-\tau)^{\frac{\alpha}{\gamma}}\right]\frac{2^{C1/W}-1}{\delta(2^{C1/\delta W}-1)} & (29) \end{aligned}$$

where Eqn. (28) results from the fact that for $0 < \delta < 1$, $2^{C2/W}-1 < (1-\delta)(2^{C2/(1-\delta)W}-1)$. Substituting Eqn. (29) in Eqn. (23) gives

$$\begin{aligned} \epsilon &= \tau^{\frac{\alpha}{\gamma}}\left[\frac{\delta(2^{C1/\delta W}-1)}{2^{C1/W}-1}\right] \\ &< \left(1-(1-\tau)^{\frac{\alpha}{\gamma}}\right)\left[\frac{2^{C1/W}-1}{\delta(2^{C1/\delta W}-1)}\right]\left[\frac{\delta(2^{C1/\delta W}-1)}{2^{C1/W}-1}\right] \\ &= 1-(1-\tau)^{\frac{\alpha}{\gamma}} < 1 & (30) \end{aligned}$$

Also, since $\tau, \delta, C1, W > 0$, Eqn. (23) implies that $\epsilon > 0$. Thus the solution for ϵ also lies in the range $(0, 1)$. This in turn implies that for any time-splitting scheme with $0 < \tau < 1$, there exists a feasible frequency-splitting scheme with $0 < \delta, \epsilon < 1$ so that the line corresponding to the time-splitting broadcast capacities and the curve corresponding to the frequency-splitting broadcast capacities intersect. Since the curve corresponding to the frequency-splitting broadcast capacities for any δ is concave, this in turn implies that there exists a non-trivial portion of this curve above the time-splitting line. As τ is varied in the range $(0, 1)$, a continuous set of broadcast capacity curves for frequency-splitting is generated for the corresponding δ's. The envelop of these frequency-splitting broadcast capacity curves strictly dominates the time-splitting broadcast capacity line since all frequency-splitting curves have some portion of them above the time-splitting line.

Finally, consider the cases when $\tau = 0$ or $\tau = 1$. When $\tau = 1$, the broadcast capacities associated with time-splitting can be compared with the frequency-splitting broadcast capacities when $\delta = 1$ and $\epsilon = 1$. This gives

$$\begin{aligned} U_1^{TS} &= C1\left[\frac{P}{WN_o(2^{C1/W}-1)}\right]^{\frac{\gamma}{\alpha}} = U_1^{FS} & (31) \\ U_2^{TS} &= 0 = U_2^{FS} & (32) \end{aligned}$$

When $\delta = 1$ and $\epsilon = 1$ in Eqns. (8) and (9), the broadcast capacities associated with frequency-splitting become the same as the time-splitting case with $\tau = 1$. Similarly, the time-splitting broadcast capacities when $\tau = 0$ can be compared with the frequency-splitting broadcast capacities when $\delta = 0$ and $\epsilon = 0$. This gives

$$\begin{aligned} U_1^{TS} &= 0 = U_1^{FS} & (33) \\ U_2^{TS} &= C2\left[\frac{P}{WN_o(2^{C2/W}-1)}\right]^{\frac{\gamma}{\alpha}} = U_2^{FS} & (34) \end{aligned}$$

which are the same as the frequency-splitting broadcast capacities when $\delta = 0$ and $\epsilon = 0$ This completes the proof. ■

F. Dominance Results for Superposition Coding

This section shows that unlike Shannon capacity where superposition coding always dominates, when the broadcast capacity is considered, frequency-splitting may dominate superposition coding under certain scenarios. In Section II-D it was noted that simultaneous transmissions at two rates is possible with superposition coding if the fraction of power alloted to transmissions with rate $C2$ satisfies $(1-\beta) < \frac{1}{2^{C1/W}}$. This section determines the conditions required for broadcast capacities achieved with superposition coding to dominate those achieved by time and frequency-splitting. Conditions determining the scenarios where frequency-splitting dominates superposition coding are also derived. Note that since the previous subsection shows that frequency-splitting dominates time-splitting, it suffices to consider the relationship between superposition coding and frequency splitting.

Result 2: If the fraction of bandwidth δ allocated to transmissions at rate $C1$ under frequency-splitting satisfies $(1-\delta)(2^{C2/(1-\delta)W}-1) \geq 2^{C1/W}(2^{C2/W}-1)$, for any broadcast capacity point (U_1^{FS}, U_2^{FS}) achievable using frequency-splitting, there exists a broadcast capacity point (U_1^{SC}, U_2^{SC}) achievable using superposition coding that dominates it in the sense

$$\begin{aligned} U_1^{SC} &\geq U_1^{FS} & (35) \\ U_2^{SC} &\geq U_2^{FS} & (36) \end{aligned}$$

where $U_1^{FS}, U_2^{FS}, U_1^{SC}$ and U_2^{SC} are given by Eqns. (8), (9),

(14) and (15) respectively. The inequalities are strictly satisfied in all cases except when $\epsilon = 0$ and $(1-\delta)(2^{C2/(1-\delta)W}-1) = 2^{C1/W}(2^{C2/W}-1)$ or $\delta = 1$ and $\epsilon = 1$ when the expressions hold with an equality.

Proof: Pick any broadcast capacity point achievable by frequency-splitting as specified by a choice of δ and ϵ with $(1-\delta)(2^{C2/(1-\delta)W}-1) > 2^{C1/W}(2^{C2/W}-1)$. Now,

$$\begin{aligned} \frac{(1-\epsilon)(2^{C2/W}-1)}{(1-\delta)(2^{C2/(1-\delta)W}-1)} &< \frac{(1-\epsilon)(2^{C2/W}-1)}{2^{C1/W}(2^{C2/W}-1)} \\ &\leq \frac{1}{2^{C1/W}} \end{aligned} \tag{37}$$

Using the result above to ensure its feasibility, select $1-\beta$ such that

$$\begin{aligned} \frac{(1-\epsilon)(2^{C2/W}-1)}{(1-\delta)(2^{C2/(1-\delta)W}-1)} < 1-\beta < \frac{1}{2^{C1/W}}\left[1-\frac{\epsilon(2^{C1/W}-1)}{\delta(2^{C1/\delta W}-1)}\right] \\ \leq \frac{1}{2^{C1/W}} \end{aligned}$$

The broadcast capacity associated with the transmissions at rate $C1$ then satisfies

$$\begin{aligned} U_1^{SC} &= C1\left[\frac{P-(1-\beta)P2^{C1/W}}{WN_o(2^{C1/W}-1)}\right]^{\frac{\gamma}{\alpha}} \\ &> C1\left[\frac{\epsilon P}{\delta WN_o(2^{C1/\delta W}-1)}\right]^{\frac{\gamma}{\alpha}} = U_1^{FS} \end{aligned} \tag{38}$$

The broadcast capacity associated with the transmissions at rate $C2$ satisfies

$$\begin{aligned} U_2^{SC} &= C2\left[\frac{(1-\beta)P}{WN_o(2^{C2/W}-1)}\right]^{\frac{\gamma}{\alpha}} \\ &> C2\left[\frac{(1-\epsilon)P}{(1-\delta)WN_o(2^{C2/(1-\delta)W}-1)}\right]^{\frac{\gamma}{\alpha}} = U_2^{FS} \end{aligned} \tag{39}$$

Next, consider the special case where $\epsilon = 0$ and $(1-\delta)(2^{C2/(1-\delta)W}-1) = 2^{C1/W}(2^{C2/W}-1)$. The broadcast capacities associated with frequency-splitting in this case can be compared with the special case of superposition coding when $1-\beta = \frac{1}{2^{C1/W}}$.

$$U_1^{FS} = 0 = U_1^{SC} \tag{40}$$

$$\begin{aligned} U_2^{FS} &= C2\left[\frac{P}{(1-\delta)WN_o(2^{C2/(1-\delta)W}-1)}\right]^{\frac{\gamma}{\alpha}} \\ &= C2\left[\frac{(1-\beta)P}{WN_o(2^{C2/W}-1)}\right]^{\frac{\gamma}{\alpha}} = U_2^{SC} \end{aligned} \tag{41}$$

Finally, when $\delta = 1$ and $\epsilon = 1$, compare the associated frequency-splitting broadcast capacities with the special case of superposition coding when $\beta = 1$. This gives,

$$U_1^{FS} = C1\left[\frac{P}{WN_o(2^{C1/W}-1)}\right]^{\frac{\gamma}{\alpha}} = U_1^{SC} \tag{42}$$

$$U_2^{FS} = 0 = U_2^{SC} \tag{43}$$

which completes the proof. ∎

The condition $(1-\delta)(2^{C2/(1-\delta)W}-1) \geq 2^{C1/W}(2^{C2/W}-1)$ which ensures the dominance of superposition coding broadcast capacities is related to the fraction of the bandwidth δ that is allocated to transmissions at rate $C1$ by the frequency-splitting scheme. Consider the broadcast capacities associated with the superposition coding scenario where $1-\beta = \frac{1}{2^{C1/W}}$ as given in Eqns. (40) and (41). Simultaneous transmissions at both rates are possible as β is increased beyond this point. The condition $(1-\delta)(2^{C2/(1-\delta)W}-1) \geq 2^{C1/W}(2^{C2/W}-1)$ ensures that even if all the available power is allocated to transmissions at rate $C2$, the available bandwidth $(1-\delta)W$ is not sufficient for the frequency-splitting broadcast capacities to dominate those of superposition coding. The following result shows the conditions under which frequency-splitting dominates superposition coding.

Result 3: If the fraction of bandwidth δ allocated to transmissions at rate $C1$ under frequency-splitting satisfies $(1-\delta)(2^{C2/(1-\delta)W}-1) < 2^{C1/W}(2^{C2/W}-1)$ and $1-\beta > \frac{1}{2^{C1/W}}$, for any broadcast capacity point (U_1^{SC}, U_2^{SC}) achievable using superposition coding, there exists a broadcast capacity point (U_1^{FS}, U_2^{FS}) that dominates it in the sense

$$U_1^{FS} \geq U_1^{SC} \tag{44}$$

$$U_2^{FS} \geq U_2^{SC} \tag{45}$$

where $U_1^{FS}, U_2^{FS}, U_1^{SC}$ and U_2^{SC} are given by Eqns. (8), (9), (14) and (15) respectively. The inequalities are strictly satisfied in all cases except when $\epsilon = 0$ and $\delta = 0$ when the expressions hold with an equality.

Proof: With $1-\beta > \frac{1}{2^{C1/W}}$, the superposition coding broadcast capacities for a given β are given by

$$U_1^{SC} = 0 \tag{46}$$

$$U_2^{SC} = C2\left[\frac{(1-\beta)P}{WN_o(2^{C2/W}-1)}\right]^{\frac{\gamma}{\alpha}} \tag{47}$$

Consider a τ such that $0 < \tau < 1-(1-\beta)^{\frac{\gamma}{\alpha}}$. The corresponding broadcast capacities for the time-splitting scheme are then

$$\begin{aligned} U_1^{TS} &= \tau C1\left[\frac{P}{WN_o(2^{C1/W}-1)}\right]^{\frac{\gamma}{\alpha}} \\ &> \left(1-(1-\beta)^{\frac{\gamma}{\alpha}}\right) C1\left[\frac{P}{WN_o(2^{C1/W}-1)}\right]^{\frac{\gamma}{\alpha}} \\ &> 0 = U_1^{SC} \end{aligned} \tag{48}$$

$$\begin{aligned} U_2^{TS} &= (1-\tau)C2\left[\frac{P}{WN_o(2^{C2/W}-1)}\right]^{\frac{\gamma}{\alpha}} \\ &> (1-\beta)^{\frac{\gamma}{\alpha}} C2\left[\frac{P}{WN_o(2^{C2/W}-1)}\right]^{\frac{\gamma}{\alpha}} \\ &= U_2^{SC} \end{aligned} \tag{49}$$

Thus there exists a set of broadcast capacities achievable with time-splitting that dominates superposition coding. From Claim 1, for any set of time-splitting broadcast capacities, there exists a set of frequency-splitting broadcast capacities that dominates it. Thus there must exist a set of frequency-splitting broadcast capacities that dominate superposition coding. ∎

III. Optimality of Superposition Coding Utilities

The previous section compared the broadcast capacities obtained by the three transmission strategies considered in this paper. This section shows that the broadcast capacities achieved by the superposition coding strategy is optimal. The optimality holds only for cases where the fraction of total power allocated to transmissions at rate $C2$ is less than $\frac{1}{2^{C1/W}}$ and thus error free transmissions at rate $C1$ are also feasible.

The set of achievable broadcast capacities with superposition coding, (U_1^{SC}, U_2^{SC}) are given in Eqns. (14) and (15) for the cases where $1-\beta < \frac{1}{2^{C1/W}}$. We then have the following result about the optimality of these broadcast capacities:

Result 4: No broadcast capacity point $\{U_1, U_2\}$ such that

$$U_1 \geq C1\left[\frac{P-(1-\beta)P2^{C1/W}}{WN_o(2^{C1/W}-1)}\right]^{\frac{\gamma}{\alpha}} \tag{50}$$

$$U_2 = C2\left[\frac{(1-\beta)P}{WN_o(2^{C2/W}-1)}\right]^{\frac{\gamma}{\alpha}} + \Delta \tag{51}$$

is feasible, where $\Delta > 0$.

Proof: We prove the result above by contradiction. Assume that the broadcast capacities given by Eqns. (50) and (51) are feasible. Since the quantities $C2$, W, N_o, γ and δ are constants, the only way to increase the broadcast capacity U_2 over that of U_2^{SC}, as given in Eqn. (15), is by increasing the power allocated to transmissions at rate $C2$. This can be seen in the following:

$$\begin{aligned} U_2 &= C2\left[\frac{(1-\beta)P}{WN_o(2^{C2/W}-1)}\right]^{\frac{\gamma}{\alpha}} + \Delta \\ &= C2\left[\frac{(1-\beta)P}{WN_o(2^{C2/W}-1)} + \Delta'\right]^{\frac{\gamma}{\alpha}} \\ &= C2\left[\frac{(1-\beta)P+\lambda P}{WN_o(2^{C2/W}-1)}\right]^{\frac{\gamma}{\alpha}} \end{aligned}$$

where $\Delta' = \left[\left(\frac{(1-\beta)P}{WN_o(2^{C2/W}-1)}\right)^{\frac{\gamma}{\alpha}} + \frac{\Delta}{C2}\right]^{\frac{\alpha}{\gamma}} - \frac{(1-\beta)P}{WN_o(2^{C2/W}-1)} > 0$ and $\Delta' = \frac{\lambda P}{WN_o(2^{C2/W}-1)} > 0$. Thus the power available for transmissions at rate $C1$ is $P-(1-\beta)P-\lambda P = (\beta-\lambda)P$. Using Eqn. (12), the maximum distance at which the transmissions at rate $C1$ can be received with arbitrarily low error rates is given by

$$\begin{aligned} d &= \left[\frac{(\beta-\lambda)P-(1-\beta+\lambda)P(2^{C1/W}-1)}{WN_o(2^{C1/W}-1)}\right]^{\frac{1}{\alpha}} \\ &= \left[\frac{P-(1-\beta+\lambda)P2^{C1/W}}{WN_o(2^{C1/W}-1)}\right]^{\frac{1}{\alpha}} \end{aligned} \tag{52}$$

The broadcast capacity of the transmissions at rate $C1$ is then given by the product $C1(d)^\gamma$ and we have

$$\begin{aligned} U_1 &= C1\left[\frac{P-(1-\beta+\lambda)P2^{C1/W}}{WN_o(2^{C1/W}-1)}\right]^{\frac{\gamma}{\alpha}} \\ &< C1\left[\frac{P-(1-\beta)P2^{C1/W}}{WN_o(2^{C1/W}-1)}\right]^{\frac{\gamma}{\alpha}} \end{aligned}$$

This is a contradiction of Eqn. (50) and the result is thus proved. ■

IV. Broadcast Scheduling Algorithm

In this section we use the results from the previous sections to develop a proportionally fair algorithm for scheduling broadcast packets. We consider a static infrastructure point that needs to broadcast packets generated by the ITS to vehicles in its vicinity. The rate which a packet may be transmitted depends on the distance to which it needs to be sent. In addition to maximizing the throughput, we also aim to maintain fairness between the vehicles in different regions with respect to the throughput they receive.

Since the information relevant to a vehicle may depend on its relative position from the infrastructure point, we assume that each packet generated by the ITS specifies the region within which it needs to be broadcast. The region around an infrastructure point is divided into k circular rings. The outer radius of the i-th region is denoted by r_i with $r_1 < r_2 < \cdots < r_k$. The inner radius of the i-th region is r_{i-1} for $1 < i \leq k$ and 0 for the first region. We assume that the transmitter at the roadside infrastructure point can transmit at two possible modulation and coding schemes corresponding to bit rates of $C1$ and $C2$ with $C1 < C2$. Given a maximum transmission power P, we denote by d_1^{max} and d_2^{max} the maximum distance till which transmissions at rate $C1$ and $C2$, respectively, can be received with arbitrarily low error rates. We assume that $r_k \leq d_1^{max}$ to keep the problem practical and concern ourselves only with scenarios where the entire region of interest is covered by the transmissions. We denote by R_i, $R_i \in \{C1, C2\}$ the highest error free transmission rate that can be sustained in the whole of region i by the infrastructure point.

The scheduler needs to decide which region to transmit the next broadcast packet to and what rate to use. Let a broadcast packet be transmitted to region i at rate $C1$ considering region i to be the primary region. The minimum fraction β of the available power P that needs to be spent can be obtained by substituting r_i for d_1 in Eqn. (10) and solving for β. We then have

$$\beta = \frac{(WN_o + Pr_i^{-\alpha})(2^{C1/W}-1)}{Pr_i^{-\alpha}2^{C1/W}} \tag{53}$$

If $1-\beta < \frac{1}{2^{C1/W}}$, simultaneous transmission at rate $C2$ using superposition coding is possible. For these cases, substituting this value of β in Eqn. (13), we obtain the distance, d_i^p, till which error free transmissions at rate $C2$ can be made given that region i can receive transmissions at rate $C1$. If $1-\beta \geq \frac{1}{2^{C1/W}}$, we have $d_i^p = 0$. On the other hand, if the transmission at rate $C1$ to region i is made considering it to be the secondary region, the minimum fraction $1-\beta$ of the available power P that needs to be spent is obtained by substituting r_i for d_2 in Eqn. (11) and solving for β. We then have

$$1-\beta = \frac{WN_o(2^{C1/W}-1)}{Pr_i^{-\alpha}} \tag{54}$$

Again, if $1-\beta < \frac{1}{2^{C2/W}}$, superposition coding is feasible. For these cases, substituting this value of β in Eqn. (12), we obtain the distance, d_i^s, till which error free transmissions at rate $C2$ can be made given that region i can receive transmissions at rate $C1$. If superposition coding is infeasible, we have $d_i^s = 0$. Combining the two cases above, the farthest region D_i^2, the whole of which can receive error free transmissions at rate $C2$ given that region i can receive transmissions at rate $C1$, is given by

$$D_i^2 = \arg\max_j \{r_j | r_j \leq \max\{d_i^p, d_i^s\}\}$$

Following a similar process, we can also obtain the farthest region D_i^1, the whole of which can receive error free transmissions at rate $C1$, given that region i is receiving transmissions at rate $C2$. We use $D_i^1 = 0$ and $D_i^2 = 0$ if no superposition coding is possible.

The basic objective of the broadcast scheme is to maintain fairness among the different regions in terms of the average throughput of each region. However, in vehicular environments, emergency messages may be generated occasionally that should be broadcast with higher priority than regular information. The second objective of the proposed broadcast scheduling algorithm is thus to provide preemptive priority to higher priority emergency messages. At the infrastructure point, separate queues are maintained for the messages destined for each region, along with a separate queue for the emergency messages. We denote the queue for region i, $1 \leq i \leq k$, by Q_i, the queue of the emergency messages by Q_p and the number of messages in these queues are denoted by $|Q_i|$ and $|Q_p|$ respectively. The task of the scheduling algorithm is then to pick the next queue to transmit from.

Consider the algorithm's operation in iteration n. The pseudo-code for the algorithm is shown in Algorithm 1. We use $T_i(n)$ to denote a measure of the number of packets broadcast to region i in a past window and t_c is a time constant that can be adjusted to select the time horizon over which to maintain fairness. The scheduling algorithm consists of the following steps:

1) If the queue corresponding to emergency messages is non-empty, pick the first packet in this queue for transmission. The transmission is scheduled at rate $R_{i^*(n)}$, where $i^*(n)$ denotes the region this message is destined for.
2) If there are more packets in emergency message queue, the scheduler tries to transmit them using superposition coding. If $R_{i^*(n)} = C1$ and $D^2_{i^*(n)} > 0$, the scheduler picks the first message whose destination region $j^*(n)$ satisfies $j^*(n) \leq D^2_{i^*(n)}$. However, if $D^2_{i^*(n)} > 0$ and no emergency messages exist such that $j^*(n) \leq D^2_{i^*(n)}$, the scheduler moves to step 4 to check for any low-priority messages that may be sent. On the other hand, if $R_{i^*(n)} = C2$ and $D^1_{i^*(n)} > 0$, the scheduler picks the first message whose destination region $j^*(n)$ satisfies $j^*(n) \leq D^1_{i^*(n)}$. Again, the scheduler moves to step 4 if no additional emergency messages can be transmitted.
3) If the queue for emergency messages is empty, from the other queues, pick the region (denoted by $i^*(n)$) that maximizes $\frac{R_i}{T_i(n)}$:
$$i^*(n) = \arg\max_{i=1,\cdots,k} \frac{R_i}{T_i(n)} \tag{55}$$
4) In this step, the scheduler checks if simultaneous transmissions using superposition coding are possible. If $R_{i^*(n)} = C1$ and $D^2_{i^*(n)} > 0$, the scheduler then selects the region where transmissions may be made at rate $C2$ according to
$$j^*(n) = \arg\max_{i=1,\cdots,D^2_{i^*(n)}} \frac{C2}{T_j(n)} \tag{56}$$
On the other hand, if $R_{i^*(n)} = C2$ and $D^1_{i^*(n)} > 0$, the scheduler selects the region where transmissions may be made at rate $C1$ using superposition coding according to
$$j^*(n) = \arg\max_{i=1,\cdots,D^1_{i^*(n)}} \frac{C1}{T_j(n)} \tag{57}$$
5) Based on the regions selected for transmission in Steps 1-4, the scheduler transmits one packet to the region chosen for transmission at rate $C1$ and $\lfloor \frac{C2}{C1} \rfloor$ packets to the region chosen for transmissions at rate $C2$, if any. If any non-emergency packets are transmitted, the throughput measure of the corresponding region, $T_i(n+1)$, is then updated as shown in Algorithm 1.

Depending on the primary region $i^*(n)$ selected by the scheduler, it may not always be feasible to use superposition coding to simultaneously broadcast packets to another region. Such a situation arises when the power required to transmit at rate $C2$ is high enough to cause significant interference to the transmissions at rate $C1$. As shown in Section II-D, this occurs when $(1-\beta) \geq \frac{1}{2^{C1/W}}$. In these situations it was shown that frequency multiplexing can lead to higher broadcast capacities. The scheduling algorithm developed in this section resorts to a special case of frequency multiplexing when superposition coding is not feasible.

V. Simulation Results

This section presents simulation results to compare the performance of the proposed scheduler with some other possibilities. The performance is compared in terms of both the fairness and the throughput. The simulations were done with a custom built simulator written in C. We use parameters corresponding to the IEEE 802.11p or Wireless Access in Vehicular Environments (WAVE) standards. The parameters used were $W = 10$MHz, $N_o = -112.9$dBm/Hz, $\alpha = 4$ and $\gamma = 1.5$. Results are reported for two sets of transmission rates: $C1 = 6$Mbps, $C2 = 30$Mbps and $C1 = 12$Mbps, $C2 = 30$Mbps. The size of each broadcast packet is assumed to be 200 bytes. The entire network is divided into ten circular regions centered at the infrastructure point. The radius r_i of region i satisfies $r_i = ir_1$. The radius of the farthest region equals the maximum distance till which transmissions at rate $C1$ may be made with arbitrarily low error rates. It is assumed that the infrastructure point always has data to send to each region, i.e., we assume a saturated traffic model. Examples of real-world applications where the scenario described above is applicable include: (1) location specific traffic advisories such as travel times to a toll booth, tunnel, bridge or point of attraction such as a stadium; (2) area specific information on nearby parking facilities; (3) information on the state (red or

Scheme	$C1 = 6$, $C2 = 30$		$C1 = 12$, $C2 = 30$	
	Throughput	Fairness	Throughput	Fairness
Maximum Throughput	30.00	0.50	30.00	0.60
Round Robin	11.58	0.99	20.70	0.99
Proportionally Fair	32.40	0.79	33.48	0.91

TABLE I
COMPARISON OF VARIOUS SCHEDULING POLICIES (THROUGHPUT VALUES ARE IN MBPS)

green) of traffic lights in a given area and the time before they change and (4) construction or accident related information such as lane closures and speed limit changes that are useful to only cars in a specific area and direction.

We use two metrics for evaluating the scheduling policies. The first is the throughput achieved by the infrastructure point measured in Mbps. The throughput reported is the average over all the ten broadcast regions served by the infrastructure point. The second metric is the fairness. For evaluating the fairness, Jain's fairness index is used [8]. If the throughput of region i at time t is denoted by $x_i(t)$, Jain's fairness index $F(t)$ at time t is then given by

$$F(t) = \frac{\left(\sum_{i=1}^{n} x_i(t)\right)^2}{n \sum_{i=1}^{n} x_i(t)^2} \tag{58}$$

which attains the value of 1 only when the allocation is totally fair $(x_1(t) = x_2(t) = \cdots = x_n(t))$. Note that the broadcast capacity of each region may be used instead of its throughput in the expression for Jain's fairness index with identical results.

Table I compares the performance of the proposed scheduler with two others: (1) a scheduler that broadcasts messages to different regions following a round robin policy and (2) a scheduler that always selects the region which supports the highest data rate. In the round robin policy, the infrastructure point schedules each region in succession. If the next region to be scheduled does not have any data to broadcast, the infrastructure point moves on to the next region in the list. For the region that is scheduled, the infrastructure point uses the maximum of the two rates $C1$ and $C2$ that it can support for that region. In the second scheduler that we consider, in order to pick the next region to schedule, the infrastructure point first lists all regions that have data queued up for broadcasting. Among these regions, the infrastructure point then picks the region for which it can support the highest data rate, i.e., have the maximum throughput. In case there are multiple regions with the same data rate, the tie is broken randomly. Note that in both the round robin and the maximum throughput schedulers at most one packet may be transmitted at any given point in time. The round robin scheduler is chosen because of its fairness properties while the other one is chosen because of its throughput performance. The results in Table I are plotted after $n = 1000$, i.e., 1000 iterations of the scheduler's operation and uses $t_c = 100$. The results show that the fairness of the proposed scheme is between that of the other two. However, while round robin scheduling can achieve excellent fairness, the proposed protocol outperforms it in terms of throughput. Also, the proposed scheduler outperforms the maximum throughput scheduler in terms of both throughput and fairness. Thus the proposed scheduler is a good compromise between throughput

t_c	$C1 = 6$, $C2 = 30$		$C1 = 12$, $C2 = 30$	
	Throughput	Fairness	Throughput	Fairness
10	31.80	0.80	33.00	0.92
100	32.40	0.79	33.48	0.91
500	35.40	0.76	35.40	0.83

TABLE II
EFFECT OF PARAMETERS ON PROPORTIONALLY FAIR SCHEDULING (THROUGHPUT VALUES ARE IN MBPS)

α	Throughput			Fairness		
	PF	MT	RR	PF	MT	RR
2.5	31.56	30.00	8.82	0.58	0.30	0.99
3.0	31.62	30.00	10.02	0.68	0.40	0.99
3.5	31.68	30.00	11.58	0.77	0.50	0.99
4.0	32.40	30.00	11.58	0.79	0.50	0.99

TABLE III
EFFECT OF α ON VARIOUS SCHEDULING POLICIES WITH $C1 = 6$MBPS, $C2 = 30$MBPS (PF: PROPORTIONALLY FAIR, MT: MAXIMUM THROUGHPUT AND RR: ROUND ROBIN). THROUGHPUT VALUES ARE IN MBPS.

and fairness. Note that the the round robin scheduler has the best fairness since it strictly rotates the broadcast region. However, the proposed scheduler prefers to pick regions which offer a chance for simultaneous transmissions (thereby improving the throughput) even through they may have been served recently, thereby compromising on the fairness.

Table II evaluates the impact of the parameter t_c on the scheduler's performance. This parameter controls the time horizon over which the scheduler maintains fairness. It can be observed that as t_c increases, the scheduler achieves lower fairness but higher throughput. The results reported here are again after 1000 rounds of the scheduler's operation. The difference in the performance in terms of both throughput and fairness, however, reduces as the scheduler runs for longer periods.

Finally, we consider the effect of the parameters γ and α on the performance of the schedulers. The throughput and fairness of all schedulers does not depend on the value γ. However, the broadcast capacity does depend on γ and increases exponentially with γ. In Tables III and IV we compare the performance of the three schedulers for $C1 = 6$Mbps, $C2 = 30$Mbps and $C1 = 12$Mbps, $C2 = 30$Mbps respectively for various values of α and the rest of the parameters were $W = 1$, $N_o = 1$, $P = 1$, $t_c = 100$ and $\gamma = 1.5$. We see that both the throughput and the fairness tends to increase with α but their values quickly saturate.

In addition to the throughput and fairness, other metrics

α	Throughput			Fairness		
	PF	MT	RR	PF	MT	RR
2.5	32.76	30.00	18.78	0.90	0.50	0.99
3.0	32.82	30.00	20.70	0.91	0.60	0.99
3.5	33.48	30.00	20.70.45	0.91	0.60	0.99
4.0	33.48	30.00	20.70	0.91	0.60	0.99

TABLE IV
EFFECT OF α ON VARIOUS SCHEDULING POLICIES WITH $C1 = 12$MBPS, $C2 = 30$MBPS (PF: PROPORTIONALLY FAIR, MT: MAXIMUM THROUGHPUT AND RR: ROUND ROBIN). THROUGHPUT VALUES ARE IN MBPS.

such as average queue lengths for each region served by the infrastructure point may be used as an evaluation metric. Since the proposed proportionally fair scheduler has maximizing the throughput subject to fairness constraints as its objective, its performance in terms of queue lengths may be worse than other schedulers. In particular, the round robin scheduler has lower average queue lengths, specially as the data arrival rates increases. This is because with round robin service, each region has a bounded waiting time before data for it is scheduled. However, the maximum throughput scheduler has the longest average queue lengths since regions with lower sustainable data rates are starved in favor of regions with higher throughputs. The performance of the proportionally fair scheduler lies between that of the other two schedulers.

To end this section, note that the round robin scheduler considered here belongs to the class of time-splitting schedulers. Also, the maximum throughput scheduler can be considered as an extreme case of frequency-splitting where the entire spectrum is allocated to one rate.

VI. CONCLUSIONS

This paper considers the problem of transmission of broadcast packets by roadside infrastructure points. Using the *broadcast capacity* as the metric, the performance of three transmission strategies is compared: time-splitting, frequency-splitting and superposition coding. It is shown that frequency-splitting always dominates time-splitting and conditions for the dominance of superposition coding are obtained. The optimality of the broadcast capacities associated with superposition coding is proved. Finally a proportionally fair scheduler for transmitting broadcast packets is proposed and its performance is compared with other schedulers. Extension of the current framework from two transmission rates to an arbitrary number of transmission rates is an interesting avenue for future work.

REFERENCES

[1] H. Alshaer and E. Horlait, "An adaptive broadcast scheme for inter-vehicle communication," *Proceedings of IEEE VTC (spring)*, Stockholm, Sweden, May 2005.

[2] P. Bergmans and T. cover, "Cooperative broadcasting," *IEEE Transactions on Information Theory*, vol. 20, no. 3, pp. 317-324, May 1974.

[3] L. Briesemeister and G. Hommel, "Role-Based Multicast in Highly Mobile but Sparsely Connected Ad Hoc Networks," *Proceedings of ACM/IEEE MOBIHOC*, pp. 45-50, Boston, MA, August 2000.

[4] N. Challa and H. Cam, "Adaptive multicasting using common spreading codes in infrastructure-to-vehicle communication networks," *Proceedings of IEEE Workshop on Mobile Networking for Vehicular Environments*, pp.61-66, Anchorage, AK, May 2007.

[5] T. Cover, "Broadcast channels," *IEEE Transactions on Information Theory*, vol. 18, no. 1, pp. 2-14, January 1972.

[6] S. Eichler, "Performance evaluation of the IEEE 802.11p WAVE communication standard," *Proceedings of IEEE VTC (fall)*, pp. 2199-2203, Baltimore, MD, October 2007.

[7] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Trans. on Information Theory*, vol. 46, no. 2, pp. 388-404, March 2000.

[8] R. Jain, D. Chiu and W. Hawe, "A quantitative measure of fairness and discrimination for resource allocation in shared computer systems," DEC Research Report TR-301, Hudson, MA, September 1984.

[9] A. Kirson, "RF data communications considerations in advanced driver information systems," *IEEE Transactions on Vehicular Technology*, vol.40, no.1, pp.51-55, February 1991.

[10] B. Kantowitz and D. LeBlanc, "Emerging technologies for vehicle-infrastructure cooperation to support emergency transportation operations," Technical report UMTRI-2006-25, University of Michigan, Ann Arbor, MI, July 2006.

[11] G. Korkmaz, E. Ekici, F. Ozguner and U. Ozguner, "Urban multi-hop broadcast protocol for inter-vehicle communication systems," *Proceedings of ACM VANET*, pp, 76-85, Philadelphia, PA 2004.

[12] J. Luo and J.-P. Hubaux, "A survey of inter-vehicle communication," *Embedded Security in Cars, Part II* pp. 111-122, Springer-Verlag, October 2005.

[13] Y. Ma, M. Chowdhury, A. Sadek and M. Jeihani, "Real-Time highway traffic condition assessment framework using vehicleinfrastructure integration (VII) with artificial intelligence (AI)," *IEEE Transactions on Intelligent Transportation Systems*, vol. 10, no. 4, pp. 615-627, December 2009.

[14] T. Mak, K. Laberteaux and R. Sengupta, "A multi-channel VANET providing concurrent safety and commercial service," *Proceedings of ACM Workshop on Vehicular Ad Hoc Networks*, pp. 1-9, Cologne, Germany, January 2005.

[15] R. Nagler, S. Eichler and J. Eberspacher, "Intelligent wireless communication for future autonomous and cognitive automobiles," *Proceedings of IEEE Intelligent Vehicles Symposium*, pp. 716-721, Istanbul, Turkey, June 2007.

[16] R. Padmasiri, T. Fujii, Y. Kamiya and Y. Suzuki, "Variable spreading method for MC/DS-CDMA road to vehicle communication system," *Proceedings of IEEE ISSSTA*, pp. 113-117, Manaus, Brazil, August 2006.

[17] C. Palazzi, M. Roccetti and S. Ferretti, "An intervehicular communication architecture for safety and entertainment," *IEEE Transactions on Intelligent Transportation Systems*, vol. 11, no. 1, pp. 90-99, March 2010.

[18] A. Reznik and S. Verdu, "On the transport capacity of a broadcast gaussian channel," *Communications in Information and Systems*, vol. 2, pp. 157-190, 2002.

[19] C. Shannon, *A Mathematical Theory of Communication*, University of Illinois Press, Urbana, IL, 1949.

[20] M. Sun, W. Feng, T. Lai, K. Yamada, H. Okada and K. Fujimura, "GPS-based message broadcast for adaptive inter-vehicle communications," *Proceedings of IEEE VTC (fall)*, pp 2685-2692, 2000.

[21] A. Vahidi and A. Eskandarian, "Research advances in intelligent collision avoidance and adaptive cruise control," *IEEE Transactions on Intelligent Transportation Systems*, vol. 4, no. 3, pp. 143-153, September 2003.

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Algorithm 1 Proportionally fair scheduling algorithm.

Initialize $T_i(0)$ to a constant value for all $1 \le i \le k$, $n = 0$ and evaluate R_i^1 and R_i^2 for all i
while (1) **do**
  if $|Q_p| > 0$ **then**
    pick the next region $i^*(n)$ to transmit: $i^*(n) =$ region of the first packet in the priority queue
  else
    pick the next region $i^*(n)$ to transmit: $i^*(n) = \arg\max_{i=1,\cdots,k} \frac{R_i}{T_i(n)}$
  end if
  additional region to transmit to: $j^*(n) = \phi$
  if $R_{i^*(n)} = C1$ **then**
    if $D^2_{i^*(n)} > 0$ **then**
      if $|Q_p| > 1$ **then**
        pick $j^*(n)$: the first packet in Q_p whose destination region $j^*(n)$ satisfies $j^*(n) \le D^2_{i^*(n)}$
      end if
      if $j^*(n) = \phi$ **then**
        pick $j^*(n)$: $j^*(n) = \arg\max_{j=1,\cdots,D^2_{i^*(n)}} \frac{C2}{T_i(n)}$
        transmit one packet at rate $C1$ to region $i^*(n)$ and $\lfloor \frac{C2}{C1} \rfloor$ packets to region $j^*(n)$ at rate $C2$
        update $T_i(n+1)$, $1 \le i \le k$: $T_i(n+1) = \begin{cases} \left(1 - \frac{1}{t_c}\right) T_i(n) + \frac{1}{t_c} & i = i^*(n) \\ \left(1 - \frac{1}{t_c}\right) T_i(n) + \frac{1}{t_c} \lfloor \frac{C2}{C1} \rfloor & i = j^*(n) \\ \left(1 - \frac{1}{t_c}\right) T_i(n) & \text{otherwise} \end{cases}$
      end if
    else
      transmit one packet at rate $C1$ to region $i^*(n)$
      update $T_i(n+1)$, $1 \le i \le k$: $T_i(n+1) = \begin{cases} \left(1 - \frac{1}{t_c}\right) T_i(n) + \frac{1}{t_c} & i = i^*(n) \\ \left(1 - \frac{1}{t_c}\right) T_i(n) & \text{otherwise} \end{cases}$
    end if
  else
    if $D^1_{i^*(n)} > 0$ **then**
      if $|Q_p| > 1$ **then**
        pick $j^*(n)$: the first packet in Q_p whose destination region $j^*(n)$ satisfies $j^*(n) \le D^1_{i^*(n)}$
      end if
      if $j^*(n) = \phi$ **then**
        pick $j^*(n)$: $j^*(n) = \arg\max_{j=1,\cdots,D^1_{i^*(n)}} \frac{C1}{T_i(n)}$
        transmit $\lfloor \frac{C2}{C1} \rfloor$ packets at rate $C2$ to region $i^*(n)$ and one packet to region $j^*(n)$ at rate $C1$
        update $T_i(n+1)$, $1 \le i \le k$: $T_i(n+1) = \begin{cases} \left(1 - \frac{1}{t_c}\right) T_i(n) + \frac{1}{t_c} \lfloor \frac{C2}{C1} \rfloor & i = i^*(n) \\ \left(1 - \frac{1}{t_c}\right) T_i(n) + \frac{1}{t_c} & i = j^*(n) \\ \left(1 - \frac{1}{t_c}\right) T_i(n) & \text{otherwise} \end{cases}$
      end if
    else
      transmit one packet at rate $C2$ to region $i^*(n)$
      update $T_i(n+1)$, $1 \le i \le k$: $T_i(n+1) = \begin{cases} \left(1 - \frac{1}{t_c}\right) T_i(n) + \frac{1}{t_c} & i = i^*(n) \\ \left(1 - \frac{1}{t_c}\right) T_i(n) & \text{otherwise} \end{cases}$
    end if
  end if
  $n = n + 1$
end while