Queueing Analysis of Polling Based Wireless MAC Protocols with Sleep-Wake Cycles

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***Abstract*—While decentralized medium access control (MAC) protocols are more popular in wireless environments, cluster based sensor networks are particularly amenable to centralized, polling based protocols. In this paper we present an analytic model to evaluate the performance of polling based MAC protocols for wireless networks in terms of the packet delay, buffer overflow rates and energy consumption. We show that polling based protocols can outperform popular decentralized MAC protocols. Simulation results are presented to validate our model and conclusions.**

***Index Terms*—MAC protocols, Performance evaluation, Queuing analysis**

I. Introduction

A key constraint that dictates the design of MAC protocols for wireless sensor networks (WSNs) is their limited on-board energy. Since the energy consumed by the nodes in idle listening of the channel causes significant battery drain (p. 75 of [1]), most MAC protocols for WSNs propose that nodes turn off their radios when not involved in ongoing transmissions. Decentralized contention based MAC protocols that use different variants of sleep-wake cycles have been studied extensively in literature [2], [3], [4]. The performance of these decentralized protocols, however, degrades as the network load increases due to the increased incidence of collisions and the associated bandwidth wastage. Cluster based WSN architectures [5], on the other hand, are particularly suitable for centralized, polling based MAC protocols.

This paper develops analytic models to evaluate the performance of polling based MAC protocols with sleep-wake cycles for WSNs. A queueing model is first developed to evaluate the average packet delays and packet loss rates due to buffer overflow, and these results are then used to evaluate the per node energy consumption rates. Simulations are used to validate the analysis and also to demonstrate the superior performance of polling based MAC protocols with sleep-wake cycles over similar decentralized protocols.

The rest of the paper is organized as follows. Section II presents the queueing model and Section III presents the model for the energy consumption rates. Section IV presents the simulation results and comparison with decentralized protocols and Section V concludes the paper.

II. Delay Model

We assume a cluster based WSN architecture wherein sensors in a geographical region select a node amongst them as the cluster head. The cluster head is responsible for communicating with other cluster heads and the sink. All other nodes are leaf nodes, and can only communicate with the cluster head in their cluster. Details of issues such as cluster formation algorithms, cluster head rotation, inter-cluster communication, routing policies etc. are outside the scope of this paper and not relevant to our analysis.

The MAC protocol's operation is divided into rounds. A round begins with the inter-cluster period where cluster heads exchange data with other cluster heads or with the sink, and leaf nodes may turn off their radios. It is followed by the intra-cluster period where cluster heads exchange data with their leaf nodes. The polling based MAC protocol applies to the intra-cluster communication and we consider two variants: **Scheme 1.** A cluster head first polls *all* its leaf nodes and then assigns them time slots to transfer their data; **Scheme 2.** A cluster head polls a node and immediately transfers data with it before moving on to poll the next node. In both cases, only nodes with data are assigned slots and the remaining nodes may sleep till the end of the round. Each slot is of the same duration and is long enough to transmit only one packet. A node is assigned at most one slot in a round. The intra-cluster period ends when all nodes have been polled in a round. Also, if none of the nodes have any data to send when polled in a round, the cluster transitions into a sleep state where all leaf nodes turn off their radios. A new round starts when the sleep period ends.

A. Polling Scheme 1

Consider a M node cluster with $M-1$ leaf nodes where each leaf node has K buffers to store packets. The channel rate is $\frac{1}{C}$ bytes/second and each data packet is of k_D bytes, requiring $T_D = k_D C$ seconds to be transmitted. During each poll, the cluster head transmits k_{P_DL} bytes to the polled node which then replies using k_{P_UL} bytes. The time to poll a node is $T_P = k_P C$ seconds where $k_P = k_{P\_DL} + k_{P\_UL}$.

The packet interarrival time distribution at each node is assumed to be a Markov modulated Poisson process (MMPP) with an arbitrary number of states, r. MMPP based arrivals are used in this paper because of their versatility in modeling traffic types such as voice, video as well as long range dependent traffic [6], [7]. The MMPP is characterized by the transition rate matrix $\mathbf{R}$ and the arrival rate matrix $\mathbf{\Lambda}$:

$$\mathbf{R} = \begin{bmatrix} -\sigma_1 & \sigma_{12} & \cdots & \sigma_{1r} \\ \sigma_{21} & -\sigma_2 & \cdots & \sigma_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{r1} & \sigma_{r2} & \cdots & -\sigma_r \end{bmatrix}, \tag{1}$$

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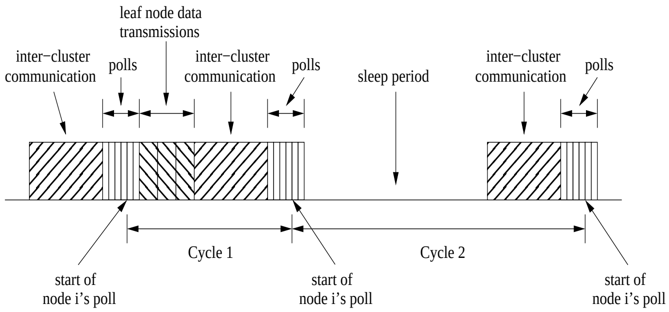

Fig. 1. Protocol operation for polling scheme 1 showing two cycles of transmission. The first cycle contains three data transmissions while in the second cycle none of the nodes have any data to send, thereby resulting in a sleep period.

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_r \end{bmatrix}. \tag{2}$$

The steady state probability vector q of the Markov chain satisfies $q\mathbf{R} = 0$ and $qe = 1$ where e is a unit vector. The average arrival rate at a node is then given by $\lambda = q\mathbf{\Lambda}$. For our analysis, we consider the system operation from leaf node i's perspective, $1 \leq i \leq M-1$, to be divided into periods of variable length, T_C, called *cycles*. A cycle begins when the polling of node i starts, and ends when node i is polled the next time. Figure 1 shows the operation of the MAC protocol under polling scheme 1 for two cycles. Note that the probability distribution of the duration of a cycle is identical to that of a round (defined earlier).

We model the MAC layer behavior of each leaf node as a MMPP/G/1/K queue. We first characterize the probability distribution function (PDF) of the service time. Consider a tagged packet arriving at leaf node i. At the instant of its arrival, the queue at node i may be in one of two states: **1. S0:** The queue is empty **2. S1:** The queue is non-empty. Next we consider the service time for the two cases.

1) Arrival at an Empty Queue: State S0: Consider the cycle in which the tagged packet arrives. The queue at leaf node i is empty when the packet arrives but may not have been so at the beginning of the cycle when it was polled. Thus we consider two subcases corresponding to whether the queue at leaf node i was empty (case C1) or not (case C2) when it was polled in this cycle.

In case C1, since node i was empty when it was polled, it cannot transmit any data in this cycle. Among the remaining $M-2$ leaf nodes, let there be k nodes that transmit data in this cycle. We first consider the cases where $k > 0$. Since each cycle also includes an inter-cluster period (of duration T_I) and the polls of M–1 leaf nodes, the length of the cycle is given by $T_C = T_I + (M-1)T_P + kT_D$. For arbitrary arrivals independent of the departures in a frame based system, an arrival is equally likely to occur anywhere in a frame [10]. In our case, *given* that an arrival occurs in a cycle, the arrival instance, t, *relative* to the start of the cycle is thus uniformly distributed in $[0, T_C]$, denoted by $U[0, T_C]$. The time the tagged arrival has to wait till the start of the next frame is $T_C - t$. If a random variable Y is uniformly distributed in the range 0 to a, then $a - Y$ is also uniformly distributed in the range 0 to a. Thus the PDF of $T_C - t$ is also $U[0, T_C]$. In the next cycle, the tagged packet first has to wait for the polls of $M-i$ nodes, including itself. Then if j of the $i-1$ leaf nodes polled before node i have data to transmit, the tagged packet has to wait for an additional jT_D seconds before it is served. Let ρ denote the probability that the queue at any leaf node is empty at an arbitrary time instant. Then j is binomially distributed with parameters $B[i-1, 1-\rho]$. The Laplace-Stieltjes Transform (LST) of the service time in case C1, $X_{i,k,S0,C1}$, is

$$\begin{aligned} H_{X_{i,k,S0,C1}}(s) &= \text{LST}\,[U[0, T_I + (M-1)T_P + kT_D] \\ &\quad + (M-i)T_P + B[i-1, 1-\rho]T_D + T_D] \\ &= \frac{1 - e^{-s(T_I + (M-1)T_P + kT_D)}}{s(T_I + (M-1)T_P + kT_D)} \times \\ &\quad e^{-s((M-i)T_P + T_D)}(\rho + (1-\rho)e^{-sT_D})^{i-1} \end{aligned} \tag{3}$$

where the first term in the equation above is the LST of $U[0, T_I + (M-1)T_P + kT_D]$, the second term is the LST of the constants $(M-i)T_P + T_D$ and the third term is the LST of $B[i-1, 1-\rho]T_D$.

In the case where $k = 0$, none of the leaf nodes have any data to send when they are polled in the current cycle. Thus the nodes enter the sleep period (of duration T_S) and the length of the cycle is $T_C = T_I +\ (M-1)T_P + T_S$. Once the cycle ends, as for cases with $k > 0$, the tagged packet waits for $(M-i)T_P + jT_D$ seconds before service, with j distributed as $B[i-1, 1-\rho]$. Unconditioning Eqn. (3) on k (distributed as $B[M-2, 1-\rho]$) and adding to it the case for $k = 0$, the LST of the service time for case C1, $X_{i,S0,C1}$, is

$$\begin{aligned} &H_{X_{i,S0,C1}}(s) \\ &= \rho^{M-2} \frac{1 - e^{-s(T_I + (M-1)T_P + T_S)}}{s(T_I + (M-1)T_P + T_S)} \frac{(\rho + (1-\rho)e^{-sT_D})^{i-1}}{e^{s((M-i)T_P + T_D)}} + \\ &\quad \sum_{k=1}^{M-2} \binom{M-2}{k} (1-\rho)^k \rho^{M-2-k} \left[\frac{1 - e^{-s(T_I + (M-1)T_P + kT_D)}}{s(T_I + (M-1)T_P + kT_D)} \right. \\ &\quad \left. \times \frac{(\rho + (1-\rho)e^{-sT_D})^{i-1}}{e^{s((M-i)T_P + T_D)}} \right]. \end{aligned} \tag{4}$$

In case C2, the tagged node was non-empty when it was polled but was empty when the tagged packet arrived in the same cycle. Thus, the tagged packet must have arrived after leaf node i transmitted its data in the current cycle. The remaining time in the cycle after the packet from leaf node i is transmitted is $kT_D + T_I + (i-1)T_P$, if k of the $M-1-i$ nodes polled after node i also transmit data in the current cycle. Now, the tagged packet has to wait for $T_C - t$ seconds for the current cycle to end. The PDF of t, given that the tagged packet arrived after node i transmitted its data in the cycle is distributed as $U[T_C - kT_D - T_I - (i-1)T_P, T_C]$. If a random variable Y has the distribution $U[a, b]$, then b–Y has the distribution $U[0, b$–$a]$. Thus $T_C - t$ is distributed as $U[0, kT_D + T_I + (i-1)T_P]$. In the next cycle, the tagged packet first waits for $M - i$ polls and possible data transmission from the $i - 1$ leaf nodes polled before node i. The LST of the service time in this case,

$X_{i,k,S0,C2}$, is given by

$$\begin{aligned} H_{X_{i,k,S0,C2}}(s) &= \text{LST}\,[U[0,kT_D+T_I+(i-1)T_P]+ \\ &\quad (M-i)T_P+B[i-1,1-\rho]T_D+T_D] \\ &= \frac{1-e^{-s(kT_D+T_I+(i-1)T_P)}}{s(kT_D+T_I+(i-1)T_P)} \times \\ &\quad e^{-s((M-i)T_P+T_D)}(\rho+(1-\rho)e^{-sT_D})^{i-1} \end{aligned} \quad (5)$$

Unconditioning Eqn. (5) on k (distributed as $B[M-1-i,1-\rho]$), the LST of the service time for case C2, $X_{i,S0,C2}$, is

$$\begin{aligned} H_{X_{i,S0,C2}}(s) &= \sum_{k=0}^{M-1-i}\binom{M-1-i}{k}(1-\rho)^k\rho^{M-1-i-k} \\ &\times\frac{1-e^{-s(kT_D+T_I+(i-1)T_P)}}{s(kT_D+T_I+(i-1)T_P)}\frac{(\rho+(1-\rho)e^{-sT_D})^{i-1}}{e^{s((M-i)T_P+T_D)}}. \end{aligned} \quad (6)$$

Now, the probabilities of cases C1 and C2 are given by $P[C1]=\rho$ and $P[C2]=1-\rho$, respectively. Combining cases C1 and C2, the LST of the service time in state S0, $X_{i,S0}$, is then given by

$$\begin{aligned} H_{X_{i,S0}}(s) &= \rho H_{X_{i,S0,C1}}(s)+(1-\rho)H_{X_{i,S0,C2}}(s) \\ &= \frac{(\rho+(1-\rho)e^{-sT_D})^{i-1}}{e^{s((M-i)T_P+T_D)}}\left[\sum_{k=0}^{M-1-i}\binom{M-1-i}{k}\left[(1-\rho)^{k+1}\right.\right. \\ &\quad \left.\rho^{M-1-i-k}\frac{1-e^{-s(kT_D+T_I+(i-1)T_P)}}{s(kT_D+T_I+(i-1)T_P)}\right]+ \\ &\quad \sum_{k=1}^{M-2}\binom{M-2}{k}(1-\rho)^k\rho^{M-1-k}\frac{1-e^{-s(T_I+(M-1)T_P+kT_D)}}{s(T_I+(M-1)T_P+kT_D)} \\ &\quad \left.+\rho^{M-1}\frac{1-e^{-s(T_I+(M-1)T_P+T_S)}}{s(T_I+(M-1)T_P+T_S)}\right]. \end{aligned} \quad (7)$$

2) Arrival at a Non-Empty Queue: State S1: For a tagged arrival at a non-empty queue, the service time starts when the last of the enqueued packets departs the queue. Once the tagged packet comes to the head of the queue, it first has to wait for the current cycle to finish. In the remainder of the current cycle, any of the remaining $M-1-i$ nodes may transmit their data, and we also have an inter-cluster period and the polls of $i-1$ leaf nodes. Before the tagged packet receives service in the next cycle, we have $M-i$ polls, including the poll of leaf node i, along with possible data transmissions from the $i-1$ leaf nodes polled before leaf node i. Since the number of nodes with data transmissions in a cycle is binomially distributed, the LST of the service time for arrivals in state S1, $X_{i,S1}$, is given by

$$\begin{aligned} H_{X_{i,S1}}(s) &= \text{LST}\,[B[M-1-i,1-\rho]T_D+T_I+(M-1)T_P \\ &\quad +B[i-1,1-\rho]T_D+T_D] \\ &= e^{-s(T_I+(M-1)T_P+T_D)}(\rho+(1-\rho)e^{-sT_D})^{M-2}. \end{aligned} \quad (8)$$

3) Overall Service Time, Delay Distribution and Loss Rates: Combining the service times for cases S0 and S1 from Eqns. (7) and (8), the LST of the service time of an arbitrary arrival at node i, X_i, is

$$H_{X_i}(s)=\rho H_{X_{i,S0}}(s)+(1-\rho)H_{X_{i,S1}}(s). \quad (9)$$

The average service time, Θ, is given by $\Theta = -\frac{d}{ds}H_{X_{i,S1}}(s)\big|_{s=0}$ and can be written as

$$\begin{aligned} \Theta &= \frac{2M-i-1}{2}T_P+\frac{T_I}{2}+\left[1+\frac{(M-3+i)(1-\rho)}{2}\right]T_D+ \\ &\quad (1-\rho)\left[\frac{(i-1)T_P+T_I+(M-1-i)(1-\rho)T_D}{2}\right]+ \\ &\quad \rho\left[\rho^{M-1}\frac{T_S}{2}+\rho\left[\frac{(M-i)T_P+(i-1)(1-\rho)T_D}{2}\right]\right]. \end{aligned}$$

To obtain the distribution of the packet delays and loss rates, the queue at each node is modeled as a MMPP/G/1/K queue whose service time distribution is given by Eqn. (9). We use the analysis for the MMPP/G/1/K queue from [8] and list the equations below for completeness. Consider the imbedded Markov chain consisting of the service completion instants at the queue. Let $\pi(k)$ (respectively, $p(k)$) be the $r-$dimensional vector whose $j-$th element is the limiting probability at the imbedded epochs (respectively, at an arbitrary time instant) of having k packets in the queue and the MMPP being in phase j, $k=0,1,\cdots,K-1$ (respectively, $k=0,1,\cdots,K$). Consider the matrix sequence $\{\mathbf{C}_k\}$ defined as

$$\mathbf{C}_{k+1}=\left[\mathbf{C}_k-\mathbf{U}\mathbf{A}_k-\sum_{\nu=1}^{k}\mathbf{C}_\nu\mathbf{A}_{k-\nu+1}\right]\mathbf{A}_0^{-1} \quad (10)$$

for $k=1,2,\cdots,K-2$, with $\mathbf{C}_0=\mathbf{I}$, $\mathbf{C}_1=(\mathbf{I}-\mathbf{U}\mathbf{A}_0)\mathbf{A}_0^{-1}$ and $\mathbf{I}$ being a $r\times r$ identity matrix. The $(k,l)-$th element of the matrix $\mathbf{A}_\nu$ denotes the conditional probability of reaching phase l and having ν arrivals at the end of a service time, starting from phase k. The matrices $\mathbf{A}_\nu$ can be easily calculated using an iterative procedure [9] (see supplemental document). The probability vectors $\pi(k)$ can then be calculated using

$$\pi(0)\left[\sum_{\nu=0}^{K-1}\mathbf{C}_\nu+(\mathbf{I}-\mathbf{U})\mathbf{A}(\mathbf{I}-\mathbf{A}+eq)^{-1}\right]=q, \quad (11)$$

and $\pi(k)=\pi(0)\mathbf{C}_k$, $k=1,2,\cdots,K-1$. The vectors $p(k)$ are given by $p(0)=\xi\pi(0)(\mathbf{\Lambda}-\mathbf{R})^{-1}\Theta^{-1}$ and

$$p(k)=\xi\left[\pi(k)+\sum_{\nu=0}^{k-1}\pi(\nu)\mathbf{U}^{k-1-\nu}(\mathbf{U}-\mathbf{I})\right](\mathbf{\Lambda}-\mathbf{R})^{-1}\Theta^{-1} \quad (12)$$

for $k=1,2,\cdots,K-1$, and $p(K)=q-\sum_{\nu=1}^{K-1}p(\nu)$ where $\xi=[1+\pi(0)(\mathbf{\Lambda}-\mathbf{R})^{-1}\Theta^{-1}e]^{-1}$. The packet blocking probability is given by

$$P_b=1-\sum_{\nu=0}^{K-1}p(\nu). \quad (13)$$

Finally, the LST of the cumulative distribution function of the packet waiting time, $W(s)$ is given by

$$W(s)=\frac{1}{1-P_b}\left[p(0)+\xi\Theta^{-1}\sum_{\nu=1}^{K-1}G_\nu(s)H_{X_i}^{K-1-\nu}(s)\mathbf{T}_{K-1-\nu}(s)\right] \quad (14)$$

where $G_j(s)=\pi(0)[\mathbf{I}-\mathbf{U}H_{X_i}(s)]-H_{X_i}^j(s)\pi(j)$, $\mathbf{T}_j(s)=\mathbf{F}(s)[-\mathbf{\Lambda}\mathbf{F}(s)]^j$ and $\mathbf{F}(s)=[s\mathbf{I}+\mathbf{R}-\mathbf{\Lambda}]^{-1}$. Moments of the packet waiting time can be easily obtained from Eqn. (14).

To complete the analysis, note that the probability that a queue is empty at an arbitrary instant of time, ρ, is given by $\rho = p(0)e$. However, ρ is used in the expression for the service time, which in turn is used to evaluate $p(0)$. To obtain ρ, we use an iterative technique: we start with an arbitrary value of ρ in $(0,1)$ and use it to compute the service time and $p(0)$. The new value of ρ given by $\rho = p(0)e$ is then used to recalculate the service time and $p(0)$. This process continues till the values of ρ and $p(0)e$ converge.

B. Polling Scheme 2

The analysis for this scheme is similar to that in Section II-A and the details have been omitted. The same definitions as in Section II-A are used for the states S0 and S1 and their subcases C1 and C2.

1) Arrival at an Empty Queue: State S0: As for polling scheme 1, in case C1, the cycle consists of an inter-cluster communication period, the time for $M-1$ polls, and the time to transmit the data packets from the other $M-2$ nodes. if k of the $M-2$ nodes transmit data in the cycle, then for $k>0$, the length of the cycle is given by $T_C = T_I + (M-1)T_P + kT_D$. Thus, t is distributed as $U[0, T_I + (M-1)T_P + kT_D]$ and the distribution of the time the tagged arrival has to wait till the start of the next frame, $T_C - t$, is also $U[0, T_I + (M-1)T_P + kT_D]$. The next cycle starts with the poll of node i and the tagged packet is then transmitted immediately, adding another $T_P + T_D$ seconds to the service time. The LST of the service time in this case, $X_{i,k,S0,C1}$, is

$$H_{X_{i,k,S0,C1}}(s) = \text{LST}\left[U[0,T_I+(M-1)T_P+kT_D]+T_P+T_D\right] = \frac{1-e^{-s(T_I+(M-1)T_P+kT_D)}}{s(T_I+(M-1)T_P+kT_D)}e^{-s(T_P+T_D)}. \quad (15)$$

In the case where $k=0$, the nodes enter a sleep period and the length of the cycle is $T_C = T_I + (M-1)T_P + T_S + jT_D$, where j, distributed as $B[i-1, 1-\rho]$, is the number of nodes that transmit before node i after the sleep period ends. Leaf node i is polled immediately after the cycle ends and the tagged packet is transmitted immediately afterwards. Then, unconditioning Eqn. (15) on k (distributed as $B[M-2, 1-\rho]$) and adding to it the case for $k=0$, the LST of the service time for case C1, $X_{i,S0,C1}$, is given by

$$H_{X_{i,S0,C1}}(s) = e^{-s(T_P+T_D)}\left[\rho^{M-2}\sum_{j=0}^{i-1}\left[\binom{i-1}{j}(1-\rho)^j \rho^{i-1-j}\frac{1-e^{-s(T_I+(M-1)T_P+T_S+jT_D)}}{s(T_I+(M-1)T_P+T_S+jT_D)}\right] + \sum_{k=1}^{M-2}\left[\binom{M-2}{k}(1-\rho)^k\rho^{M-2-k}\frac{1-e^{-s(T_I+(M-1)T_P+kT_D)}}{s(T_I+(M-1)T_P+kT_D)}\right]\right]. \quad (16)$$

In case C2, the remaining time in the cycle after the packet from node i is transmitted is $kT_D+T_I+(M-2)T_P$, if k of the $M-2$ nodes polled before leaf node i is polled again, transmit data in the current cycle. The PDF of t is then distributed as $U[T_C - kT_D - T_I - (M-2)T_P, T_C]$. Thus the time that the tagged packet has to wait before the current cycle ends, $T_C - t$ is distributed as $U[0, kT_D + T_I + (M-2)T_P]$. In the next cycle, node i is polled first and it immediately transmits the tagged packet, adding $T_P + T_D$ seconds to the service time. Now, k is distributed as $B[M-2, 1-\rho]$. The LST of the service time in this case, $X_{i,S0,C2}$, is then

$$H_{X_{i,S0,C2}}(s) = \text{LST}\left[\sum_{k=0}^{M-2} U[0, kT_D + T_I + (M-2)T_P] + T_P + T_D\right] \quad (17)$$

$$= e^{-s(T_P+T_D)}\sum_{k=0}^{M-2}\left[\binom{M-2}{k}(1-\rho)^k\rho^{M-2-k}\frac{1-e^{-s(kT_D+T_I+(M-2)T_P)}}{s(kT_D+T_I+(M-2)T_P)}\right]. \quad (18)$$

Now, the probabilities of cases C1 and C2 are given by $P[C1] = \rho$ and $P[C2] = 1-\rho$, respectively. Combining cases C1 and C2, the LST of the service time in state S0, $X_{i,S0}$, is then given by

$$H_{X_{i,S0}}(s) = e^{-s(T_P+T_D)}\left[\sum_{k=0}^{M-2}\left[\binom{M-2}{k}(1-\rho)^{k+1}\rho^{M-2-k}\frac{1-e^{-s(kT_D+T_I+(M-2)T_P)}}{s(kT_D+T_I+(M-2)T_P)}\right] + \sum_{j=0}^{i-1}\left[\binom{i-1}{j}(1-\rho)^j \rho^{M+i-2-j}\frac{1-e^{-s(T_I+(M-1)T_P+T_S+jT_D)}}{s(T_I+(M-1)T_P+T_S+jT_D)}\right] + \sum_{k=1}^{M-2}\left[\binom{M-2}{k}(1-\rho)^k\rho^{M-1-k}\frac{1-e^{-s(T_I+(M-1)T_P+kT_D)}}{s(T_I+(M-1)T_P+kT_D)}\right]\right] \quad (19)$$

2) Arrival at a Non-Empty Queue: State S1: Once the tagged packet comes to the head of the queue and starts its service, it first waits for the polls and data transmissions of $M-2$ nodes and an inter-cluster communication period before the current cycle ends. In the next cycle, node i is polled first and the tagged packet is transmitted immediately. The LST of the service time for arrivals in state S1, $X_{i,S1}$, is then

$$H_{X_{i,S1}}(s) = \text{LST}\left[B[M-2,1-\rho]T_D + T_I + (M-1)T_P + T_D\right] = e^{-s(T_I+(M-1)T_P+T_D)}(\rho + (1-\rho)e^{-sT_D})^{M-2}. \quad (20)$$

3) Overall Service Time, Delay Distribution and Loss Rates: Combining cases S0 and S1, the LST of the service time of an arbitrary arrival at node i, X_i, is given by $H_{X_i}(s) = (1-\rho)H_{X_{i,S0}}(s) + \rho H_{X_{i,S1}}(s)$ where $H_{X_{i,S0}}(s)$ and $H_{X_{i,S1}}(s)$ are given in Eqn. (19) and (20) respectively. The waiting time distribution and the loss rates can then be evaluated using Eqns. (14) and (13) and the methodology of Section II-A3.

III. Energy Consumption Model

The energy consumption of the MAC protocols depend on the time spent by each node in transmitting, receiving or in the sleep period, in addition to the energy dissipation characteristics of the radios used by the nodes. To model the radio power usage at the nodes, we use the radio energy dissipation model in [5]. The model assumes that the radio dissipates E_{elec} Joules/bit (J/bit) to run the transmitter or receiver circuitry and E_{amp} J/m^2 for the transmitter amplifier to achieve an acceptable signal to noise ratio. Assuming d^2 energy loss in the channel, to send a k bits message to a distance d, the radio expends

$$E_{Tx}(k,d) = kE_{elec} + kd^2E_{amp}, \tag{21}$$

where the first term is the energy spent in the transmitter electronic circuits and the second part represents the energy consumed by the transmitter amplifier and includes the radio transmit power. To receive this message, the radio expends

$$E_{Rx}(k,d) = kE_{elec}. \tag{22}$$

where we only need to account for energy spent in the receiver circuits. To obtain the energy consumption rate, for each of the two polling schemes, we first evaluate the average cycle time. If k nodes transmit data in a cycle with $k > 0$, we have $T_C = T_I + (M-1)T_P + kT_D$. The expected cycle time is then

$$\begin{aligned} E[T_C|k>0] &= T_I + (M-1)T_P + \sum_{i=1}^{M-1}\Bigg[iT_D\binom{M-1}{i} \\ &\qquad (1-\rho)^i\rho^{M-1-i}\frac{1}{1-\rho^{M-1}}\Bigg] \\ &= T_I + (M-1)T_P + \frac{(M-1)(1-\rho)}{1-\rho^{M-1}}T_D \end{aligned} \tag{23}$$

If $k=0$, the length of the cycle is $T_C = T_I + (M-1)T_P + T_S$. Thus the expected cycle length is

$$\begin{aligned} E[T_C] &= \rho^{M-1}(T_I + (M-1)T_P + T_S) + (1-\rho^{M-1}) \times \\ &\quad \left[T_I + (M-1)T_P + \frac{(M-1)(1-\rho)}{1-\rho^{M-1}}T_D\right] \\ &= T_I + (M-1)T_P + \rho^{M-1}T_S + (M-1)(1-\rho)T_D \end{aligned} \tag{24}$$

At any instant, a leaf node may be either in the polling, data transmission, inter-cluster or sleep period. During the polls in a cycle, each leaf node spends $k_{P\_UL}(E_{elec} + E_{amp}d^2) + k_{P\_DL}E_{elec}$ J on its own poll and $(M-2)k_PE_{elec}$ J listening to the polls of other nodes. Since polls occur every cycle, this energy is expended every $E[T_C]$ seconds. Also, the rate at which packets are accepted in the queue of each node is $\lambda(1-P_b)$ where the packet blocking probability, P_b, is given by Eqn. (13). In a stable system, the rate at which packets depart is thus also $\lambda(1-P_b)$. For each packet transmitted, a leaf node expends $E_{elec}k_D + E_{amp}k_Dd^2$ J. Finally, a node does not spend any energy during the inter-cluster and sleep periods. Using the expression for $E[T_C]$ from Eqn. (24), the total rate at which a leaf node spends energy is

$$\begin{aligned} E_{avg} &= \frac{(M-1)k_PE_{elec} + k_{P\_UL}E_{amp}d^2}{T_I + (M-1)T_P + \rho^{M-1}T_S + (M-1)(1-\rho)T_D} \\ &\quad + \lambda(1-P_b)(E_{elec}k_D + E_{amp}k_Dd^2). \end{aligned} \tag{25}$$

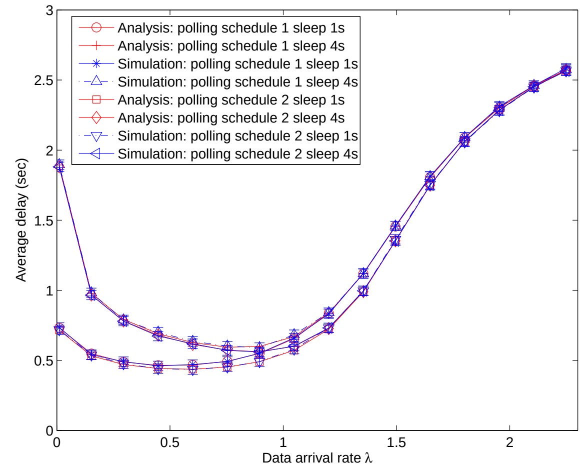

Fig. 2. Average packet delay at leaf node 5 in a cluster with 9 leaf nodes.

Note that ρ and P_b for the two polling schemes is different, leading to different energy consumption rates.

IV. Simulation Results

The two polling based MAC schemes were implemented in the NS-2 simulator and this section presents simulation results to verify our analysis and compare their performance against decentralized protocols. The length of each simulation run was 2000 seconds, and each result is the average of 20 runs. The channel data rate is 20Kbps, $T_P = 0.004$sec, $T_I = 0.4$sec and $T_D = 0.0256$sec. There are 9 leaf nodes in each cluster, and results for other cluster sizes are similar and omitted due to space constraints. A 2-state MMPP with transition rates of $\sigma_{12} = 3.15$ and $\sigma_{21} = 1.94$ and the ratio $\lambda_1 = 1.6\lambda_2$ was used [6] for the arrival process. The radio parameters were $E_{elec} = 50nJ/bit$ and $E_{amp} = 100pJ/m^2$ [5].

Figures 2 and 3 show the close match between the analytic and simulation results for the packet delay and energy consumption rate for different traffic loads and sleep periods. Figure 2 shows that the minimum delay is not achieved at low arrival rates but at moderate loads. At low data rates, a large fraction of the arrivals occur when the system is in the sleep state. These arrivals need to wait for the relatively large sleep period to finish before they can be transmitted. As the arrival rate increases, the probability that an arrival occurs in a sleep period decreases, thereby reducing the delays due to sleep periods. At high arrival rates, the queuing delay becomes dominant and the packet delay increases again. Consequently, there exists an unique arrival rate, typically at moderate loads, where the overall delay is lowest.

Figure 3 shows that increasing the sleep time reduces the energy consumption in both polling schemes. However, this decrease cannot continue unboundedly because as the sleep time becomes longer, each sleep period has a larger number of packet arrivals. These arrivals will queue up and consequently, the subsequent active periods also become longer. Also, at high loads, the system does not enter the sleep period and each node

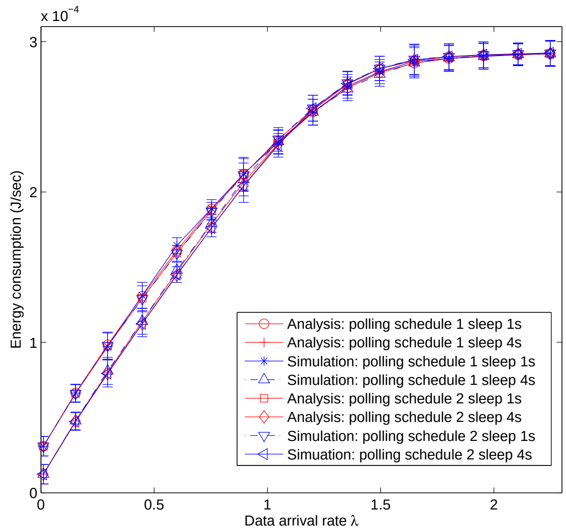

Fig. 3. Average rate of energy consumption at leaf node 5 in a cluster with 9 leaf nodes.

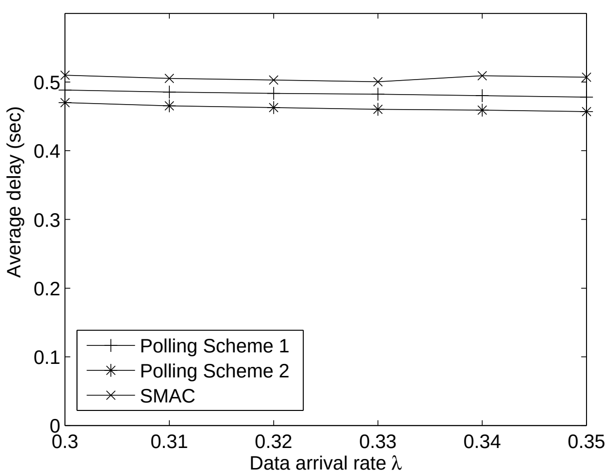

Fig. 4. Comparison of packet delays between SMAC and polling based MAC protocols. The channel data rate was 2Mbps and we used SMAC with a duty cycle of 10% and $T_P = 0.00004$ sec, $T_I = 0.4$ sec, $T_S = 1$ sec, and $T_D = 0.000256$ sec for the polling based schemes.

almost always transmits a packet in a cycle. Thus cycle lengths are almost constant and the energy consumption rates saturate for both polling schemes and all sleep periods.

Table I shows the results for packet loss rates of the two polling schemes for $T_S = 1$ sec. Scheme 2 has slightly lower loss rates (specially at lower arrival rates) because it does not poll all nodes at the beginning of the intra-cluster phase, allowing later arrivals to get served in the same round. At larger arrival rates the performance of both polling schemes is almost the same since the systems are saturated.

Finally, we compare the polling based schemes with a popular decentralized protocol with sleep-wake cycles: SMAC [2]. In the SMAC protocol nodes periodically alternate between two modes: sleep and wake. Nodes turn off their transceiver

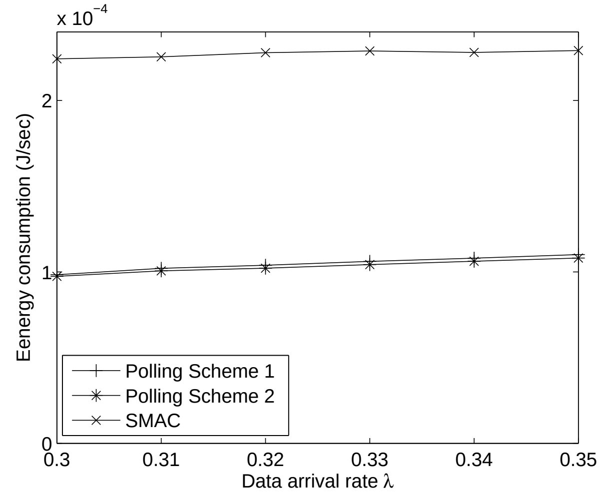

Fig. 5. Comparison of energy consumption between SMAC and polling based MAC protocols. The parameters used are the same as those for Figure 4.

in the sleep period, thereby saving energy. The nodes wake-up at the end of the sleep period and use carrier sense multiple access with collision avoidance (CSMA/CA) to send and receive packets. The period length and duty-cycle values are system parameters to be set by the network manager. We compare the performance at low data rates since the contention based SMAC has high collision rates at high traffic loads and its performance degrades. The results are shown in Figures 4 and 5. To compare the protocol performance in similar settings, parameters (see figures) were selected such that the packet delays of the protocols are similar. Both polling strategies outperform SMAC in terms of the delay as well as the energy consumption. Interestingly, the polling based schemes have at least 100% lower energy consumption as compared to SMAC and the difference is larger at higher loads. This is because SMAC: (1) does not adapt to the changing traffic conditions, resulting in energy wastage and (2) wastes energy through the collisions due to its contention based MAC protocol.

V. Conclusion

This paper presents models to evaluate the delay, loss rates and energy consumption of polling based MAC protocols with sleep-wake cycles. The performance of polling based MAC protocols is compared against similar decentralized protocols and is shown to be superior in terms of both delay and energy.

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Data arrival rate λ	Polling Scheme 1						Polling Scheme 2					
	$K=1$		$K=5$		$K=10$		$K=1$		$K=5$		$K=10$	
	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.
1.048	0.290	0.311	0.002	0.003	0.000	0.000	0.274	0.281	0.001	0.001	0.000	0.000
1.354	0.353	0.362	0.026	0.028	0.003	0.002	0.334	0.341	0.019	0.020	0.001	0.001
1.648	0.409	0.410	0.117	0.124	0.094	0.091	0.389	0.395	0.111	0.122	0.092	0.090
1.954	0.461	0.484	0.234	0.246	0.228	0.247	0.442	0.462	0.233	0.252	0.228	0.232
2.249	0.505	0.515	0.329	0.331	0.327	0.321	0.487	0.497	0.329	0.330	0.327	0.324

TABLE I
PACKET LOSS RATES FOR THE TWO POLLING SCHEMES FOR BUFFER SIZES OF 1, 5 AND 10.

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