

# A Queueing Model for Polled Service in WiMAX/IEEE 802.16 Networks

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**Abstract**—This paper presents a queueing model for the polling based service classes of WiMAX/IEEE 802.16 based wireless networks. Models are presented for both single channel and multiple channel (OFDMA based) operations. The models evaluate the MAC layer packet delays as a function of various system parameters.

**Index Terms**—IEEE 802.16, WiMAX, delay analysis, modeling

## I. INTRODUCTION

The WiMAX/IEEE 802.16 standard for point to multipoint broadband wireless access is expected to provide ubiquitous broadband wireless access supporting fixed, nomadic, portable and fully mobile operations offering integrated voice, video and data services. The IEEE 802.16e standard supports five scheduling service classes for QoS (UGS, rtPS, nrtPS, ertPS and BE) and includes a request-grant mechanism for uplink transmissions from a Subscriber Station (SS) to its Base Station (BS). This paper proposes analytic models for evaluating the performance of the polling based request-grant mechanisms in terms of the average packet delay.

Existing literature has evaluated many aspects of WiMAX/IEEE 802.16. Simulation based evaluation of various service classes are presented in [1], [2]. The binary exponential backoff of IEEE 802.16 is modeled in [3]. Delay bounds for OFDMA-TDMA and OFDMA systems for some specific burstiness control schemes are developed in [4]. Connection-level characteristics of IEEE 802.16 under call admission control and bandwidth allocation schemes developed by the authors are modeled in [5], [6]. However, analytic models for polled services classes have not been proposed in literature.

This paper presents queueing models specific to the case of polling based service classes in the MAC layer of WiMAX/IEEE 802.16. The MAC layer delay is an important factor in the overall performance and capacity utilization of the system and accurate characterization of this delay is critical to meeting performance goals of delay-sensitive applications. Our models derive expressions for the packet delays at each SS as a function of various systems parameters.

The rest of the paper is organized as follows. Section II introduces the queueing model starting with the single channel physical (PHY) layer, and then extends the analysis to the IEEE 802.16 OFDMA PHY. Section III presents the simulation results and Section IV concludes the paper.

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## II. DELAY ANALYSIS

We consider a single BS serving  $n$  SSs through a TDMA/TDD, single channel air-interface. Each frame is of duration  $T_S$  and is divided into uplink and downlink subframes of duration  $T_{UL}$  and  $T_{DL}$ , respectively. No scheduling algorithm is specified in the IEEE 802.16 standard and it is left to the vendor. We assume that a SS transmits a single packet in a frame if it made a bandwidth request in the previous frame.

We assume that nodes are polled sequentially at the end of every uplink subframe. Our model can be easily extended for other polling schemes. Packet interarrival times at a SS are assumed to be exponentially distributed with rate  $\lambda$ . Results with this Poisson assumption provide lower bounds on the delays under more bursty and correlated traffic models, as shown in Section III. The utilization factor of the queue at each SS is  $\rho = \lambda T_S$ . The time taken for polling a SS and transmitting a packet are  $T_P$  and  $L$ , respectively.

Consider a tagged packet arriving at SS  $i$ ,  $1 \leq i \leq n$ . At the instance of its arrival, the queue at the SS may be in one of three states: **1. S0**: The queue is empty. **2. S1**: The queue is non-empty but no bandwidth was reserved when the SS was last polled, i.e., the queue was empty when it was last polled. **3. S2**: The queue is non-empty and bandwidth was reserved when the SS was last polled. Given that the queue is non-empty, let the probability of it being in states S1 and S2 be  $p$  and  $1 - p$  respectively.

**A. Arrival at an Empty Queue: State S0:** We consider two subcases: arrival before (case C1) and after (case C2) SS  $i$  has been polled in the current frame. In case C1, since the SS has not been polled yet, a reservation can be made in this frame for transmitting the tagged packet in the next frame. For exponential arrivals independent of the departure process in a slotted departure system, an arrival is equally likely to occur anywhere in a slot [7]. In our case, given that an arrival occurs in a frame, the arrival instance,  $t$ , relative to the start of the frame is thus uniformly distributed in  $[0, T_S]$ . SS  $i$  is polled  $(n-i+1)T_P$  seconds before the frame ends. The time the SS waits before it sends the bandwidth request is thus  $T_S - (n-i+1)T_P - t$ . The probability distribution function (PDF) of  $t$  given that the arrival is of case C1,  $P[t \leq \tau \mid t \leq T_S - (n-i+1)T_P]$ , is given by

$$\begin{aligned} P[t \leq \tau \mid t \leq T_S - (n-i+1)T_P] &= \frac{P[t \leq \tau, t \leq T_S - (n-i+1)T_P]}{P[t \leq T_S - (n-i+1)T_P]} \\ &= \frac{\tau}{T_S - (n-i+1)T_P}, \end{aligned} \quad (1)$$

which is an Uniform distribution:  $U[0, T_S - (n-i+1)T_P]$ . If a random variable  $Y$  is distributed as  $U[0, a]$ , then  $a - Y$  is also distributed as  $U[0, a]$ . Thus the PDF of  $T_S - (n-i+1)T_P - t$  is also  $U[0, T_S - (n-i+1)T_P]$ . After SS  $i$  is polled,  $(n-i+1)T_P$  seconds pass before the frame ends. The packet is transmitted in the next frame, following the downlink subframe of  $T_{DL}$  seconds and the packets of any of the  $i-1$  SSs that were polled before the  $i$ -th SS. Let  $j$  of the  $i-1$  SSs polled before SS  $i$  also transmit data in the next frame. The expected waiting time for the tagged packet is thus

$$\begin{aligned} E[X_{i,j,C1}] &= E[T_S - (n-i+1)T_P - t] + (n-i+1)T_P \\ &\quad + T_{DL} + jL \\ &= \frac{T_S + (n-i+1)T_P}{2} + T_{DL} + jL. \end{aligned} \quad (2)$$

In a frame, the probability that a SS has a packet for which bandwidth was previously reserved is  $\rho(1-p)$ .  $j$  is thus binomially distributed with parameters  $B[i-1, \rho(1-p)]$ . Unconditioning Eqn. (2) on  $j$  we have

$$\begin{aligned} E[X_{i,C1}] &= \sum_{j=0}^{i-1} E[X_{i,j,C1}] \binom{i-1}{j} (\rho(1-p))^j \\ &\quad \times (1-\rho(1-p))^{i-j-1} \\ &= \frac{T_S + (n-i+1)T_P}{2} + T_{DL} \\ &\quad + (i-1)\rho(1-p)L. \end{aligned} \quad (3)$$

For case C2, the packet first waits for the current frame to be over ( $T_S - t$ ). The SS makes a bandwidth request in the next frame and the packet is transmitted in the subsequent frame. The PDF of  $t$  given that it arrived after SS  $i$  was polled (i.e. in the last  $(n-i+1)T_P$  seconds of the frame) is  $U[T_S - (n-i+1)T_P, T_S]$ . If a random variable  $Y$  has the distribution  $U[a, b]$ , then  $b - Y$  has the distribution  $U[0, b-a]$ . Thus  $T_S - t$  is distributed as  $U[0, (n-i+1)T_P]$ . With  $j$  of  $i-1$  SSs also sending data in the frame with the tagged packet, the expected waiting time is given by

$$\begin{aligned} E[X_{i,j,C2}] &= E[T_S - t] + T_S + T_{DL} + jL \\ &= \frac{(n-i+1)T_P}{2} + T_S + T_{DL} + jL \end{aligned} \quad (4)$$

Since  $j$  is distributed as  $B[i-1, \rho(1-p)]$ , unconditioning on  $j$  gives

$$\begin{aligned} E[X_{i,C2}] &= \sum_{j=0}^{i-1} E[X_{i,j,C2}] \binom{i-1}{j} (\rho(1-p))^j \\ &\quad \times (1-\rho(1-p))^{i-j-1} \\ &= \frac{(n-i+1)T_P}{2} + T_S + T_{DL} \\ &\quad + (i-1)\rho(1-p)L. \end{aligned} \quad (5)$$

Now, the probabilities of cases C1 and C2 are given by

$$P[C1] = \frac{T_S - (n-i+1)T_P}{T_S}, \quad (6)$$

$$P[C2] = \frac{(n-i+1)T_P}{T_S}. \quad (7)$$

The expected waiting time in state S0,  $D_{i,S0}$ , is then given by

$$\begin{aligned} E[D_{i,S0}] &= E[X_{i,C1}]P[C1] + E[X_{i,C2}]P[C2] \\ &= \frac{T_S}{2} + (n-i+1)T_P + T_{DL} \\ &\quad + (i-1)\rho(1-p)L. \end{aligned} \quad (8)$$

**B. Arrival at a Non-Empty Queue: State S1:** Let the number of packets seen by a tagged arrival at a non-empty queue be  $N_{NQ}$ . Since no bandwidth was reserved at the last poll, SS  $i$  does not transmit in this frame. As in state S0, we consider the same subcases C1 and C2. In case C1, a bandwidth request is sent in the current frame and the queued packets are transmitted in the next  $N_{NQ}$  frames. As in state S0,  $T_S - (n-i+1)T_P - t$  has the distribution  $U[0, T_S - (n-i+1)T_P]$ . The expected waiting time is then

$$\begin{aligned} E[X_{i,j,C1}] &= E[T_S - (n-i+1)T_P - t] + (n-i+1)T_P \\ &\quad + N_{NQ}T_S + T_{DL} + jL \\ &= \frac{T_S + (n-i+1)T_P}{2} + E[N_{NQ}]T_S + T_{DL} + jL. \end{aligned}$$

Again,  $j$  follows a Binomial distribution with parameters  $B[i-1, \rho(1-p)]$ . Unconditioning on  $j$  gives

$$\begin{aligned} E[X_{i,C1}] &= \frac{T_S + (n-i+1)T_P}{2} + E[N_{NQ}]T_S + T_{DL} \\ &\quad + (i-1)\rho(1-p)L. \end{aligned} \quad (9)$$

For case C2, bandwidth for the first of the  $N_{NQ}$  enqueued packets can only be reserved in the next frame. Another  $N_{NQ}$  frames are required to transmit these packets. The expected waiting time is then

$$\begin{aligned} E[X_{i,j,C2}] &= E[T_S - t] + T_S + N_{NQ}T_S + T_{DL} + jL \\ &= \frac{(n-i+1)T_P}{2} + (E[N_{NQ}] + 1)T_S + T_{DL} + jL. \end{aligned}$$

Unconditioning on  $j$  gives

$$\begin{aligned} E[X_{i,C2}] &= \frac{(n-i+1)T_P}{2} + (E[N_{NQ}] + 1)T_S + T_{DL} \\ &\quad + (i-1)\rho(1-p)L. \end{aligned} \quad (10)$$

An arrival sees that no request was made in the previous poll only if all queued packets arrived after the poll. Since  $(n-i+1)T_P$  seconds remain in the previous frame after the poll, the duration from the poll to the tagged arrival is  $(n-i+1)T_P + t$ . Given  $t$ , the probability that all arrivals occurred after the poll is

$$P[S1, C1 | t] = \int_0^{(n-i+1)T_P + t} \frac{\lambda(\lambda x)^{E[N_{NQ}]-1} e^{-\lambda x}}{\Gamma(E[N_{NQ}])} dx, \quad (11)$$

where  $\Gamma(z) = \int_0^\infty y^{z-1} e^{-y} dy$ . The tagged arrival occurs before SS  $i$  is polled in the current frame if it arrives in the first  $T_S - (n-i+1)T_P$  seconds of the frame. Then, unconditioning on  $t$  gives

$$\begin{aligned} P[S1, C1] &= \int_0^{T_S - (n-i+1)T_P} \frac{1}{T_S} \int_0^{(n-i+1)T_P + t} \\ &\quad \frac{\lambda(\lambda x)^{E[N_{NQ}]-1} e^{-\lambda x}}{\Gamma(E[N_{NQ}])} dx dt. \end{aligned} \quad (12)$$

For case C2, the duration between the last poll and the tagged arrival is  $t - T_S + (n-i+1)T_P$  and lies in the range  $[0, (n-i+1)T_P]$ . The probability that all  $N_{NQ}$  enqueued packets occur in this interval is then

$$P[S1, C2] = \int_0^{(n-i+1)T_P} \frac{1}{T_S} \int_0^t \frac{\lambda(\lambda x)^{E[N_{NQ}]-1} e^{-\lambda x}}{\Gamma(E[N_{NQ}])} dx dt. \quad (13)$$

Combining the cases C1 and C2, the expected waiting in state S1 is given by

$$E[D_{i,S1}] = E[X_{i,C1}]P[S1, C1] + E[X_{i,C2}]P[S1, C2]. \quad (14)$$

Finally, the probability that an arbitrary arrival sees a non-empty queue where no bandwidth has been reserved in the previous poll,  $p$ , is given by  $p = P[S1, C1] + P[S1, C2]$ .

C. Arrival at a Non-Empty Queue: State S2: We consider two subcases: the tagged arrival occurs before SS  $i$ 's opportunity to transmit a packet in the current frame (C1) or after (C2). Let  $k$  of  $i-1$  SSs before SS  $i$  transmit data in the current frame. The time from the start of the frame when SS  $i$  may transmit a packet is then  $T_{DL} + kL$ . The PDF of the tagged packet's arrival time  $t$  in case C1 is then  $U[0, T_{DL} + kL]$ . Thus the PDF of  $T_{DL} + kL - t$  is also  $U[0, T_{DL} + kL]$ . The remaining time in the frame after SS  $i$  transmits its packet is  $T_S - T_{DL} - kL$ . One of the  $N_{NQ}$  enqueued packets gets transmitted in the current frame and another  $N_{NQ} - 1$  frames must pass before the tagged packet is served. Following the analysis of cases C0 and C1, the expected time in the frame where the tagged packet gets served is  $(i-1)\rho(1-p)L$ . The expected waiting time is then

$$\begin{aligned} E[X_{i,k,C1}] &= E[T_{DL} + kL - t] + (T_S - T_{DL} - kL) \\ &\quad + (N_{NQ} - 1)T_S + T_{DL} + (i-1)\rho(1-p)L \\ &= -\frac{T_{DL} + kL}{2} + E[N_{NQ}]T_S + T_{DL} \\ &\quad + (i-1)\rho(1-p)L. \end{aligned} \quad (15)$$

In case C2, that tagged packet arrives in the last  $T_S - T_{DL} - kL$  seconds of the frame and  $t$  thus has the distribution  $U[T_{DL} + kL, T_S]$ . Consequently,  $T_S - t$  is uniformly distributed in  $U[0, T_S - T_{DL} - kL]$ . Also, another  $N_{NQ}$  frames are needed to transmit the enqueued packets. The expected waiting time is then

$$\begin{aligned} E[X_{i,k,C2}] &= E[T_S - t] + N_{NQ}T_S + T_{DL} + (i-1)\rho(1-p)L \\ &= \frac{T_S - T_{DL} - kL}{2} + E[N_{NQ}]T_S + T_{DL} \\ &\quad + (i-1)\rho(1-p)L. \end{aligned}$$

The probabilities of cases C1 and C2 are given by

$$P[C1] = \frac{T_{DL} + kL}{T_S}, \quad (16)$$

$$P[C2] = \frac{T_S - T_{DL} - kL}{T_S}. \quad (17)$$

Combining cases C1 and C2 and unconditioning on  $k$ , the

expected waiting time is given by

$$\begin{aligned} E[D_{i,S2}] &= \sum_{k=0}^{i-1} (E[X_{i,j,C1}]P[C1] + E[X_{i,j,C2}]P[C2]) \\ &\quad \times \binom{i-1}{k} (\rho(1-p))^k (1-\rho(1-p))^{i-k-1} \\ &= \frac{T_S}{2} + E[N_{NQ}]T_S. \end{aligned}$$

D. Overall Delay: The expected queue length seen by an arbitrary arrival,  $\bar{E}[N]$  is related to  $E[N_{NQ}]$  by

$$\begin{aligned} E[N_{NQ}] &= \sum_{i=0}^{\infty} \frac{iP[N=i, \text{NEQ}]}{P[\text{NEQ}]} \\ &= \sum_{i=1}^{\infty} \frac{iP[N=i]}{\rho} = \frac{E[N]}{\rho}, \end{aligned} \quad (18)$$

where  $P[\text{NEQ}]$  is the probability that the queue is non-empty. From Little's Law  $E[N] = \lambda D_i$  and thus  $E[N_{NQ}] = \frac{\lambda D_i}{\rho}$ . Using this in the equations for  $E[D_{i,S1}]$  and  $E[D_{i,S2}]$  and combining the results for states S0, S1 and S2, the expected delay at SS  $i$  is

$$E[D_i] = E[D_{i,S0}](1-\rho) + E[D_{i,S1}]\rho + E[D_{i,S2}]\rho(1-p) + L. \quad (19)$$

It may also be of interest to evaluate the average delay across all nodes in the network. This can be obtained using

$$E[D] = \frac{1}{n} \sum_{i=1}^n E[D_i]. \quad (20)$$

The key feature in the model above that differentiates it from models for other frame based polling schemes such as IEEE 802.11 point coordination function (PCF), protocols for satellite networks etc. is that if a packet arrives after the node is polled in a frame in IEEE 802.16, it may have to wait for the next frame to be polled, and then transmit the data in the subsequent frame (case C2 in the analysis for states S1 and S2). In traditional frame based polling schemes, a packet that arrives after the poll in the current frame gets polled in the next frame and also transmits its data in the same frame. Thus IEEE 802.16 has an additional delay in certain cases. However, it is non-trivial to compute the probability of the cases where an additional component is added as can be seen from Eqn. (13).

#### A. Multichannel Scenario

This section extends the analysis to IEEE 802.16 operation over an OFDMA PHY. The OFDMA PHY is modeled as a set of  $m$  orthogonal subchannels (each consisting of multiple OFDM subcarriers) in the frequency domain. The analysis closely follows the structure developed earlier and the main difference is that now  $m$  SSs may transmit at the same time. Thus the time before SS  $i$  is polled relative to the start of polling is  $\lfloor \frac{i}{m} \rfloor T_P$  and if  $j$  SSs transmit their data before SS  $i$ , SS  $i$  has to wait for  $\lfloor \frac{j}{m} \rfloor L$  seconds before it transmits its own packet. The rest of the analysis stays the same and the details are thus omitted.

*A. Arrival at an Empty Queue: State S0:* The expressions for the expected delay for tagged arrivals before SS  $i$  is polled in the current frame (case C1) and after (case C2) now become

$$E[X_{i,C1}] = T_S - \frac{T_{DL} + \lfloor \frac{i-1}{m} \rfloor T_P}{2} + T_{DL} + \left\lceil \frac{n}{m} \right\rceil T_P + \left\lfloor \frac{(i-1)\rho(1-p)}{m} \right\rfloor L, \quad (21)$$

$$E[X_{i,C2}] = \frac{T_S - T_{DL} - \lfloor \frac{i-1}{m} \rfloor T_P}{2} + T_S + T_{DL} + \left\lceil \frac{n}{m} \right\rceil T_P + \left\lfloor \frac{(i-1)\rho(1-p)}{m} \right\rfloor L. \quad (22)$$

The probabilities of cases C1 and C2 are given by

$$P[C1] = \frac{T_{DL} + \lfloor \frac{i-1}{m} \rfloor T_P}{T_S} \quad (23)$$

$$P[C2] = \frac{T_S - T_{DL} - \lfloor \frac{i-1}{m} \rfloor T_P}{T_S}. \quad (24)$$

Combining cases C1 and C2, the expected waiting time,  $D_{i,S0}$ , is given by

$$E[D_{i,S0}] = \frac{3}{2}T_S + \left( \left\lceil \frac{n}{m} \right\rceil - \left\lfloor \frac{i-1}{m} \right\rfloor \right) T_P + \left\lfloor \frac{(i-1)\rho(1-p)}{m} \right\rfloor L. \quad (25)$$

*B. Arrival at a Non-Empty Queue: State S1:* For a tagged arrival before (case C1) and after (case C2) SS  $i$  is polled in the current round, the expected delays are

$$E[X_{i,C1}] = -\frac{T_{DL} + \lfloor \frac{i-1}{m} \rfloor T_P}{2} + (E[N_{NQ}] + 1)T_S + T_{DL} + \left\lceil \frac{n}{m} \right\rceil T_P + \left\lfloor \frac{(i-1)\rho(1-p)}{m} \right\rfloor L, \quad (26)$$

$$E[X_{i,C2}] = \frac{T_S - T_{DL} - \lfloor \frac{i-1}{m} \rfloor T_P}{2} + (E[N_{NQ}] + 1)T_S + T_{DL} + \left\lceil \frac{n}{m} \right\rceil T_P + \left\lfloor \frac{(i-1)\rho(1-p)}{m} \right\rfloor L, \quad (27)$$

with the corresponding probabilities

$$P[S1, C1] = \int_0^{T_{DL} + \lfloor \frac{i-1}{m} \rfloor T_P} \frac{1}{T_S} \int_0^{T_S - T_{DL} - \lfloor \frac{i-1}{m} \rfloor T_P + t} \frac{\lambda(\lambda x)^{E[N_{NQ}]-1} e^{-\lambda x}}{\Gamma(E[N_{NQ}])} dx dt, \quad (28)$$

$$P[S1, C2] = \int_0^{T_S - T_{DL} - \lfloor \frac{i-1}{m} \rfloor T_P} \frac{1}{T_S} \int_0^t \frac{\lambda(\lambda x)^{E[N_{NQ}]-1} e^{-\lambda x}}{\Gamma(E[N_{NQ}])} dx dt. \quad (29)$$

Combining cases C1 and C2, the expected waiting time,  $D_{i,S1}$ , is given by  $E[D_{i,S1}] = E[X_{i,C1}]P[S1, C1] + E[X_{i,C2}]P[S1, C2]$ . Finally,  $p$  is given by  $p = P[S1, C1] + P[S1, C2]$ .

*C. Arrival at a Non-Empty Queue: State S2:* For tagged arrivals before (case C1) and after (case C2) SS  $i$  is served

in the current frame with  $k$  SSs transmitting before SS  $i$ , the expected delays are given by

$$E[X_{i,k,C1}] = -\frac{T_{DL} + \lfloor \frac{n}{m} \rfloor T_P + \lfloor \frac{k}{m} \rfloor L}{2} + E[N_{NQ}]T_S + T_{DL} + \left\lceil \frac{n}{m} \right\rceil T_P + \left\lfloor \frac{(i-1)\rho(1-p)}{m} \right\rfloor L,$$

$$E[X_{i,k,C2}] = \frac{T_S - T_{DL} - \lfloor \frac{n}{m} \rfloor T_P - \lfloor \frac{k}{m} \rfloor L}{2} + E[N_{NQ}]T_S + T_{DL} + \left\lceil \frac{n}{m} \right\rceil T_P + \left\lfloor \frac{(i-1)\rho(1-p)}{m} \right\rfloor L,$$

and the probabilities of cases C1 and C2 are

$$P[C1] = \frac{T_{DL} + \lfloor \frac{n}{m} \rfloor T_P + \lfloor \frac{k}{m} \rfloor L}{T_S}$$

$$P[C2] = \frac{T_S - T_{DL} - \lfloor \frac{n}{m} \rfloor T_P - \lfloor \frac{k}{m} \rfloor L}{T_S}.$$

Combining the two cases and unconditioning on  $k$ , the expected waiting time,  $D_{i,S2}$ , is

$$E[D_{i,S2}] = \frac{T_S}{2} + E[N_{NQ}]T_S. \quad (30)$$

*D. Overall Delay:* Using  $E[N_{NQ}] = \frac{\lambda D_i}{\rho}$  and combining the expressions for cases S0, S1 and S2, the expected delay  $E[D_i]$  at SS  $i$  is given by

$$E[D_i] = E[D_{i,S0}](1-\rho) + E[D_{i,S1}]\rho + E[D_{i,S2}]\rho(1-p) + L. \quad (31)$$

Also, the average delay across all nodes in the network is given by

$$E[D] = \frac{1}{n} \sum_{i=1}^n E[D_i]. \quad (32)$$

### III. SIMULATION RESULTS

This section verifies the accuracy of our models by comparing them against simulations. The simulations were carried out using a NS-2 based IEEE 802.16 module developed by the WiMAX Forum. The following parameters were used:  $T_S = 5\text{ms}$ ,  $T_{DL} = 3.4\text{ms}$ ,  $T_{UL} = 1.6\text{ms}$ ,  $T_P = 10\mu\text{s}$  and a bit rate of 50Mbps.

Figure 1 demonstrates the closeness in the simulation and analytic results for the single carrier PHY with SSs polled at the end of the uplink subframe. The analytic results from our model when it is extended for polling at the beginning of the uplink subframe are shown in Figure 2. The figure also shows the delays (through simulations) when SSs send piggybacked bandwidth requests embedded in any data packet they transmit. Polling at the beginning (end) of the subframe maximizes (minimizes) the likelihood that an arrival misses the poll in the frame of its arrival thereby increasing (decreasing) the delays. Our models thus form upper and lower bounds on the delay for piggybacked operation, as verified in Figure 2.

Figure 3 evaluates the effectiveness of our model in the presence of non-Poisson traffic. Specifically, traffic at each SS was generated according to a Pareto on-off model with average burst times (on and off) of 250msec and shape parameter of 10. The analytic results were generated using

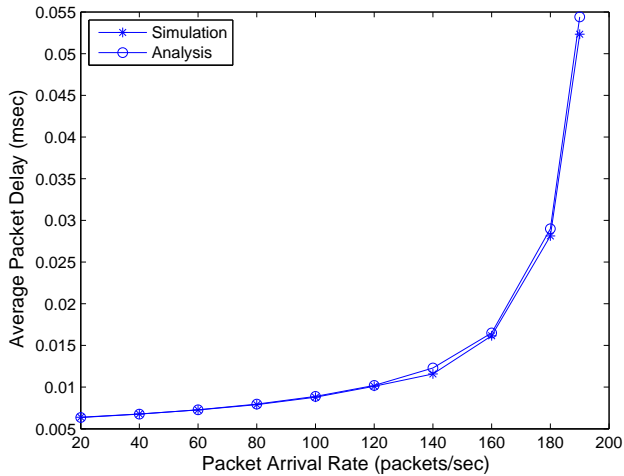


Fig. 1. Polling at End of Uplink Subframe: Average delay,  $n = 20$ ,  $i = 10$ .

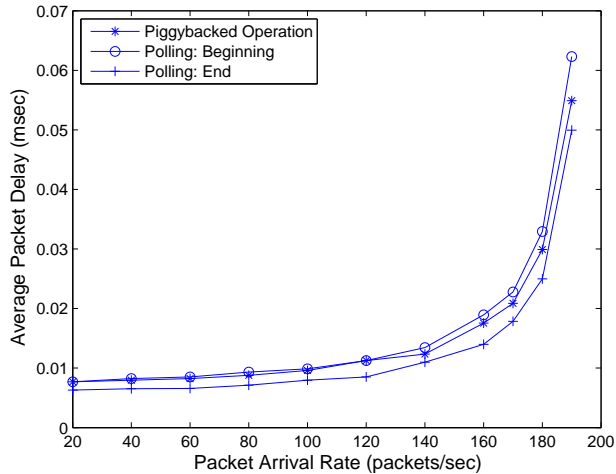


Fig. 2. Comparison with Piggybacked Operation: Average Delay,  $n = 20$ ,  $i = 10$ .

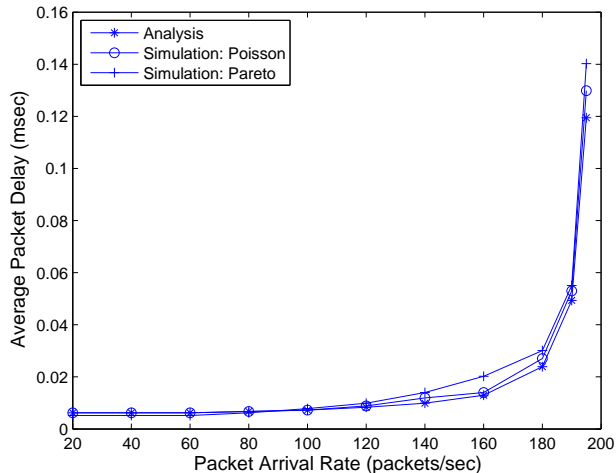


Fig. 3. Comparison with Pareto Traffic: Average Delay,  $n = 10$ ,  $i = 5$ .

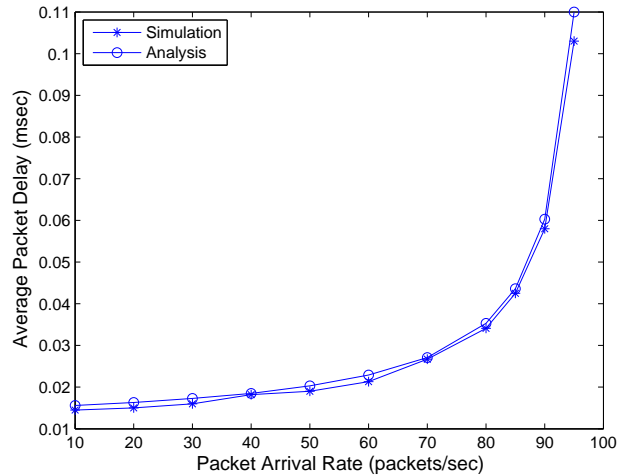


Fig. 4. Multichannel Operation with Polling at End of Uplink Subframe: Average Delay,  $n = 20$ ,  $i = 10$ ,  $T_S = 10\text{ms}$ ,  $T_{DL} = 6.7\text{ms}$ .

the Poisson traffic based model with the average arrival rate equal to that of the Pareto on-off process. Note that our model performs reasonably well in this scenario. The comparisons between simulation and analysis for the multichannel OFDMA operation are presented in Figure 4. It was assumed that  $m = 5$  subchannels were available for the polled Ss and again we note the close match between the simulation and analytic results.

#### IV. CONCLUSIONS

This paper presented queuing models to evaluate the packet delays in the polling based operation of WiMAX/IEEE 802.16 networks. We considered single channel as well as OFDMA based PHY layers. The models were verified in diverse scenarios, including non-Poisson traffic, using simulations.

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