Throughput Guarantee for Maximal Schedulers in Sensor Networks with Cooperative Relays

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***Abstract*—This paper addresses the question of throughput guarantees through distributed scheduling in sensor networks with relay based cooperative communications. We prove that in a single frequency network with bidirectional, equal power communication and low complexity distributed maximal scheduling attains a guaranteed fraction of the maximum throughput region in arbitrary wireless networks. We also show that the guarantees are tight in the sense that they cannot be improved any further with maximal scheduling. Simulation results are also provided to show the performance of a distributed, maximal scheduling algorithm under different network settings.**

***Index Terms*—Maximal scheduler, wireless sensor network, cooperative relay, throughput guarantee**

I. Introduction

Communication technologies that use relays or cooperative transmissions have received considerable attention, particularly due to their ability to increase a wireless sensor network's (WSN's) range and capacity [1]. Existing research has shown that cooperative diversity gains can be achieved in distributed WSNs where nodes help each other by relaying transmissions [2]. This paper focuses on the performance of the *scheduling algorithm* used to control the channel access at the medium access control (MAC) layer in WSNs with cooperative relays. We focus on the throughput guarantees that may be provided by distributed schedulers for WSNs with cooperative relays and prove that maximal schedulers can achieve a guaranteed fraction of the maximum throughput region in arbitrary wireless networks.

The communication theory aspect of cooperative relaying, such as energy efficiency, bit error rate, forwarding strategies (e.g. decode and forward, amplify and forward) have been widely investigated [2], [3]. However, the performance of upper layer protocols, such as MAC layer schedulers, that use cooperative relay based communication technologies has not been investigated in detail. For wireless networks without cooperative relays, [4] presents the maximum achievable throughput region and an algorithm for attaining it, although the centralized nature and computational complexity of the scheduler limits its applicability. Instead, we focus on *maximal scheduling*, which is equivalent to solving the *Maximal Independent Set* (MIS) problem in graph theory. While it is known that a simple randomized distributed MIS algorithm for an arbitrary graph of size n, including exchange of messages, can be done in time $O(log^2 n)$ [5], [6], their performance in terms of the achievable throughput in cooperative relay based WSNs is unknown.

It has been shown in [7] that for wireless networks with direct transmissions (i.e. no cooperative communications), maximal scheduling is guaranteed to achieve a fraction of the maximum throughput region and the fraction (of value $1/8$) is decided by the maximum "conflict degree" of the network. The conflict degree of a transmitter-receiver pair (u, v) is defined as the number of transmitter-receiver pairs that interfere with (u, v) but not with each other. Using this notion of conflict degree, we prove that in a network with cooperative relays, any distributed maximal scheduling algorithm can achieve at least $1/10$ of the maximum throughput region and this guarantee cannot be improved any further. Finally we note that MAC protocols focusing on the implementation details have been proposed for cooperative communications [8]. Our contribution is that we characterize the attainable maximum throughput region while considering a general maximal scheduler as the baseline.

The rest of the paper is organized as follows. Section II describes the network and conflict models and a performance guarantee on maximal throughput scheduling is presented in Section III. Simulation results are presented in Section IV. Finally Section V concludes the paper.

II. Network Model

We consider a network where sensors share a single frequency and have the same transmission power. We consider a WSN with simple cooperative communication and a discrete memoryless three-terminal relay model as in [1]. Every session I with a packet transmission involves three nodes: the source S, the destination D and the relay R. Thus a session may be represented as a 4-tuple (I, S, D, R). The distance and channel gains between two nodes i and j are represented by $d(i, j)$ and $\lambda_{i,j}$ respectively.

A discrete slotted-time model is assumed where each slot is long enough so that a source and a relay can cooperatively transmit a single packet to the destination. The message exchanges among the source, relay and destination are considered to be bi-directional; the source broadcasts the *data* to the destination and the relay; the relay retransmits the *data* to the destination, and the destination replies with an *ACK* if it successfully receives the packet. The decode-and-forward relay strategy is assumed in this paper [3].

We model a wireless network as a graph $G = (V, E)$, where V is the set of nodes (i.e. sensors in the network) and E is the set of links. If sensor A is in the transmission range of sensor B, then A is B's *neighbor*. By assuming bidirectional

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symmetric communication, B is A's neighbor too. If A and B are neighbors, there is a link $(A,B) \in E$. We denote the neighborhood of A as N_A, defined as the set of nodes that are in A's transmission range. In addition, it is assumed that each node has a single transceiver (transmitter/receiver), thus each node can only participate in one session at a time. Then a session (I_i, S_i, D_i, R_i) is successful when none of the nodes in this session is participating in other sessions *and* if none of the neighbors of S_i, D_i, and R_i transmit in this slot. The *conflict set* of session I_i is then,

$$\begin{aligned} C(I_i) = \{I_j &: I_j \text{ shares a common node with } I_i, \\ &(S_j, \text{or } R_j \text{ or } D_j) \in (N_{S_i} \cup N_{D_i} \cup N_{R_i})\}. \quad (1) \end{aligned}$$

If $I_j \in C(I_i)$ and $I_i \in C(I_j)$, sessions I_i and I_j are also defined as *neighbors*.

III. Maximal Scheduling Performance Guarantee

Let the number of sessions in conflict set $C(I_i)$ that can be scheduled at the same time (but not with session I_i) be defined as the *conflict degree* of conflict set $C(I_i)$. Denote the maximum conflict degree in the network as $K(\mathcal{N})$. It is proved in [7] that for the single frequency, bi-directional, equal-power, two-terminal communication network model (i.e. no relays), the performance of an arbitrary maximal scheduling algorithm is guaranteed to achieve $1/K(\mathcal{N})$ of the maximum throughput region. In this paper we extend this result for networks with cooperative relays and note that the extension is non-trivial. Further, we show that for a three-terminal relay network, at least $1/10$ of the maximum throughput region is attained. We also show that this guarantee cannot be improved because there exist network topologies where at most $1/10$ of the maximum throughput region is achieved.

In a wireless network $\mathcal{N}$, let λ_i be the *arrival rate* of session I_i, $i = 1, \cdots, N$. Define $\vec{\lambda}$ as the N-dimensional *arrival rate vector* whose components are the arrival rates of the sessions. A network is said to be *stable* if the arrival rate of each session equals its departure rate. The *throughput region* of a maximal scheduling policy π_{MS}, denoted as Λ^{MS}, is the set of arrival rate vectors such that the network is stable under π_{MS}. Also, an arrival rate vector $\vec{\lambda}$ is said to be *feasible* if it falls in the throughput region of some scheduling policy. The *maximum throughput region* of the network $\mathcal{N}$ is the set of all feasible rate vectors from all possible policies and is denoted by Λ. We then use the following result from [7]:

Theorem 1 [7]: In any wireless network $\mathcal{N}$, if $\vec{\lambda} \in \Lambda$ in $\mathcal{N}$, then $\vec{\lambda}/K(\mathcal{N}) \in \Lambda^{MS}$ in $\mathcal{N}$.

The formal proof is detailed in [7] for two terminal communication networks and all statements hold for networks with cooperative relays. The theorem states that, for any wireless network $\mathcal{N}$, at least $1/K(\mathcal{N})$ of the maximum throughput is guaranteed given any maximal scheduling policy. For an arbitrary session I_i in $\mathcal{N}$, there are at most $K(\mathcal{N})$ sessions in $I_i \cup C(I_i)$ that can be scheduled simultaneously. Thus, the sum of departure rates and the sum of the feasible arrival rates for $I_i \cup C(I_i)$ is at most $K(\mathcal{N})$. For an arrival rate vector $\vec{\lambda}/K(\mathcal{N})$, the sum of arrival rates for $I_i \cup C(I_i)$ is at most 1. With maximal scheduling, one session is always scheduled among $I_i \cup C(I_i)$. Thus, with $\vec{\lambda}/K(\mathcal{N})$, the departure rates are greater than or equal to the arrival rates and the network is stable.

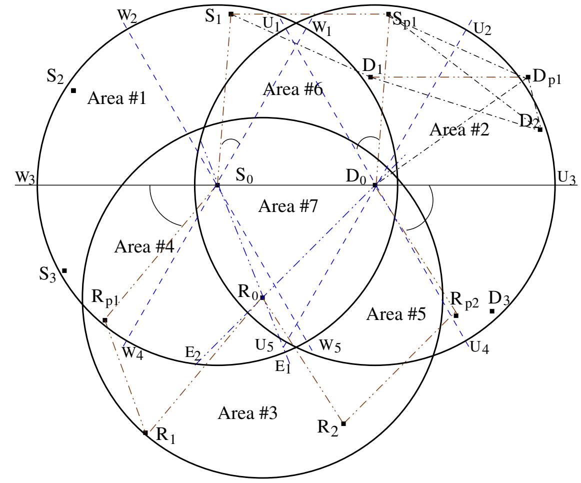

Fig. 1. A session (I_0, S_0, R_0, D_0) in the Euclidean plane, showing the division of $Area\#1, \cdots, Area\#7$, the sectorization of $\overline{b(S_0)}$ and $\overline{b(D_0)}$, and the location of other nodes.

We now state and prove the main result of this paper that quantifies $K(\mathcal{N})$ and thus the guaranteed throughput region for maximal scheduling.

Lemma 1: For any wireless network $\mathcal{N}$ with relay usage, if the same frequency and equal power are used in nodes, and bi-directional communication is involved, then $K(\mathcal{N}) \leq 10$.

Proof: Let the transmission range of each sensor be d_{max}. In a two-dimensional Euclidean plane, the neighborhood area of a node A is equivalent to the closed circle centered at A with radius d_{max}, denoted as $\overline{b(A)}$. The *coverage area* of any session (I_0, S_0, D_0, R_0) is then the union of the areas $\overline{b(S_0)}$, $\overline{b(D_0)}$ and $\overline{b(R_0)}$, as shown in Fig. 1. If a session $I_j \in C(I_0)$, at least one terminal of session I_j falls in the coverage area of session (I_0, S_0, D_0, R_0). Thus to find the maximum $K(\mathcal{N})$, it is sufficient to find the maximum number of nodes that can be contained in $\overline{b(S_0)} \cup \overline{b(D_0)} \cup \overline{b(R_0)}$, such that the distance between any two of these nodes is greater than d_{max}.

Without loss of generality (wlog), we can assume that node S_0 and D_0 lie on the x axis. For the sake of convenience, we divide $\overline{b(S_0)} \cup \overline{b(D_0)} \cup \overline{b(R_0)}$ into 7 sub-areas, labeled $Area\#1$ through $Area\#7$ in Fig. 1. For example, $\overline{b(S_0)} = Area\#1 \cup Area\#4 \cup Area\#6 \cup Area\#7$. $Area\#1$ is the area where $\overline{b(S_0)}$ does not intersect with either $\overline{b(D_0)}$ or $\overline{b(R_0)}$; $Area\#4$ is the area where $\overline{b(S_0)}$ intersects with $\overline{b(R_0)}$ but not with $\overline{b(D_0)}$; $Area\#6$ is the area where $\overline{b(S_0)}$ intersects with $\overline{b(D_0)}$ but not with $\overline{b(R_0)}$; and $Area\#7$ is the area where $\overline{b(S_0)}$ intersects with both $\overline{b(D_0)}$ and $\overline{b(R_0)}$. We use U_n to denote the number of nodes lying in $Area\#n$. We first formulate the following geometric facts, statements and intermediate results:

$$U_1 + U_2 + \sum_{n=4}^{7} U_n \leq 8, U_1 + \sum_{n=3}^{7} U_n \leq 8, \sum_{n=2}^{7} U_n \leq 8 \quad (2)$$

$$U_1 + U_4 + U_6 + U_7 \leq 5, U_2 + \sum_{n=5}^{7} U_n \leq 5, \sum_{n=3}^{5} U_n + U_7 \leq 5 \quad (3)$$

$$U_1 + U_4 \leq 4, U_2 + U_5 \leq 4 \quad (4)$$

$$U_2 + U_6 \leq 4, U_3 + U_4 \leq 4 \quad (5)$$

$$U_3 + U_5 \leq 4, U_1 + U_6 \leq 4 \quad (6)$$

where Eqn. (2) is proved in Lemma 3 of [7] and states that in the coverage area of any two-node session, there can be at most 8 nodes that are in conflict with the session but not with each other. Eqn. (3) follows the geometric argument and Lemma 18 of [7] and states that at most 5 nodes can be located in a circle ($\overline{b(S_0)}$, $\overline{b(D_0)}$, or $\overline{b(R_0)}$) such that the distance between any two nodes is greater than the radius. The two inequalities in Eqn. (4) are based on the fact that $\overline{b(S_0)}$ and $\overline{b(D_0)}$ intersect with each other with the distance between S_0 and D_0 satisfying $|S_0D_0| \leq d_{max}$ and the region $\overline{b(S_0)}\backslash\overline{b(D_0)}$ is covered by 4 $\pi/3$ sectors. The same arguments hold for $\overline{b(D_0)}\backslash\overline{b(S_0)}$. Also as shown in the proof of Lemma 3 in [7], the two equalities in Eqn. (4) can not be achieved at the same time. Similar arguments hold for Eqn. (5) and (6) as well. The rest of the proof proceeds in two steps.

Step 1: We prove by contradiction that the number of nodes not interfering with each other in $Area\#1 \cup Area\#2 \cup Area\#3$ is at most 8:

$$U_1 + U_2 + U_3 \leq 8. \quad (*)$$

Since each scenario with $U_1+U_2+U_3 \geq 9$ can be reduced to a scenario with $U_1+U_2+U_3 = 9$ by eliminating some sessions, it suffices to consider only the scenarios with $U_1+U_2+U_3 = 9$.

$$\textit{Contradicting assumption:} \quad U_1 + U_2 + U_3 = 9. \quad (A1)$$

Definition 1: Consider a session (I_i, S_i, D_i, R_i). For each session in $C(I_i)$ but not in conflict with each other, choose one of its terminals that falls in the coverage area of session I_i. Denote the set of chosen nodes by $\mathcal{U}$, and let $U = |\mathcal{U}|$. Given a node $A \in \mathcal{U}$, $A \in \overline{b(x)}$ and $A \notin \overline{b(y)}$ with $x, y = \{S_i, D_i, R_i\}$ and $x \neq y$, define the *distance* from A to $\overline{b(y)}$ as $\min\{|AB| : B \in \mathcal{U}, B \in \overline{b(y)}, y \notin \overline{b(x)}\}$. The smaller the distance, the *closer* node A is to disk $\overline{b(y)}$.

Since $0 \leq U_n \leq 4$ for $n = 1, 2, 3$, at least two areas have 3 or more nodes. Because of symmetry, wlog, we can assume that $Area\#1$ and $\#2$ are the two areas with 3 or more nodes. Then we divide $\overline{b(S_0)}$ and $\overline{b(D_0)}$ into six $\pi/3$ sectors respectively as shown in Fig. 1 with the dashed lines.

case (i): $U_1 = U_2 = 3$. Thus $U_3 = 3$ by assumption A1. Let S_1, S_2, S_3 be the nodes on $Area\#1$, with S_1 being the *closest* to $\overline{b(D_0)}$ and S_3 being the *closest* to $\overline{b(R_0)}$. Similarly, let D_1, D_2, D_3 be the nodes on $Area\#2$, with D_1 being the *closest* to $\overline{b(S_0)}$ and D_3 being the *closest* to $\overline{b(R_0)}$. Finally, let R_1, R_2, R_3 be the nodes on $Area\#3$, with R_1 being the *closest* to $\overline{b(S_0)}$ and R_2 being the *closest* to $\overline{b(D_0)}$. Note that the distance between any two nodes is greater than d_{max} and the angle subtended at S_0 or D_0 by any two nodes in adjacent sectors is greater than $\pi/3$. Then S_3 can only be on either sector $W_3S_0W_4$ or sector $W_4S_0W_5$ and D_3 can only be on sector $U_3D_0U_4$ or sector $U_4D_0U_5$.

To obtain the maximum value of $U_1 + U_2 + U_3$, we can assume that S_1 is on sector $W_1S_0W_2$ (Argument: Since $U_1 = 3$ and $Area\#1$ is covered by four $\pi/3$ sectors, whatever be the spread of the three nodes S_1, S_2, S_3, at least one node falls on either sector $W_1S_0W_2$ or sector $W_4S_0W_5$. Due to symmetry, wlog, we can assume that there is one node on sector $W_1S_0W_2$.). Now, choose the point S_{p1} to make $S_0S_1S_{p1}D_0$ a parallelogram, choose R_{p1} to make $S_0R_0R_1R_{p1}$ a parallelogram, and choose R_{p2} to make $D_0R_0R_2R_{p2}$ a parallelogram. First we claim that,

$$\angle S_{p1}D_0D_2 > \pi/3, \angle R_{p1}S_0S_2 > \pi/3, \angle D_2D_0R_{p2} > \pi/3 \quad (7)$$

$$\angle S_1S_0W_1 + U_1D_0S_{p1} = \pi/3. \quad (8)$$

To see these, wlog, we can assume that D_1 has a smaller y-coordinate than S_1. If node D_1 lies outside the parallelogram or on the line $S_{p1}D_0$, $\angle S_{p1}D_0D_2 > \angle D_1D_0D_2 > \pi/3$. If node D_1 lies inside the parallelogram $S_0S_1S_{p1}D_0$, to see that $\angle S_{p1}D_0D_2 > \pi/3$, we choose D_{p1} such that $S_1S_{p1}D_{p1}D_1$ is a parallelogram, as shown in Fig. 1. We then have $|S_{p1}D_{p1}| = |S_1D_1| > d_{max}$. Join D_{p1} with D_0 and D_2. By construction, D_1D_{p1} is parallel to the x axis. Then D_2 must have a y-coordinate lower than D_1 or D_{p1} since it is easy to see that there is no point in sector $U_2D_0U_3$ with y-coordinate higher than D_1 or D_{p1} and whose distance from D_1 is greater than d_{max}. Also, by construction, $|D_1D_{p1}| = |S_1S_{p1}| = |S_0D_0| \leq d_{max}$. Then it is easy to see that D_{p1} must lie outside $\overline{b(D_0)}$. Thus line segment $S_{p1}D_2$ intersects with line segment D_1D_{p1}. In triangle $D_1D_{p1}D_2$, $|D_1D_2| > d_{max} \geq |D_1D_{p1}|$, and thus $\angle D_1D_{p1}D_2 > \angle D_{p1}D_2D_1$. Since $\angle S_{p1}D_{p1}D_2 > \angle D_1D_{p1}D_2 > \angle D_{p1}D_2D_1 > \angle D_{p1}D_2S_{p1}$, in triangle $S_{p1}D_{p1}D_2$, we have $|S_{p1}D_2| > |S_{p1}D_{p1}| > d_{max}$. Since S_1 is in sector $W_1S_0W_2$, it follows that S_{p1} must lie on sector $U_1D_0U_2$. Since both S_{p1} and D_2 are in $\overline{b(D_0)}$, we have $\angle S_{p1}D_0D_2 > \pi/3$.

Similarly, we have $\angle R_{p1}S_0S_2 > \pi/3$ and $\angle D_2D_0R_{p2} > \pi/3$. Then, since $\angle S_1S_0W_1 = \angle S_{p1}D_0U_2$ and $\angle U_1D_0S_{p1} + \angle S_{p1}D_0U_2 = \pi/3$, and we have Eqn. (8).

Next, we claim that,

$$\angle R_1R_0R_2 < 2\pi/3. \quad (9)$$

To see this, extend the line S_0R_0 to point E_1 and line D_0R_0 to point E_2. Then,

$$\begin{aligned}\angle R_1R_0R_2 &= \angle R_1R_0E_1 + \angle E_2R_0R_2 - \angle E_2R_0E_1\\ &= \angle R_{p1}S_0E_1 + \angle E_2D_0R_{p2} - \angle S_0R_0D_0\\ &= \angle R_{p1}S_0R_0 + \angle R_0S_0D_0 + \angle S_0D_0R_0\\ &\quad + \angle R_0D_0R_{p2} - \pi\\ &= \pi - \angle R_{p1}S_0W_3 - \angle U_3D_0R_{p2}. \end{aligned} \quad (10)$$

To show $\angle R_1R_0R_2 < 2\pi/3$, it is enough to show that

$$\angle R_{p1}S_0W_3 + \angle U_3D_0R_{p2} > \pi/3. \quad (11)$$

Since $\angle R_{p1}S_0S_2 > \pi/3$ (from Eqn. (7)), R_{p1} can only lie in sector $W_3S_0W_4$ or sector $W_4S_0W_5$. If it is in sector $W_4S_0W_5$, $\angle R_{p1}S_0W_3 > \pi/3$, and we have Eqn. (11). Similarly, if R_{p2} is in sector $U_4D_0U_5$, we have Eqn. (11). On the other hand, if R_{p1} lies in sector $W_3S_0W_4$ and R_{p2} lies in sector $U_3D_0U_4$, since $\angle S_2S_0S_1g > \pi/3$ and $\angle R_{p1}S_0S_2 > \pi/3$, we have $\angle W_4S_0R_{p1} + \angle S_1S_0W_1 < \pi/3$. Thus,

$$\angle R_{p1}S_0W_3 = \pi/3 - \angle W_4S_0R_{p1} > \angle S_1S_0W_1. \quad (12)$$

Additionally, since $\angle S_2S_0S_1g > \pi/3$, we have $\angle W_3S_0S_2 + \angle S_1S_0W_1 \leq \pi/3$. Since $\angle S_3S_0S_2 = \angle S_3S_0W_3 +$

$\angle W_3S_0S_2 > \pi/3$, we have

$$\angle S_3S_0W_3g > \angle S_1S_0W_1.$$

Likewise, we have

$$\angle U_3D_0R_{p2} = \pi/3 - \angle R_{p2}D_0U_4 > \angle U_1D_0S_{p1}. \quad (13)$$

$$\angle U_3D_0D_3 > \angle U_1D_0S_{p1}. \quad (14)$$

Combining Eqns. (12), (13) and (8), we have

$$\angle R_{p1}S_0W_3 + \angle U_3D_0R_{p2} > \angle S_1S_0W_1 + \angle U_1D_0S_{p1} = \pi/3. \quad (15)$$

However, by assumption we must have $U_3 = 3$. Thus there is a third node, say R_3, in $Area\#3$, which satisfies $\angle R_1R_0R_3 > \pi/3$ and $\angle R_3R_0R_2 > \pi/3$ at the same time. However, this is a contradiction with Eqn. (9). Thus, if $U_1 = U_2 = 3$, $U_3 \leq 2$ and $U_1 + U_2 + U_3 \leq 8$.

Case (ii): $U_1 = 3, U_2 = 4$. Let S_1, S_2 and S_3 be the nodes in $Area\#1$ with positions as defined for case (i). Since $U_2 = 4$ and $Area\#2$ is covered by four sectors, each sector contains exactly one node. Let D_1 be on sector $U_1D_0U_2$, D_2 be on sector $U_2D_0U_3$, D_3 be on sector $U_3D_0U_4$ and D_4 be on sector $U_4D_0U_5$. Similar to Eqn. (7) we then have,

$$\angle R_{p2}D_0D_3 > \pi/3. \quad (16)$$

From Eqns. (14) and (16),

$$\angle U_3D_0R_{p2} = \angle U_3D_0D_3 + \angle D_3D_0R_{p2} > \angle U_1D_0S_{p1} + \pi/3. \quad (17)$$

Substituting Eqns. (12), (17) and (8) into Eqn. (10), we have

$$\angle R_1R_0R_2 < \pi/3. \quad (18)$$

However, based on our assumption, $|R_1R_2| > d_{max}$, which contradicts Eqn. (18). Thus, if $U_1 = 3$ and $U_2 = 4$, $U_3 \leq 1$ and $U_1 + U_2 + U_3 \leq 8$.

Since the equalities in Eqn. (4) cannot be achieved at the same time (i.e. $\overline{b(S_0)}\backslash\overline{b(D_0)}$ and $\overline{b(D_0)}\backslash\overline{b(S_0)}$ cannot each contain 4 nodes at the same time), there is no case where $U_1 = U_2 = 4$. Therefore, for any case, $U_1+U_2+U_3 \leq 8$ holds.

Step 2: Following Eqn. (*) and constrains in Eqns. (2) to (6), we now traverse the cases for all possible values of U_1 to show that $K(\mathcal{N}) \leq 10$ always holds.

Case (i): $U_1 = 4$. Then $U_7 \leq 1$ by Eqn. (3), $U_4 = U_6 = 0$ by Eqn. (4) and (6), and $U_2 + U_3 \leq 4$ by Eqn. (*). Then we have the following scenarios:

1) If $U_2 \leq 1$, $K \leq 10$ since $U_3 + U_4 + U_5 + U_7 \leq 5$ from Eqn. (3).
2) If $3 \leq U_2 \leq 4$, $U_3 \leq 1$. Then $K \leq 9 < 10$ from Eqn. (2).
3) If $U_2 = 2$, $U_3 \leq 2$ and $U_5 \leq 1$ since the equalities in Eqn. (4) cannot be achieved at the same time. Thus $K \leq 4 + 2 + 2 + 1 + 1 = 10$.

Case (ii): $0 \leq U_1 \leq 2$. Since $\sum_{n=2}^{7} U_n \leq 8$ by Eqn. (2), $K \leq 2 + 8 = 10$.

Case (iii): $U_1 = 3$. Then $U_2 + U_3 \leq 5$ by Eqn. (*). Then we have the following scenarios as all possible U_2 and U_3 combinations.

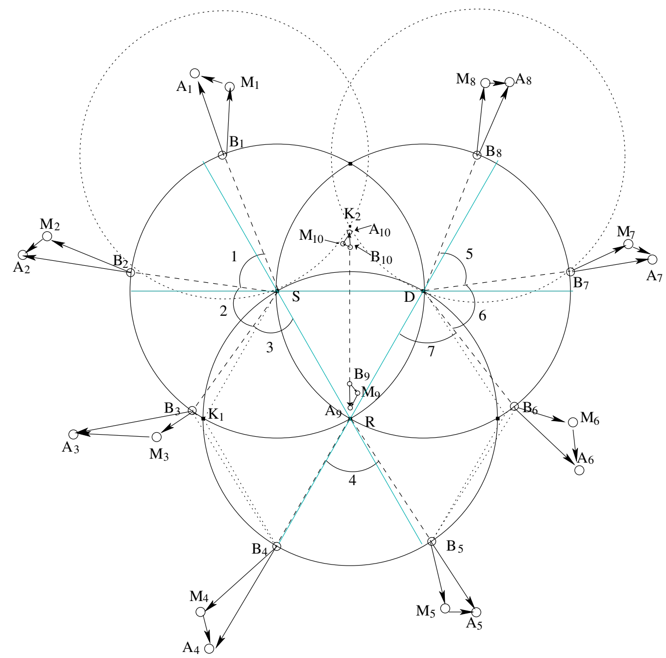

Fig. 2. Example of a network with $K(\mathcal{N}) = 10$.

Note that bidirectional symmetric communication is considered in this article and with the defintion of the conflict set (Eqn. (1)), in the context where we are looking for the maximum conflict degree, the role of source, destination or relay is equivalent and interchangable while Theorem 1 will still hold. Thus, in the Euclidean plane model of a session, the result concluded for a scenario of $U_1 = u$, where $1 \leq u \leq 4$, will hold for cases of $U_2 = u$ and $U_3 = u$, vice verse.

1) $U_2 = 4, U_3 \leq 1$. Cases can be broken down as in Case (i) $U_1 = 4$ and the result $K \leq 10$ holds.
2) $U_2 = 3, U_3 \leq 2$ or $U_2 \leq 2, U_3 = 3$. These scenarios are similar to Case (ii). If $U_3 \leq 2$, since $U_1 + U_2 + \sum_{n=4}^{7} U_n \leq 8$ by Eqn.(2), we have $K \leq 2 + 8 = 10$. In the same way, if $U_2 \leq 2$, $K \leq 10$.
3) $U_2 \leq 1, U_3 = 4$. Since $U_3 = 4$, $K \leq 10$ can be proved in the same way as in Case (i) $U_1 = 4$.

In conclusion, $\max K(\mathcal{N}) \leq 10$ for all possible cases. ■

Next we show that the performance guarantee is tight by demonstrating the existence of a network with $K(\mathcal{N}) = 10$.

Lemma 2: There exists a wireless network with relay usage that uses a single frequency with equal power in all nodes and bi-directional communication such that $K(\mathcal{N}) = 10$.

Proof: We prove the result using construction. An example network with $K(\mathcal{N}) = 10$ is shown in Fig. 2. To construct the network, consider a session (I, S, D, R) with $|SD| = |SR| = |DR| = d_{max}$. Let B_i, A_i, M_i, $i = 1, \cdots, 10$ be the transmitter, receiver, and relay respectively of session I_i.

The nodes B_i, $i = 1, \cdots, 8$ are located respectively at the edges of $\overline{b(S_i)} \cup \overline{b(D_i)} \cup \overline{b(R_i)}$ as shown. Thus, $I_i \in C(I)$. Specifically, $\angle B_1SD = 117°$ and $\angle B_1SB_2 = \angle B_2SB_3 = \angle B_3SR = 61°$. Thus, $|B_1B_2| = |B_2B_3| > d_{max}$. Also, we have $\angle B_4RS = 120°$ and $\angle B_4RB_5 = 61°$. Thus, $|B_4B_5| > d_{max}$. Finally, we have $\angle B_8DS = 116°$, $\angle B_8DB_7 = \angle B_7DB_6 = 61°$ and $\angle B_6DR = 62°$. Thus,

Algorithm 1 Maximal Scheduling Algorithm

loop
  {# comment: at each time slot, one single phase}
  Each undecided session I_i chooses a random number $r(I_i) \in (0, 1)$ and sends it to all its neighbors.
  If $r(I_i) < r(I_j)$ for all sessions in $C(I_i)$, session I_i is picked to be scheduled and informs all its neighbors.
  If one of I_i's neighbor is scheduled, I_i decides not to transmit.
  If all sessions reach their decisions, the scheduling is done. Otherwise, enter the next phase.
end loop

$|B_8B_1| > d_{max}$, $|B_8B_7| = |B_7B_6| > d_{max}$.

We now show that $|B_3B_4| > d_{max}$ and $|B_5B_6| > d_{max}$. Denote one of the intersection points of $\overline{b(S)}$ and $\overline{b(R)}$ as K_1 (the other is D). Then SRB_4K_1 is a parallelogram and $|B_4K_1| = d_{max}$. Thus in triangle $K_1B_3B_4$, $\angle B_4K_1B_3 > \pi/2$ and $|B_4B_3| > |B_4K_1| = d_{max}$. Similarly, $|B_5B_6| > d_{max}$.

Next, suppose $\overline{b(B_1)}$ and $\overline{b(B_8)}$ intersect with each other at point K_2 as shown. Choose A_{10}, B_{10} and M_{10} such that none of them is in $\overline{b(B_1)}$ or $\overline{b(B_8)}$. More specifically, let A_{10} and B_{10} lie on the line K_2R with $|A_{10}K_2| = \epsilon_1$ and $|A_{10}B_{10}| = \epsilon_2$. Then we have $|B_1A_{10}| > d_{max}$, $|B_8A_{10}| > d_{max}$, $|B_1B_{10}| > d_{max}$, $|B_8B_{10}| > d_{max}$, $|B_1M_{10}| > d_{max}$ and $|B_8M_{10}| > d_{max}$. In the same way, construct A_9, B_9 and M_9. Let $|B_9R| = \epsilon_3$ and $|A_9R| = \epsilon_4$. Choose ϵ_1, ϵ_2, ϵ_3 and ϵ_4 small enough such that that $|B_{10}B_9| > d_{max}$. Thus, I_i, $i = 1, \cdots, 10$ do not conflict with each other and can be scheduled at the same time, but all are in the conflict set of session I, so $K(\mathcal{N}) = 10$. ■

IV. Simulation and Results

In this section we present simulation results to evaluate the throughput achieved by maximal scheduling under different networking conditions. The simulations were done using a simulator written in C. All simulations were run for a duration of 10000 time units and each result shown is the average of 10 simulation runs with different seeds. A packet size of 256 bytes is assumed, unless otherwise noted, and 16-QAM (quadrature amplitude modulation) is used by the source and relay nodes. For our simulations, we consider a simple, distributed collision-free maximal scheduler based on the well known solution for maximal independent sets [6]. We use the randomized distributed algorithm shown in Algorithm 1 which for a graph of size n, has a time complexity $O(log^2 n)$ [6]. Note that in this algorithm multiple sessions contend for the transmission using a random access mechanism and distributed information is allowed to be updated in the same time slot.

To evaluate the throughput, for each packet we first check if the transmission was successful or not by using the bit error rate (BER) associated with the transmission. For every session, we assume that the channels between the sensors are mutually independent Rayleigh fading channels with average channel powers $\lambda_{S,D}$, $\lambda_{S,R}$ and $\lambda_{R,D}$ [3]. We use the log-distance path loss model where $P(receiver) = P(transmitter)/d^{\alpha}$, where d is the distance between the transmitter and the receiver and α is the path loss exponent. Typical values of α equal 2 for free space and $2.7 \sim 3.5$ for urban areas [9]. In our simulations, α is set to 3. Finally, we assume equal power allocation between the source and the relay. If the power budget for a transmission is PmW, then we assume that the source and the relay each consume $P/2mW$. Assuming that the signal at the destination is combined by using maximal ratio combining, we use the closed form expressions for the BER of Decode-and-Forward relaying for phase-shift keying (PSK) or QAM given in [3] to evaluate the probability that a packet is successfully delivered.

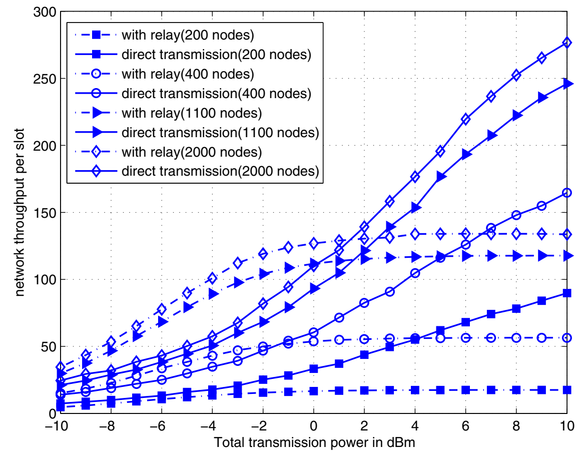

Fig. 3. Per slot throughput compared with direct transmission in networks with 200, 400, 1100, and 2000 nodes. Two packets are transmitted in each slot with direct transmission.

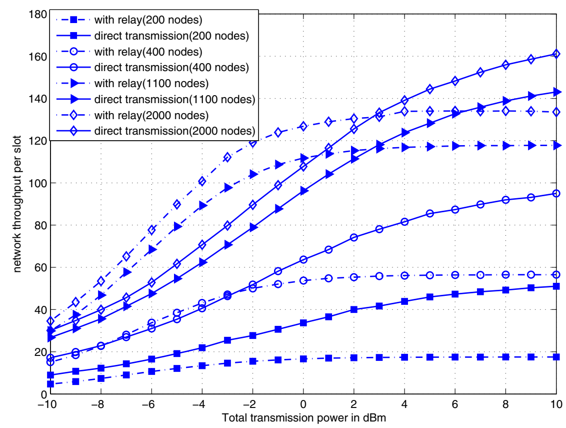

Fig. 4. Per slot throughput compared with direct transmission in networks with 200, 400, 1100, and 2000 nodes. The modulation data rate with direct transmission is half of that with relay usage.

A. Results

We simulate a network where nodes are randomly distributed in a $2000m \times 2000m$ square region. We assume a noise level $N_0 = -90dBm$ and transmission range $d_{max} = 100m$. We also assume saturated traffic conditions where in every time slot, each node always has a packet to send. A scheduled transmission is counted only if it is successfully decoded using the BER calculations.

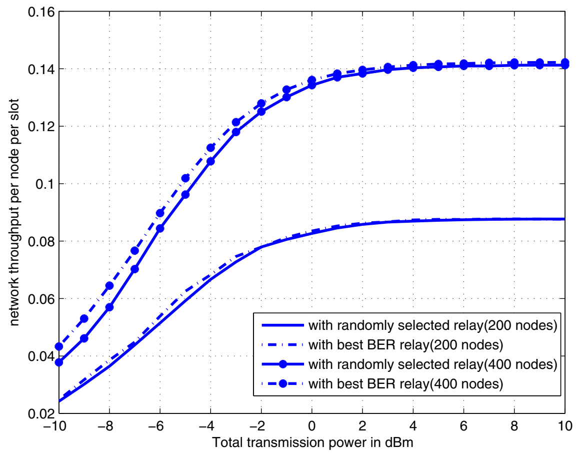

Fig. 5. Per node throughput in for two different relay selection policies in networks with 200 and 400 nodes.

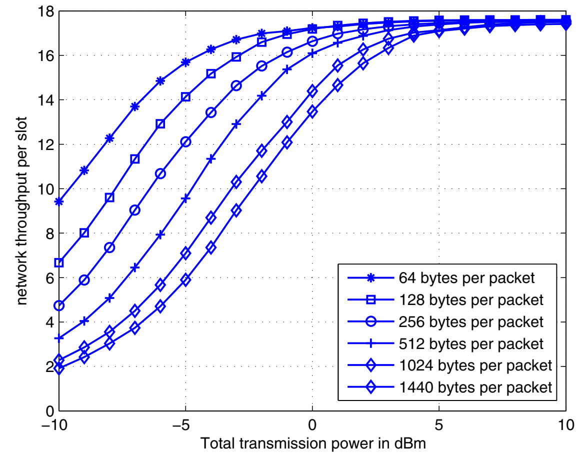

Fig. 6. Per node throughput for different packet lengths in a network with 200 nodes.

Figure 3 compares the network throughput per slot of networks using relays and direct transmissions, with 200, 400, 1100 and 2000 randomly distributed nodes. If a source has multiple relays to choose from, it picks one randomly in each slot. Each slot is assumed to be long enough so that the packet is broadcast to the relay and the relay is able to re-encode and transmit the data to the destination. For direct transmissions without relays, we assume that a source is able to transmit two packets in a slot of same length, for a given data rate. From Fig. 3, we observe that when relays are used, as the node density increases, the network throughput increases (but not necessarily the throughput per node due to higher channel contention). We also note that as the transmission power increases, the throughput saturates and the improvement slows down as the size of the network increases. For networks with 200 nodes, direct transmissions outperform relays over the power range considered. As the network size increases, relays achieve better throughput than direct transmissions when the transmission power is low. The power range where relays have better throughput increases (but saturates) as the size of the network increases. One of the reasons behind this is that in a network with sparse node density, there are much fewer three-terminal links compared to two-nodes links. However, at high node densities, the difference in the number of available links decreases and almost every node in the network has at least one neighbor that may relay its transmissions.

Figure 4 also compares the network throughput per slot for relays and direct transmissions. However, here direct transmissions use a lower data rate modulation compared to the relays. Specifically, 16-QAM is used by both the source and the relay for cooperative communication, while 4-QAM is used if the source transmits the packet on its own. Thus in a slot, a single packet is transmitted by both relays and direct transmissions. Compared to Fig. 3, we observe that the throughput of direct transmissions saturates faster as the transmission power increases and the network with relays achieves a slightly wider power range where it outperforms the network with direct transmission.

Figure 5 compares the performance of two different strategies that may be used by source nodes to pick the relay nodes. Instead of randomly picking a relay in each slot as considered in Figs. 3 and 4, we consider another policy where the source always chooses the relay that has the smallest calculated BER. It can be seen that choosing the relay with the lowest BER increases the throughput. However, this also leads to a faster battery consumption in the selected relays, as compared to random relay selection.

Figure 6 shows the impact of the packet size on the throughput in a 200 node relay based network. At relatively smaller transmission powers, the throughput per node decreases as the packet size increases due to the higher packet error rate. When the transmission power is high enough, the packet error rate is negligible for all packet sizes and the throughput is limited by the requirement to pick conflict-free sessions for transmission.

Finally we note that the scheduling algorithm used in the simulations is a general maximal scheduling algorithm. The throughput and fairness can possibly be improved by more sophisticated schedulers that take into account additional information such as the network traffic etc..

V. Conclusions

While a number of cooperative communication schemes have been proposed for wireless networks, the performance of upper layer protocols under such communication paradigms is largely unknown. This paper considers the problem of the achievable maximum throughput region of maximal schedulers in WSNs with cooperative relays. We show that distributed maximal scheduling algorithms can achieve a guaranteed fraction of the maximum throughput region in arbitrary wireless networks. It was also shown that the guarantees are tight in the sense that they cannot be improved any further with maximal scheduling.

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