A Population Based Approach to Model Network Lifetime in Wireless Sensor Networks*

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Extended Abstract

The physical constraints of battery-powered sensors impose limitations on their processing capacity and longetivity. As battery power in the nodes decays, certain parts of the network may become disconnected or the coverage may shrink, thereby reducing the reliability and the potency of the sensor network. Since sensor networks operate unattended and without maintainence, it is imperative that network failures are detected early enough so that corrective measures can be taken.

Existing research has primarily concentrated on developing algorithms, be it distributed or centralized, to optimize network longetivity metrics. For instance, [4, 5] propose MAC layer optimizations to prolong longetivity, while [7, 6] look at the problem from a Layer 3 perspective. Works along the lines of actually building network models for energy consumption are addressed in [2], [3], but these models fail to capture the interplay between a node's spatial location and it's energy consumption.

In our current work, we develop an unifying framework to characterize the lifetime of such energy constrained networks, and obtain insights into their working. In particular, we employ a framework similar to population models for biological systems, to model the network lifetime. We consider both *spatial* scenarios, where a node's power consumption is governed by it's position in space as well as *non spatial* scenarios, where the node's location and power consumption model are independent entities.

MODEL

To model the lifetime of such energy constrained networks, we propose a generalization of Leslie's population matrix [1], which is used to study populations structured by age. The "age" of a node in our model corresponds to the amount of the battery power consumed, with one unit of power expended per packet transmitted, and the "age" of any node lies in one of the $m+1$ possible intervals; $0, 1, \cdots, m$. In other words, we assume that each sensor has enough energy to transmit m packets and the nodes in the network are structured based on this value. Our model makes the following assumptions

1. Power mainly expended to transmit packets
2. Network Lifetime can be discretised into "cycles", wherein each cycle spans a communication round among the nodes
3. Probability of receiving i packets, $i = 0, 1, \cdots, m$, to transmit is same in all cycles

Let $n_i(t)$ denote the number of nodes in age group i at time t, i.e. $n_i(t)$ denotes the number of nodes which have used up i units of the total quota of m. Let p_i, $0 \leq i \leq m$ denote the probability that a node consumes i units of energy in a time unit (we derive expressions for p_i below). Then, the number of nodes at each energy level at an arbitrary time step is given by

$$\begin{aligned} n_0(t+1) &= p_0 n_0(t) \\ n_1(t+1) &= p_0 n_1(t) + p_1 n_0(t) \\ &\vdots \\ n_{m-1}(t+1) &= p_0 n_{m-1}(t) + p_1 n_{m-2}(t) + \cdots + p_{m-1} n_0(t) \\ n_m(t+1) &= n_m(t) + \sum_{i=1}^{m} p_i n_{m-1}(t) + \sum_{i=2}^{m} p_i n_{m-2}(t) \\ &+ \cdots + \sum_{i=m-1}^{m} p_i n_1(t) + p_m n_0(t) \end{aligned}$$

The rationale behind the above formulation can be justified as follows: a node with full power at time t (class n_0) will retain it's entire battery reserve only if it receives no packets to route for the duration of the cycle. The probability of this event is p_0, and by mean-field analysis the expected number of nodes receiving zero packets is $p_0 n_0(t)$, which in turn is the count of nodes with full battery power at time $t+1$. Similarly the number of nodes in class n_1 at time $t+1$ is the sum of nodes in class n_1 that route zero packets, and the nodes in class n_0 that spend one unit of energy at time t. For enumerating the nodes of class m, note that a sensor in class n_i $i = 0, \cdots, m-1$ will power down if it receives more than $m-i$ routing packets and the probability of this event is given by $\sum_{k=(m-i)}^{m} p_i$ $i = 0, \cdots, m$. Also, in the scenario where expended energy is not replenished, a sensor that had no battery power during the cycle starting at t will continue to remain powered down at $t+1$ and hence the equation for $n_m(t+1)$. The above formulation can also be expressed in a vector-matrix form. To this end, we define

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the $(m+1)$-dimensional column vector $\vec{\rho_t}$ and matrix A as follows

$$\vec{\rho_t} = \begin{pmatrix} n_0(t) \\ n_1(t) \\ \vdots \\ n_m(t) \end{pmatrix}$$

$$A = \begin{pmatrix} p_0 & 0 & 0 & 0 & \dots & 0 & 0 \\ p_1 & p_0 & 0 & 0 & \dots & 0 & 0 \\ p_2 & p_1 & p_0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ p_m & \sum_{m-1}^{m} p_i & \sum_{m-2}^{m} p_i & \dots & \dots & \sum_{1}^{m} p_i & 1 \end{pmatrix}$$

The model then can be expressed as the vector difference equation $\vec{\rho}_{t+1} = A\vec{\rho_t}$. This formulation is equivalent to a discrete time Markov chain as the number of nodes at a particular energy level is dependent only on the number at the previous cycle. The solution of this difference equation is easily obtained, using a recursive definition, as: $\vec{\rho}_{t+1} = A^{t+1}\vec{\rho_0}$; where $\vec{\rho_0}$ is the initial distribution of nodes among the various energy levels. In our simulations we assume that at time $t = 0$, all the nodes are fully powered, i.e. $n_i = 0 \ \forall i > 0$ and $n_0 = N$. What now remains is determining the probabilities for the energy consumption during a cycle.

The potency of the framework developed here lies in it's inherent ability to be abstracted to networks with varied node deployment as well as routing schemes. In the current work we highlight and investigate the interplay between a node's geographical co-ordinates in space and it's power consumption under the aegis of shortest path routing by considering two scenarios: **(1)** a spatial model where the sensor nodes are located at the vertices of a finite grid and **(2)** a non-spatial model where nodes are randomly and homogeneously distributed such that traffic conditions at each node are statistically identical.

0.1 Spatial Network

To consider a node's spatial location on its energy consumption rates and node lifetime, we consider a deployment scenario where the sensor nodes are placed at the vertices of a finite grid, as shown in Fig. (1). The co-ordinates of node i, $i = 1, \cdots, N$ in the grid (x_i, y_i) is determined as follows: $x_i = (i-1)/\sqrt{N}$ and $y_i = (i-1)\%\sqrt{N}$.

We now incorporate the contribution of a node's geographic location into the derivation of the power consumption probabilities under the assumption that the network employs *shortest path routing.* The following probability is assumed to be known: p_s, the probability that in a given cycle a sensor node (say i) has a new packet to send to another node (say j) in the grid. The probability that a node i has a packet to transmit during a cycle is the probability of the union of two mutually exclusive events: the event of a node initiating a communication session and the event where it receives a routing request. The probability of the latter, p_{ri}, can be obtained by using the conditional probability of it receiving a packet, given two nodes in the network communicate. Mathematically, for node i

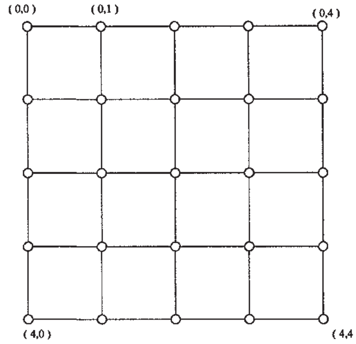


Figure 1: An example of a grid topology for sensor networks.

$$p_{ri} = 2\left[\sum_{\substack{j=1 \\ j\neq i}}^{N-1} \sum_{\substack{k=1 \\ k\neq i, k>j}}^{N} Pr\{\text{session between } j \text{ and } k \text{ is through } i\} \times Pr\{j \text{ and } k \text{ communicate}\}\right] \quad (1)$$

Note that, for each pair (j,k), the expression for (k,j) communicating through node i has the same numerical value since the grid is symmetric and hence the summation in Eqn (1) is multiplied by a factor of two. Now, the probability that two particular nodes, say j and k communicate is: $Pr\{j \text{ and } k \text{ communicate}\} = \frac{1-(1-p_s)^2}{\binom{N-1}{2}}$. In other words, the pair (j,k) can be selected from $(N-1)$ nodes (since node i is not a candidate) in $\binom{N-1}{2}$ ways and for nodes j and k to communicate, it is sufficient if either initiates a session. The expression for $Pr\{\text{session between } j \text{ and } k \text{ is through } i\}$ is derived as follows. Let $(x_i, y_i), (x_j, y_j), (x_k, y_k)$ denote the co-ordinates of nodes i, j and k respectively. Defining, $\Delta x_{i,j} \triangleq |x_i - x_j|$ and $\Delta y_{i,j} \triangleq |y_i - y_j|$, we obtain: $r_{i,j} = \Delta x_{i,j} + \Delta y_{i,j}$. Similarly values for $r_{j,k}$ and $r_{k,i}$ can be obtained using the previous definition. Now,

$$\begin{matrix} Pr\{\text{session between } j \\ \text{and } k \text{ is through } i\} \end{matrix} = \begin{cases} \frac{L_{j,k,i}}{L_{j,k}} & if \ r_{i,k} + r_{i,j} = r_{j,k} \\ 0 & otherwise \end{cases} \quad (2)$$

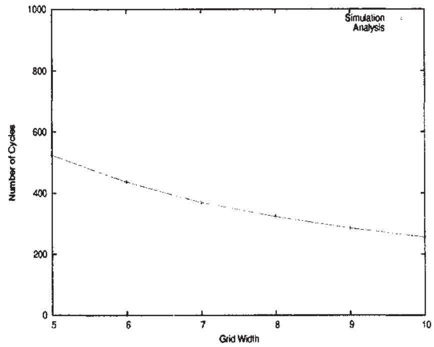
where

$$L_{j,k,i} = \binom{r_{i,j}}{\Delta x_{i,j}}\binom{r_{i,k}}{\Delta x_{i,k}} \quad \text{and} \quad L_{j,k} = \binom{r_{j,k}}{\Delta x_{j,k}}$$

Given the probability of a sensor initiating a session, p_s, each cycle sees an average of Np_s sessions. To obtain the state probabilities, p_i, $i = 0, \cdots, m$, we again condition on the node's geographic location.

$$Pr\{\text{a node transmits } i \text{ packets}\} = \sum_{k=1}^{N} Pr\{\text{a node transmits } i \text{ packets}|\text{node id} = \text{k}\} \times Pr\{\text{node id} = \text{k}\} \quad (3)$$

Note that, a node k transmits $i, i > 0$ packets during a routing cycle if it either receives i routing packets and does not



(a) Grid Model: Analysis vs. Simulation

Figure 2: Simulation results

initiate a session or starts a communication session and receives $i-1$ routing requests. In our model, we analyse the sytem with the number of communication sessions per cycle at Np_s, though theoretically the upper bound is N. The simulations validate our intuition that the expected number is a good approximation of the underlying communication process. Denoting $Pr\{\text{node id} = \text{k}\}$ by $\mathcal{P}_k$, the state probabilities can be expressed as follows:

$$p_i = \begin{cases} \left\{(1-p_s)(1-p_{rk})^{Np_s}\right\}\mathcal{P}_k & i = 0 \\ \left\{(1-p_s)\binom{Np_s}{i}p_{rk}^{i}(1-p_{rk})^{Np_s-i} \right. & \\ \left. +p_s\binom{Np_s-1}{i-1}p_{rk}^{(i-1)}(1-p_{rk})^{Np_s-i}\right\}\mathcal{P}_k & 0 < i \leq Np_s \\ 0 & \textit{otherwise} \end{cases}$$

Also, the evaluation of $Pr\{\text{node id} = \text{k}\}$ has two possibilities: one where the choice of a node is equally likely among the N nodes present and the second, where the selection of the node is governed by it's location. Assuming shortest path routing, we approximate the likelihood of the node being chosen by the number of shortest paths it lies on. That is

$$Pr\{\text{node id} = \text{k}\} = \frac{\sum_{\substack{i=1 \\ i\neq k}}^{N-1}\sum_{\substack{j=i+1 \\ j\neq k}}^{N} 1\{r_{i,k}+r_{i,j}=r_{j,k}\}}{\sum_{k=1}^{N}\sum_{\substack{i=1 \\ i\neq k}}^{N-1}\sum_{\substack{j=i+1 \\ j\neq k}}^{N} 1\{r_{i,k}+r_{i,j}=r_{j,k}\}} \quad (4)$$

where $1\{r_{i,k}+r_{i,j}=r_{j,k}\} = 1$ if $r_{i,k}+r_{i,j}=r_{j,k}$, 0 otherwise.

In Fig. (2(a)) we compare the results for the number of cycles till the first node goes down from our analytic model with simulation results for grid sizes ranging from five to ten with $p_s = 0.34$ and $m = 1000$. We see that the analytic results match closely with the simulations.

0.2 Non-spatial Homogeneous Networks

In the case of scenarios where the sensor network is homogeneous and is either assumed to span an extremely (ideally infinitely) large space or to be very densely deployed, the traffic conditions at each node can be approximated to be statistically identical. To qualitatively evaluate the node lifetimes in these scenarios, we consider a model where the number of packets transmitted by each node during a time cycle follows a Poisson distribution with mean λ, irrespective of its geographical location. Let N denote the initial number of nodes deployed and m denote the number of packets each node can transmit before dying out. The power consumption probabilities p_i are given by: $p_i = \frac{e^{-\lambda}\lambda^i}{i!}$. Since the power consumption probabilities wholly characterize the network evolution, the system is completely determined. Results and further derivations for this section have been omitted in the extended abstract due to space limitations.

Summary

In this paper we have motivated the need and importance of analyzing the network lifetime by quantifying the connectivity and coverage characteristics as a function of time and energy consumption. The impact of packet arrival rate at the sensor nodes on the energy consumption was studied and an analytical model for the network lifetime incorporating a sensor node's geographic location was presented.

1. REFERENCES

[1] P. Leslie, "On the use of matrices in certain population mathematics," *Biometrika,* vol. 33, no. 3. pp, 183-212, November 1945.

[2] Zhao, Y.J., Govindan, R., and Estrin, D., "Residual Energy Scans for Monitoring Wireless Sensor Networks," *IEEE Wireless Communications and Networking Conference (WCNC),* Orlando, Florida, 2002.

[3] Raquel, M., Antonio, A., Ferreira, L., Badri N., "A More Realistic Energy Dissipation Model for Sensor Nodes," *SBRC,* 2004

[4] Ye, W., Heidemann, J., and Estrin, D., "An Energy-Efficient MAC Protocol for Wireless Sensor Networks," *Proceedings of the IEEE Infocom,'* 2002.

[5] Heinzelman, R.W., Chandrakasan, A., and Balakrishnan, H., "Energy Efficient Communication Protocol for Wireless Microsensor Networks (LEACH)," *Proceedings of Hawaaian International Conference on Systems Science,* January 2000.

[6] Misra, A., and Banerjee, S., "MRPC: Maximising Network Lifetime for Reliable Routing in Wireless Networks," *IEEE Wireless Communications and Networking Conference (WCNC),* Orlando, Florida, 2002.

[7] Shah, R.C, and Rabaey, J.M, "Energy Aware Routing for Low Energy Ad-hoc Sensor Networks," *IEEE Wireless Communications and Networking Conference (WCNC),* Orlando, Florida, 2002.