Outage Estimation for Solar Powered Cellular Base Stations

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***Abstract*—Solar powered cellular base stations are emerging as a key solution in green cellular networks. A major challenge in the design of such a base station (BS) is finding the optimal cost configuration of the photo-voltaic (PV) panel size and number of batteries which meets a tolerable outage probability with the least cost. One of the fundamental steps in this process is to calculate the outage probability associated with a particular PV panel size and battery size configuration. To address this issue, this paper proposes an analytic model to evaluate the outage probability of a solar powered BS. The proposed model factors in the daily and hourly variations in the harvested solar energy and the traffic dependent BS load, and develops a discrete-time Markov process to model the battery level and thus the outage probability of the BS. Simulation results with empirical solar irradiance data for three different locations are used to validate the proposed model and demonstrate its accuracy.**

I. Introduction

With the increasing number of cellular subscribers and traffic handled by cellular base stations, the number of base stations has been increasing at a rapid rate. This increase in the number of BSs has contributed not only to the increased energy consumption of the telecom industry (currently 3% of worldwide energy consumption), but also to the industry's contribution to the global carbon footprint (2% of the worldwide emissions) [1]. This has stimulated the interest of researchers and telecom operators across the globe to explore ways to reduce the power consumption in cellular networks, and develop greener cellular networks.

Base stations consume a large fraction of the energy in a cellular network, accounting for around 60-80% of the total [2]. Thus one of the promising ways to make the operation of cellular networks greener is to use renewable energy (e.g. solar energy) to power the base stations [3]. Such an approach not only cuts the cost of operating the base station, but also has the advantage of being environment-friendly, since there are no carbon emissions in the use of such energy.

In addition to the Base Transceiver system (BTS), a solar powered base station also consists of Photovoltaic (PV) panels and batteries, which are meant to harvest and store the solar energy. A part of the harvested solar energy is used to meet the instantaneous power requirements of the BS, and the excess energy is stored in the batteries. One of the major challenges in the design of such a base station is to determine the cost optimal dimensions of the PV panel and number of batteries. Choosing a large PV panel size/number of batteries makes the system more reliable, but has the disadvantage of very high capital cost. On the other hand, if the PV panel size and number of batteries is too small, it can lead to frequent outages when the BS runs out of energy, thereby reducing its reliability. Thus PV panel size and number of batteries have to be carefully chosen in order to ensure that the performance is above a certain tolerable outage level. Further, among the feasible configurations of PV panel size and number of batteries meeting that meet a given outage criteria, the configuration with the least cost is chosen as the optimal configuration. Note that in this paper an *outage* event refers to the situation when the BS does not have adequate energy to operate.

The problem of evaluating the optimal PV panel size and number of batteries has been addressed in [3], [4]. A key step in this process is to evaluate the outage probability associated with a particular configuration of PV panel size and number of batteries. The traditional way to do so is to conduct simulations using long-term solar irradiance data (either real or synthetically generated) [4]. This process is based on simulating the system for many years, so as to obtain the outage probability associated with a given PV panel and battery size configuration. Such an approach does not provide any insights into the performance of the system, or the relationships between the system parameters and the system performance. To the best of our knowledge no analytic model exists to evaluate the outage probability in solar powered base stations. To address this issue, this paper proposes an analytic framework to model the battery levels at the BS and use it to determine the outage probability. In addition, the proposed model can be used to quantify the relationship between the system parameters such the PV panel size, battery size, harvested solar energy profile as well as the load profile on the outage related performance. The proposed model is based on developing a discrete time Markov chain for capturing the battery level at the BS. The model accommodates the hourly variations in harvested solar energy as well as the network traffic (i.e. the load). The accuracy of the proposed model has been verified by comparing it with results obtained using real traces of solar irradiance data for three geographically diverse locations.

The rest of the paper is organized as follows. Section II presents the system model and the background material. Section III present the proposed model for evaluating the outage probability at the BS while Section IV presents the simulation results. Finally, Section V concludes the paper.

II. Background and System Model

This section presents the system model assumed in this paper and an overview of the background material.

A. Base Station Power Consumption

This paper considers a Long Term Evolution (LTE) base station. The power consumed by a base station consists of two parts [5]: the fixed part (e.g. due to air conditioners, losses in cable feeders etc.), and the variable part, which depends of the instantaneous traffic at a given point in time. In this paper, we consider a macro base station. The power consumption of such a base station can be modeled as [5]

$$P_{BS} = N_{TRX}(P_0 + \Delta_\rho P_{max}\rho), \quad 0 \leq \rho \leq 1 \qquad (1)$$

where N_{TRX} is the number of transceivers, P_0 is the power consumption at no load (zero traffic), Δ_ρ is the slope of the load dependent power consumption, P_{max} is the output of the power amplifier at the maximum traffic, and ρ is the normalized traffic at the given time. The typical values of P_0, P_{max} and Δ_ρ for a macro base station are 118.7 W, 40 W and 2.66, respectively [5].

For modeling the traffic, we use call based models as proposed in [6]. The call arrivals are modeled as a Poisson process and the call duration has been taken to be exponentially distributed. This model is used to generate calls on minutely basis, with rate depending on the hour of the day. Each call is assigned a call duration which is exponentially distributed and has a mean value of two minutes [7]. The normalized traffic denoted by ρ is then obtained by normalizing the number of users at a given instant of time by the maximum number of calls the base station can support at any point of time. This can thus be used in Eqn. (1) to calculate the instantaneous power consumption of the BS.

B. Solar Energy Resource and Batteries

This paper uses statistical weather data provided by National Renewable Energy Laboratory (NREL), USA [8]. The data consists of hourly traces of solar irradiance data for a given location. In particular, we use 10 years of solar data for three locations: San Diego (USA), Las Vegas (USA) and Jaipur (India). This data is further fed to the System Advisor Model (SAM) [9] developed by NREL, to yield the hourly energy generated by a PV panel of a given rating. The paper assumes a PV panel with a DC-AC loss factor of 0.77 and tilt of the PC panel as the latitude of the location, which are the default values [10].

This paper assumes that the base station uses lead acid batteries to store the excess energy harvested by the PV panels. Lead acid batteries are a popular choice in storage applications because they are a time tested option, and are also much cheaper than other storage options.

III. A Model for Evaluating BS Outage Probability

This section presents the proposed analytic framework for evaluating the outage probability of a solar powered cellular base station.

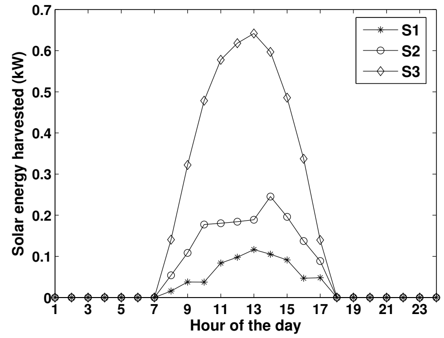


Fig. 1. Average hourly values of the solar energy harvested for the 3 day types for the Month of February for San Diego (PV panel rating = 1 kW)

A. System Resources

Let n_{PV} denote the number of PV panels used by the BS. We denote the DC rating of each of the PV panels by E_{panel}. Thus the overall DC rating, PV_w, is given by

$$PV_w = n_{PV}E_{panel}. \qquad (2)$$

Similarly, we denote the number of batteries used in the BS by n_b. We denote the storage capacity of each battery as E_{bat}. The overall battery storage capacity, K_{cap}, is then given by

$$K_{cap} = n_b E_{bat}. \qquad (3)$$

Next we consider the problem of obtaining the outage probability associated with a given choice of PV_w and K_{cap}. An outage is defined as the event that the charge level of the batteries supporting the BS falls below a predefined depth of discharge. In such a scenario, the batteries are disconnected from the BS and there is an outage event. To facilitate the calculation of the outage probability, we characterize the solar energy, the load, and the battery state as a discrete time Markov process (with time granularity as one day). These models are now described in detail.

B. Model for Harvested Solar Energy

It has been shown in previous work that the solar energy profile of any location may be modeled as a Markov process [11]. In the proposed model, we classify any given day being in one of the three categories which we denote by $S1$, $S2$ and $S3$. As a benchmark, we take the solar energy harvested by a PV panel with DC rating 1 kW to classify a given day among one of the day types. The bad weather days strongly influence the PV panel and battery size configuration requirement, since they are the ones which lead to outages. Thus in our model we use two of the solar types, $S1$ and $S2$, for the bad weather days. Among them, days in category $S1$ correspond to days with very low solar energy harvested. The days with daily harvested energy below a threshold α_1 are classified into the state $S1$. Next, days with harvested solar energy greater than α_1 but below another threshold α_2 are classified as type $S2$. Days in category $S2$ correspond to days which have a little higher solar irradiance than $S1$, but still not enough to power the base station (with a considerable PV panel size) for an entire day. The remaining days are classified as $S3$.

To model the daily transitions in the day types, for any given location, we use ten year's statistical data of solar irradiation to calculate the transition probability from a given day type to another. The transition probability matrix for the Markov process corresponding to the daily variations in the solar irradiance is given by

$$T_S = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \tag{4}$$

where p_{11} is the probability of a day of type $S1$ remaining in the same state on the next day, whereas p_{12} (resp. p_{13}) is the probability of transition from a day type $S1$ to day type $S2$ (resp. $S3$) on the next day. The other variables are defined similarly.

In addition, the average profile of solar energy for each of these day types can be evaluated using the historical solar irradiance data. The solar energy profile for each day type consists of the average solar energy harvested in each hour of the day by a PV panel with rating 1 kW. The solar energy profile is then given given by a vector S as

$$S = (s_1, s_2, ..., s_{24}).$$

where s_1 is the average solar energy harvested in the first hour and so on. As there are three solar day types, we have three possible values of the vector S which can be expressed as

$$S : S \in \{S_{S1}, S_{S2}, S_{S3}\}, \tag{5}$$

where S_{S1}, S_{S2} and S_{S3} are the average harvested energy profiles for day type $S1$,$S2$ and $S3$ respectively. Figure 1 shows the harvested energy profile for these three day types for San Diego for the month of March (using data from 2000 to 2009). The procedure of estimating the transition probabilities and the average solar profile has been described in the section III-F. For a given PV panel size, the profile of harvested solar energy can then be represented as

$$E(t) = PV_w S. \tag{6}$$

C. Model for BS Load

The load of a base station is usually dependent on the day of the week and is typically lower on the weekends as compared to the weekdays [12]. Thus we consider two load types: low ($L1$) and high ($L2$) depending on the day of the week. We approximate the base station load as a Markov process where the daily changes in the load type are assumed to occur as a two-state Markov process. The transition probability matrix of this Markov process is given by

$$T_L = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \tag{7}$$

where q_{11} (resp. q_{22}) is the probability of transition from a low load (high load) day to a low load (high load) day, and $q_{12} = 1 - q_{11}$ (resp. $q_{21} = 1 - q_{22}$) is the probability of

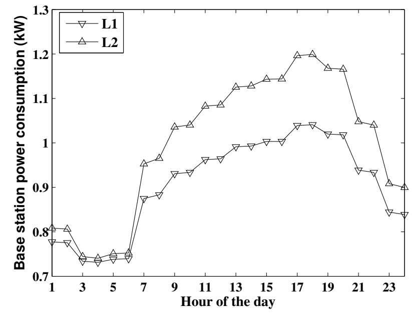


Fig. 2. Average hourly values of BS power consumption.

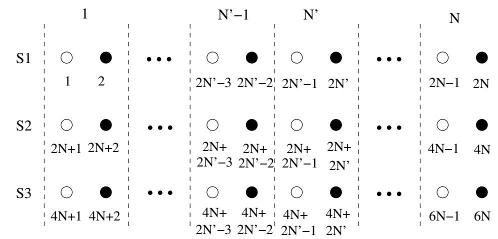


Fig. 3. States of the system

transition from low load day to a high load day. Note that the transition probabilities may be chosen such that on an average, five weekdays are followed by two weekend days.

As with the harvested solar energy, we define the load profile for high and low load days as the average load during each hour of the day. The load profile is given by

$$L = (l_1, l_2, ..., l_{24}).$$

where l_1 is the average BS load for the first hour and so on. As there are two load types, we have two possible values of the vector L which can be expressed as

$$L : L \in \{L_{L1}, L_{L2}\}, \tag{8}$$

where L_{L1} and L_{L2} are the average load profile vectors for a low load day and a high load day, respectively. The methodology for obtaining these vectors is given in Section III-F. Figure 2 shows the average load profile for a low load and a high load day.

D. Model for Battery Level

To develop a tractable model for the battery level, we discretize the possible battery levels into blocks of 1 kW. First, we round the battery bank capacity, K_{cap}, to the closest integer value above it. This gives us the number of levels in our battery model and we denote this value by N with

$$N = \lceil K_{cap} \rceil. \tag{9}$$

At any given point of time, the battery can be in either of the N possible battery levels. Though the accuracy of the model may be increased by making the granularity even finer, our experiments show that by doing so, there is only marginal improvement and thus the inclusion of the additional states and the associated complexity is not justified.

E. BS Outage Probability

To model the outage probability at the BS, we first define the *state* of the system. The state of the BS on a given day is characterized by three factors: the solar day type, the load type and the battery level. For simplicity of notation, we denote the state of the system by i where i is given by

$$i = 2(k-1) + y + 2N(x-1) \\ k \in \{1,2,...,N\}, x \in \{1,2,3\}, y \in \{1,2\} \quad (10)$$

where k is the battery level, x is the solar day type ($x = 1$ for $S1$, $x = 2$ for $S2$ and $x = 3$ for $S3$) and y is the load type ($y = 1$ for low load and $y = 2$ for high load).

As we have three possible choices for the solar day type, two choices for load type and N choices for battery level, there are $6N$ possible states. These states are shown in Figure 3. Based on our notation, the first row corresponds to day type $S1$ ($x = 1$), and the second and third rows correspond to day type $S2$ and $S3$, respectively. Further the odd positions in each row (shown by empty circles) denote low load and the even positions (shown by dark circles) denote high load.

We model the transitions in the system state at the beginning of each day (or every 24 hour period). Note that the state on a given day only depends on the battery level of the previous day, the energy harvested in the previous day (i.e. the day type) and the energy consumed in the previous day (i.e. the load type). Thus the system state constitutes a discrete time Markov chain and its transition probability matrix may be written as

$$T_B = \begin{bmatrix} b_{(1,1)} & \cdots & b_{(1,6N)} \\ \vdots & \ddots & \vdots \\ b_{(6N,1)} & \cdots & b_{(6N,6N)} \end{bmatrix} \quad (11)$$

where $b_{(i,j)}$ is the probability of transition from state i to j.

To avoid battery degradation which happens due to the battery level going to very low values, we disconnect the batteries from the load when the charge level goes below a specified depth of discharge. For a depth of discharge threshold ν, the batteries are disconnected when the battery charge level goes below νK_{cap}. Since the battery level never goes below this value at any point in time, the system states with battery level below this level are *redundant* and never used. This boundary battery level is denoted by N' and is given by

$$N' = \lceil \nu K_{cap} \rceil. \quad (12)$$

Thus the feasible states which the system can take at any point of time can be expressed as

$$i : i \in \{(2N'-1, 2N) \cup (2N'-1+2N, 4N) \\ \cup (2N'-1+4N, 6N)\} \quad (13)$$

Figure 3 depicts this battery level N'.

Given that we begin a day with a certain battery level, solar day type and load type, the battery level at the next day depends on the solar energy profile and load profile over the day. In addition, the outage experienced by the BS depends on the battery level during each hour of the day. To determine the next battery level as well as the outage probability, we define a function $F(i)$ as shown in Algorithm 1. The function accepts the state of the system i at the beginning of the day as an argument. Based on the value of i, it first extracts the harvested solar energy profile, load profile, and battery level k for that particular state. The variable k' meant to track the battery level is initialized to k. Next, the battery level is updated for each hour during a 24 hour period using the variables representing the solar energy and load profiles. During this calculation, the battery level is not allowed to go above K_{cap} or to go below νK_{cap}. The function returns k' rounded off to the closest integer, as we allow only discrete battery levels for a state. Also, in case of an outage event, the outage status for the particular state is recorded using the variable O which is stored to be used later for outage calculation. Thus the next battery level for any state i can be written as

$$k' = F(i) \quad (14)$$

Since the next battery level just depends on the state i, the next state can only be one amongst one of six states (based on the solar day type and load type of the next day). Let the system be in state i and let x and y denote the solar day type and load type for that particular state. The transition

Algorithm 1 Calculation of Battery Level

```
function F(i)
    if 1 ≤ i ≤ 2N then
        S = S1;
        k = ⌈i/2⌉;
    else if 2N + 1 ≤ i ≤ 4N then
        S = S2;
        k = ⌈(i − 2N)/2⌉;
    else
        S = S3;
        k = ⌈(i − 4N)/2⌉;
    end if
    if i%2 == 1 then
        L = L1;
    else
        L = L2;
    end if
    initialize: k' = k , O(i) = 0
    for t = 1 : 24 do
        k' = k' + E(t) − L(t);
        if k' > K_cap then
            k' = K_cap;
        else if k' < νK_cap then
            k' = νK_cap;
            O(i) = 1;
        end if
    end for
    return: round(k')
end function
```

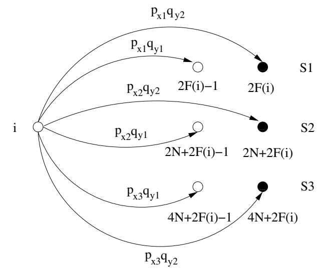


Fig. 4. Transition from a given state i.

probability from state i to and state j can be given as

$$P(i,j) = \begin{cases} p_{x1}q_{y1} & j = 2F(i) - 1 \\ p_{x1}q_{y2} & j = 2F(i) \\ p_{x2}q_{y1} & j = 2F(i) + 2N - 1 \\ p_{x2}q_{y2} & j = 2F(i) + 2N \\ p_{x3}q_{y1} & j = 2F(i) + 4N - 1 \\ p_{x3}q_{y2} & j = 2F(i) + 4N \\ 0 & \text{otherwise} \end{cases} . \quad (15)$$

Figure 4 shows the different possible state transitions from any state i. For each feasible state i, Eqn. (15) is then used to evaluate the transition probabilities for the transition matrix given in Eqn. (11). Then, the limiting steady state probabilities of being in a given state, π, can be obtained by solving

$$\pi = \pi T_B \quad (16)$$

for π.

Note that the function $F(i)$ computes and stores the outage status (1 for outage and 0 for no outage) for each of the states in the vector O. The dot product of the vectors O and π gives the outage probability, Ω. This can be expressed as

$$\Omega = O \cdot \pi. \quad (17)$$

For each value of PV_w and n_b, the above analysis can be done so as to evaluate the outage probability associated with the configuration. Assume that the tolerable outage constraint given by the telecom operator is given by β. Thus the feasible dimensioning solutions are all configurations of PV_w and n_b satisfying the constraint

$$\Omega \leq \beta. \quad (18)$$

F. Parameter Estimation

To obtain the parameters for the solar model described in III-B, we use ten years' historical data of solar irradiance levels from NREL. This data is used as an input to the SAM tool to generate the solar energy harvested by a PV panel of rating $1kW$. We parse the data on a monthly basis and for a given month, we use ten months data (one from each year) to characterize the solar energy harvested in that month. First, from the ten months data for the given month, we calculate the daily solar radiation for each of the days. The days with daily solar energy harvested below α_1 are considered as type $S1$. We use the average value of solar energy harvested in each of the hours for the days of type $S1$ to obtain the harvested energy profile in state $S1$ for this month. Days with daily solar energy between α_1 and α_2 are characterized as $S2$ and the average solar energy harvested (on hourly basis) for these days is used to compute the harvested energy profile for $S2$. Days with solar energy more than α_2 are classified as $S3$ and their harvested energy profile are obtained similarly. Also, the data is used to calculate the transition probability from a given solar day type to another in order to populate the state transition matrix in Eqn. (4).

To generate the load profiles, we first generate traces of call arrivals and their holding times as per the model described in Section II-A. These call volumes are then normalized and used in Eqn. (1) to determine the load at the BS. These traces for the load are then used to calculate the average hourly load profiles for the high and low load days. Also the transition probability associated with transitions among low load and high load days are computed so as to populate the load state transition matrix in Eqn. (7).

IV. Results

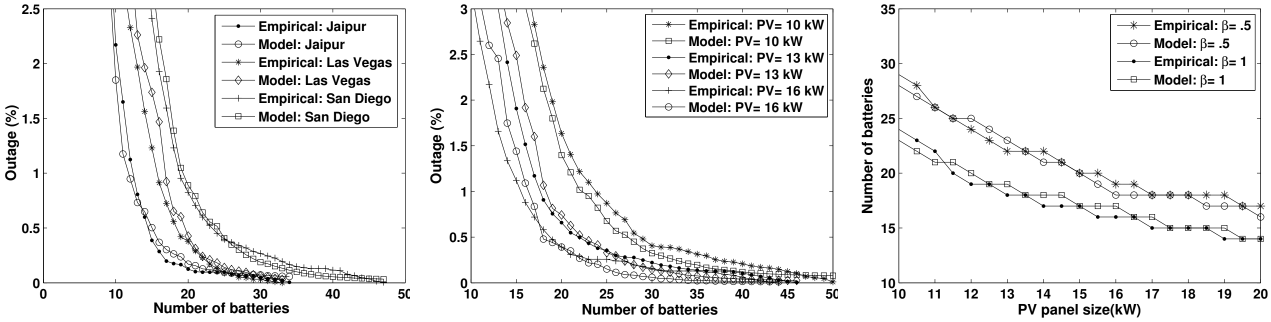
In this section we verify the outage probability estimation and solar powered BS dimensioning framework developed in this paper.

A. Simulation Setup

To validate the proposed model, we consider a LTE base station with 10 MHz Bandwidth and 2×2 Multi Input Multi Output configuration. Three sectors have been assumed for the BS, each with two transceivers, thus giving us $N_{TRX} = 6$. The traffic is modeled using the methodology described in Section II and the hourly normalized traffic rates are used to determine the hourly base station power consumption using Eqn. (1). We assume that 12 V, 205 Ah flooded lead acid batteries are used by the base station. For deployment, we consider three locations: San Diego (USA), Las Vegas (USA) and Jaipur (India). For each of these locations we use ten years solar data from NREL database to obtain the various parameters required to characterize our model for the harvested solar energy, as described in Section III-B. We used $\alpha_1 = 1$ kW and $\alpha_2 = 2$ kW for all the three locations. To validate the proposed framework, we compare our results against those obtained from simulations based on the empirical solar energy data.

B. Outage Statistics

We first evaluate how the outage probability varies with the number of batteries. Figure 5(a) shows the outage for different battery sizes for three different locations for a PV wattage of 12 kW. We observe that the outage probabilities predicted by our model match closely with that obtained from simulations using empirical data. It can also be seen that below a particular battery size, the outage increases very rapidly. This is because in this case the batteries are too small to hold adequate charge, even when there is sufficient solar energy. Thus any occurrence of bad solar days leads to outages. Also for very low outage probabilities (less than 0.25), we observe that the number of



(a) (b) (c)

Fig. 5. (a) Outage vs number of batteries required for a PV wattage of 12 kW (b) Outage vs batteries required for various PV panel sizes: San Diego (c) Number of batteries vs PV wattage required for different outage probabilities (β): San Diego.

batteries required becomes very large. This is because for any location, one may occasionally have consecutive bad weather days. To sustain the BS's operation during these periods, we need very large battery storage.

C. Outage Variation with PV Panel Size

Next we analyze the performance of the model for different PV dimensions. We consider three different PV panel sizes: 10 kW, 13 kW and 16 kW and consider how the outage probability varies with the number of batteries. Figure 5(b) shows the number of batteries required to achieve a particular outage as a function of the PV panel value for San Diego. We can see that as the PV dimension increases, the number of batteries required to achieve a tolerable outage becomes lower. The results for Las Vegas and Jaipur are similar.

D. PV Battery Configuration for a Given Outage Constraint

In this section we evaluate the minimum number of batteries required to meet a tolerable outage constraint for different values of PV wattage. Figure 5(c) shows the number of batteries required to achieve two different tolerable outage values ($\beta = 0.5$ and 1) for San Diego. The results for Las Vegas and Jaipur have been tabulated in Table I. With these configurations available, for a given desired tolerable outage constraint, a telecom operator can compute the cost associated with each of the configurations and the one with the lowest cost can be chosen as the cost optimal configuration.

V. Conclusion

In this paper we proposed a model for estimating the outage probability of a solar powered base station. In the proposed methodology we model the solar energy, BS load and the battery state as Markov processes which are then used to evaluate the outage probability for a given configuration of PV panel size and number of batteries. The results from the proposed model have been verified with results obtained from empirical data for three different locations.

TABLE I
PV-Battery Dimensioning

PV Size (kW)	Number of Batteries			
	$\beta = .5\%$		$\beta = 1\%$	
	Empirical	Model	Empirical	Model
Las Vegas				
10	21	22	18	19
12	19	20	16	17
14	18	18	15	16
16	17	17	14	15
18	16	16	13	14
20	15	15	12	13
Jaipur				
10	19	18	16	15
12	15	16	13	12
14	13	14	11	10
16	11	12	10	9
18	10	11	9	9
20	10	10	9	9

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