A Performance Guarantee for Maximal Schedulers in Sensor Networks with Cooperative Relays

Huijiang Li and Biplab Sikdar
Department of ECSE, Rensselaer Polytechnic Institute, Troy, NY 12180

***Abstract*— This paper addresses the question of throughput guarantees through distributed scheduling in wireless sensor networks (WSNs) with relay based cooperative communications. We prove that in a single frequency network with bidirectional, equal power communication, low complexity distributed maximal scheduling attains a guaranteed fraction of the maximum throughput region in arbitrary wireless networks. We also show that the guarantees are tight in the sense that they cannot be improved any further with maximal scheduling. Simulation results are also provided to show the performance of a distributed, maximal scheduling algorithm under different network settings.**

I. Introduction

The use of relays or cooperative communications for WSNs has received considerable attention in the recent past, particularly due to its ability to increase a WSN's range and capacity [1]. Existing research has shown that cooperative diversity gains can be achieved in distributed WSNs where nodes help each other by relaying transmissions [2]. This paper focuses on the performance of the *scheduling algorithm* used to control the channel access at the medium access control (MAC) layer in WSNs with cooperative relays. We focus on the throughput guarantees that may be provided by distributed schedulers for WSNs with cooperative relays and prove that maximal scheduler can achieve a guaranteed fraction of the maximum throughput region in arbitrary wireless networks.

The communication theory aspect of cooperative relaying, such as energy efficiency, bit error rate, forwarding strategies (e.g. decode and forward, amplify and forward) have been widely investigated [2], [3]. However, the performance of upper layer protocols, such as MAC layer schedulers, that use cooperative relay based communication technologies has not been investigated in detail. For wireless networks without cooperative relays, [4] presents the maximum achievable throughput region and an algorithm for attaining it, although the centralized nature and computational complexity of the scheduler limits its applicability. Instead, we focus on *maximal scheduling*, which is equivalent to solving the *Maximal Independent Set* (MIS) problem in graph theory. While it is known that a simple randomized distributed MIS algorithm for an arbitrary graph of size n including exchange of messages can be done in time $O(log^2 n)$ [5], [6], their performance in terms of the achievable throughput in cooperative relay based WSNs is unknown.

It has been shown in [7] that for wireless networks with direct transmissions (i.e. no cooperative communications), maximal scheduling is guaranteed to achieve a fraction of the maximum throughput region and the fraction (of value $1/8$) is decided by the maximum "conflict degree" of the network. The conflict degree of a transmitter-receiver pair (u, v) is defined as the number of transmitter-receiver pairs that interfere with (u, v) but not with each other. Using this notion of conflict degree, we prove that in a network with cooperative relays, any distributed maximal scheduling algorithm can achieve at least $1/10$ of the maximum throughput region and this guarantee cannot be improved any further.

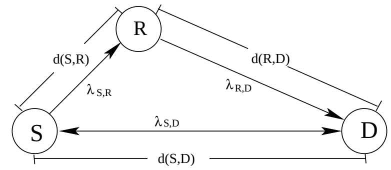

Fig. 1. Three-terminal relay model.

The rest of the paper is organized as follows. Section II describes the network and conflict models and Section III presents a performance guarantee on maximal scheduling. Simulation results showing the achieved throughput using maximal scheduling under different network scenarios are presented in Section IV. Finally Section V concludes the paper.

II. Network Model

In this paper, we consider a network where sensors share a single frequency and have the same transmission range. We consider a WSN with simple cooperative communication and a discrete memoryless three-terminal relay model as shown in Fig. 1. Every session I with a packet transmission involves three nodes: the source S, the destination D and the relay R. Thus a session may be represented as a 4-tuple (I, S, D, R). The distance and channel gain between two nodes i and j are represented by $d(i, j)$ and $\lambda_{i,j}$ respectively.

A discrete slotted-time model is assumed where each time slot is long enough so that a source node and a relay node can cooperatively transmit a single packet to the destination. The message exchanges among the source, relay and destination are considered to be bi-directional; the source broadcasts the *data* to the destination and the relay; the relay retransmits the *data* to the destination, and the destination replies with an *ACK* if it successfully receives the packet. The decode-and-forward relay strategy is assumed in this paper [3].

We model a wireless network as a graph $G = (V, E)$, where V is the set of nodes (i.e. sensors in the network) and E is the set of links. If sensor A is in the transmission range of sensor B, then B is A's *neighbor*. By assuming bidirectional symmetric communication, A is B's neighbor too. If A and

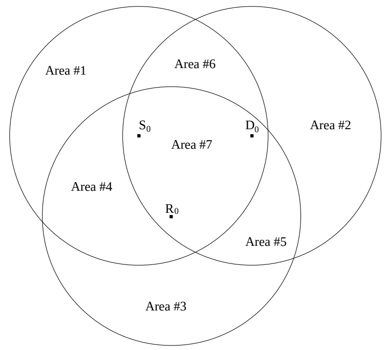

Fig. 2. A session (I_0, S_0, D_0, R_0) in the Euclidean plane.

B are neighbors, there is a link $(A, B) \in E$. We denote the neighborhood of node A as N_A, defined as the set of nodes that are in A's transmission range. In addition, it is assumed that each node has a single transceiver (transmitter/receiver), thus each node can only participate in one session at a time. Then a session (I_i, S_i, D_i, R_i) is successful when none of the nodes in this session is participating in other sessions *and* if none of the neighbors of S_i, D_i, and R_i transmit in this slot. The *conflict set* of session I_i is then,

$$C(I_i) = \{I_j\text{: } I_j \text{ shares a common node with } I_i, \\ (S_j, \text{or } R_j \text{ or } D_j) \in (N_{S_i} \cup N_{D_i} \cup N_{R_i})\} \quad (1)$$

If $I_j \in C(I_i)$ and $I_i \in C(I_j)$, sessions I_i and I_j are also defined as *neighbors*.

III. Maximal Scheduling Performance Guarantee

Let the number of sessions in conflict set $C(I_i)$ that can be scheduled at the same time (but not with session I_i) be defined as the *conflict degree* of conflict set $C(I_i)$. Denote the maximum conflict degree in the network as $K(\mathcal{N})$. It is proved in [7] that for the single frequency, bi-directional, equal-power, two-terminal communication network model (i.e. no relays), the performance of an arbitrary maximal scheduling algorithm is guaranteed to achieve $1/K(\mathcal{N})$ of maximum throughput region. In this paper we extend this result for networks with cooperative relays. Further, we show that for a three-terminal relay network, at least $1/10$ of the maximum throughput region is attained. We also show that this guarantee cannot be improved because there exist network topologies where at most $1/10$ of the maximum throughput region is achieved.

Lemma 1: For any wireless network $\mathcal{N}$ with relay usage, if the same frequency and equal power are used in nodes, bi-directional communication is involved, then $K(\mathcal{N}) \leq 10$.

Proof: Let the transmission range of each sensor in the WSN be d_{max}. In a two-dimensional Euclidean plane, the neighborhood area of a node A is equivalent to the closed circle centered at A with radius d_{max}, denoted as $\overline{b(A)}$. The *coverage area* of any session (I_0, S_0, D_0, R_0) is then the union of the areas $\overline{b(S_0)}$, $\overline{b(D_0)}$ and $\overline{b(R_i)}$, as shown in Fig. 2. If a session $I_j \in C(I_0)$, at least one terminal of session I_j falls in the coverage area of session (I_0, S_0, D_0, R_0). Thus to find the maximum $K(\mathcal{N})$, it is sufficient to find the maximum number of nodes that can be contained in $\overline{b(S_0)} \cup \overline{b(D_0)} \cup \overline{b(R_0)}$, such that the distance between any two of these nodes is greater than d_{max}.

Without loss of generality (wlog), we can assume that node S_i and D_i lie on the x axis. For the sake of convenience, we divide $\overline{b(S_0)} \cup \overline{b(D_0)} \cup \overline{b(R_0)}$ into 7 sub-areas, labeled $Area\#1$ through $Area\#7$ in Fig. 2. We use U_n to denote the number of nodes lying in $Area\#n$.

To start with, we formulate the geometric facts, statements and intermediate results proved in [7] as follows:

$$U_1 + U_2 + \sum_{n=4}^{7} U_n \leq 8, U_1 + \sum_{n=3}^{7} U_n \leq 8, \sum_{n=2}^{7} U_n \leq 8 \quad (2)$$

$$U_1 + U_4 + U_6 + U_7 \leq 5, U_2 + \sum_{n=5}^{7} U_n \leq 5, \sum_{n=3}^{5} U_n + U_7 \leq 5 \quad (3)$$

$$U_1 + U_4 \leq 4, U_2 + U_5 \leq 4 \quad (4)$$

$$U_2 + U_6 \leq 4, U_3 + U_4 \leq 4 \quad (5)$$

$$U_3 + U_5 \leq 4, U_1 + U_6 \leq 4 \quad (6)$$

where Equation (2) is proved in Lemma 3 of [7] and states that in the coverage area of any two-node session, there can be at most 8 nodes that are in conflict with the session but not with each other. Equation (3) follows the geometric argument and Lemma 18 of [7] and states that at most 5 nodes can be located in a circle ($\overline{b(S_0)}$, $\overline{b(D_0)}$, or $\overline{b(R_0)}$) such that the distance between any two nodes is greater than the radius. The two inequalities in Equation (4) are based on the fact that $\overline{b(S_0)}$ and $\overline{b(D_0)}$ intersect with each other with the distance between S_0 and D_0 satisfying $|S_0D_0| < d_{max}$ and the region $\overline{b(S_0)} \backslash \overline{b(D_0)}$ is covered by 4 $\pi/3$ sectors. The same arguments hold for $\overline{b(D_0)} \backslash \overline{b(S_0)}$. Also as shown in the proof of Lemma 3 in [7], the two equalities in Equation (4) can not be achieved at the same time. Similar arguments hold for Equations (5) and (6) as well. The rest of the proof proceeds in two steps.

Step 1: We prove by contradiction that the number of nodes not interfering with each other in $Area\#1 \cup Area\#2 \cup Area\#3$ is at most 8.

$$U_1 + U_2 + U_3 \leq 8 \quad (*)$$

Assumption:

$$U_1 + U_2 + U_3 = 9 \quad \text{(A1)}$$

Definition 1: Consider a session (I_i, S_i, D_i, R_i). For each session in $C(I_i)$ but not in conflict with each other, choose one of its terminals that falls in the coverage area of session I_i. Denote the set of chosen nodes by $\mathcal{U}$, and let $U = |\mathcal{U}|$. Given a node $A \in \mathcal{U}$, $A \in \overline{b(x)}$ and $A \notin \overline{b(y)}$ with $x, y = \{S_i, D_i, R_i\}$ and $x \neq y$, define the $distance$ from A to $\overline{b(y)}$ as $\min\{|AB| : B \in \mathcal{U}, B \in \overline{b(y)}, y \notin \overline{b(x)}\}$. The smaller the distance, the $closer$ node A is to Disk $\overline{b(y)}$.

Since $0 \leq U_n \leq 4$ for $n = 1, 2, 3$, at least two areas have 3 or more nodes. Because of symmetry, we can assume that $Area\#1$ and $\#2$ are the two areas with 3 or more nodes. Then we divide $\overline{b(S_0)}$ and $\overline{b(D_0)}$ into 6 $\pi/3$ sectors respectively as shown in Figure 3 with the dashed lines.

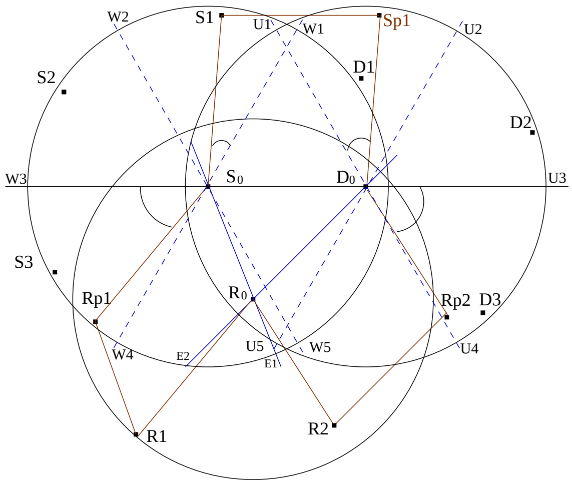

Fig. 3. Topology showing the source, relay and destination nodes (S_0, R_0, D_0)), the sectorization of $\overline{b(S_0)}$ and $\overline{b(D_0)}$, and the location of other nodes.

case (i): $U_1 = U_2 = 3$. Thus $U_3 = 3$ by assumption A1. Let S_1, S_2, S_3 be the nodes on $Area\#1$, with S_1 being the *closest* to $\overline{b(D_0)}$ and S_3 being the *closest* to $\overline{b(R_0)}$. Similarly, let D_1, D_2, D_3 be the nodes on $Area\#2$, with D_1 being the *closest* to $\overline{b(S_0)}$ and D_3 being the *closest* to $\overline{b(R_0)}$. Finally, let R_1, R_2, R_3 be the nodes on $Area\#3$, with R_1 being the *closest* to $\overline{b(S_0)}$ and R_2 being the *closest* to $\overline{b(D_0)}$. Note that the distance between any two nodes is greater than d_{max} and the angle subtended at S_0 or D_0 by any two nodes in adjacent sectors is greater than $\pi/3$. Then S_3 can only be on either sector $W_3S_0W_4$ or sector $W_4S_0W_5$ and D_3 can only be on sector $U_3D_0U_4$ or sector $U_4D_0U_5$.

To obtain the maximum value of $U_1 + U_2 + U_3$, we can assume that S_1 is on sector $W_1S_0W_2$ (Argument: Since $U_1 = 3$ and $Area\#1$ is covered by 4 $\pi/3$ sectors, whatever be the spread of the three nodes S_1, S_2, S_3, at least one node falls on either sector $W_1S_0W_2$ or sector $W_4S_0W_5$. Due to symmetry, we can assume that there is one node on sector $W_1S_0W_2$.).

Now, choose the point S_{p1} to make $S_0S_1S_{p1}D_0$ a parallelogram, choose R_{p1} to make $S_0R_0R_1R_{p1}$ a parallelogram, and choose R_{p2} to make $D_0R_0R_2R_{p2}$ a parallelogram. First we claim that,

$$\angle S_{p1}D_0D_2 > \pi/3, \angle R_{p1}S_0S_2 > \pi/3, \angle R_{p2}D_0D_2 > \pi/3 \quad (7)$$

$$\angle S_1S_0W_1 + U_1D_0S_{p1} = \pi/3 \quad (8)$$

To see these, *wlog*, we can assume that D_1 has a smaller y-coordinate than S_1. If node D_1 lies outside the parallelogram or on the line $S_{p1}D_0$, $\angle S_{p1}D_0D_2 > \angle D_1D_0D_2 > \pi/3$. If not, node D_1 must lie inside the parallelogram $S_0S_1S_{p1}D_0$, and it is proved in [7] that $\angle S_{p1}D_0D_2 > \pi/3$. Therefore, $\angle S_{p1}D_0D_2 > \pi/3$.

In the same way, we have $\angle R_{p1}S_0S_2 > \pi/3$ and $\angle R_{p2}D_0D_2 > \pi/3$. Then, since $\angle S_1S_0W_1 = \angle S_{p1}D_0U_2$ and $\angle U_1D_0S_{p1} + \angle S_{p1}D_0U_2 = \pi/3$, we have Equation (8).

Next, we claim that,

$$\angle R_1R_0R_2 < 2\pi/3 \quad (9)$$

To see this, extend the line S_0R_0 to point E_1 and line $D_0R_{)}$ to point E_2. Then,

$$\begin{aligned}\angle R_1R_0R_2 &= \angle R_1R_0E_1 + \angle E_2R_0R_2 - \angle E_2R_0E_1\\ &= \angle R_{p1}S_0E_1 + \angle E_2D_0R_{p2} - \angle S_0R_0D_0\\ &= \angle R_{p1}S_0R_0 + \angle R_0S_0D_0 + \angle S_0D_0R_0\\ &\quad + \angle R_0D_0R_{p2} - \pi\\ &= \pi - \angle W_3S_0R_{p1} - \angle R_{p2}D_0U_3 \quad (10)\end{aligned}$$

To show $\angle R_1R_0R_2 < 2\pi/3$, it is enough to show that

$$\angle W_3S_0R_{p1} + \angle R_{p2}D_0U_3 > \pi/3 \quad (11)$$

Since $\angle R_{p1}S_0S_2 > \pi/3$ (from Eqn.(7)), point R_{p1} can only lie in sector $W_3S_0W_4$ or sector $W_4S_0W_5$. If it is in sector $W_4S_0W_5$, $\angle W_3S_0R_{p1} > \pi/3$, and we have Eqn. (11). Similarly, if R_{p2} is in sector $U_4D_0U_5$, we have Eqn. (11).

On the other hand, if R_{p1} lies in sector $W_3S_0W_4$ and R_{p2} lies in sector $U_3D_0U_4$, since $\angle S_1S_0S_2 > \pi/3$ and $\angle R_{p1}S_0S_2 > \pi/3$, we have $\angle R_{p1}S_0W_4 + \angle S_1S_0W_1 < \pi/3$. Thus,

$$\angle W_3S_0R_{p1} = \pi/3 - \angle R_{p1}S_0W_4 > \angle S_1S_0W_1. \quad (12)$$

Additionally, since $\angle S_1S_0S_2 > \pi/3$, we have $\angle W_3S_2 + \angle S_1S_0W_1 \leq \pi/3$. Since $\angle S_3S_0S_2 = S_3S_0W_3 + W_3S_0S_2 > \pi/3$, we have

$$\angle W_3S_0S_3 > \angle S_1S_0W_1.$$

Likewise, we have

$$\angle U_3D_0R_{p2} = \pi/3 - \angle R_{p2}D_0U_4 > \angle U_1D_0S_{p1}. \quad (13)$$

$$\angle U_3D_0D_3 > \angle U_1D_0S_{p1}. \quad (14)$$

Combining Equations (12), (13) and (8), we have

$$\angle W_3S_0R_{p1} + \angle U_3D_0R_{p2} > \angle S_1S_0W_1 + \angle U_1D_0S_{p1} = \pi/3. \quad (15)$$

However, by assumption we must have $U_3 = 3$. Thus there is a third node, say R_3, in $Area\#3$, which satisfies $\angle R_1R_0R_3 > \pi/3$ and $\angle R_3R_0R_2 > \pi/3$ at the same time. However, this is a contradiction with Equation (9). Thus, if $U_1 = U_2 = 3$, $U_3 \leq 2$ and $U_1 + U_2 + U_3 \leq 8$.

Case (ii): $U_1 = 3, U_2 = 4$. Let S_1, S_2 and S_3 be the nodes in $Area\#1$ with positions as defined for case (i). Since $U_2 = 4$ and $Area\#2$ is covered by four sectors, each sector contains exactly one node. Let D_1 be on sector $U_1D_0U_2$, D_2 be on sector $U_2D_0U_3$, D_3 be on sector $U_3D_0U_4$ and D_4 be on sector $U_4D_0U_5$. Similar to Equation (7) we then have,

$$\angle R_{p2}D_0D_3 > \pi/3 \quad (16)$$

From Equations (14) and (16),

$$\angle U_3D_0R_{p2} = \angle U_3D_0D_3 + \angle D_3D_0R_{p2} > \angle U_1D_0S_{p1} + \pi/3 \quad (17)$$

Substituting Eqns. (12), (17) and (8) into Eqn. (10), we have

$$\angle R_1R_0R_2 < \pi/3 \quad (18)$$

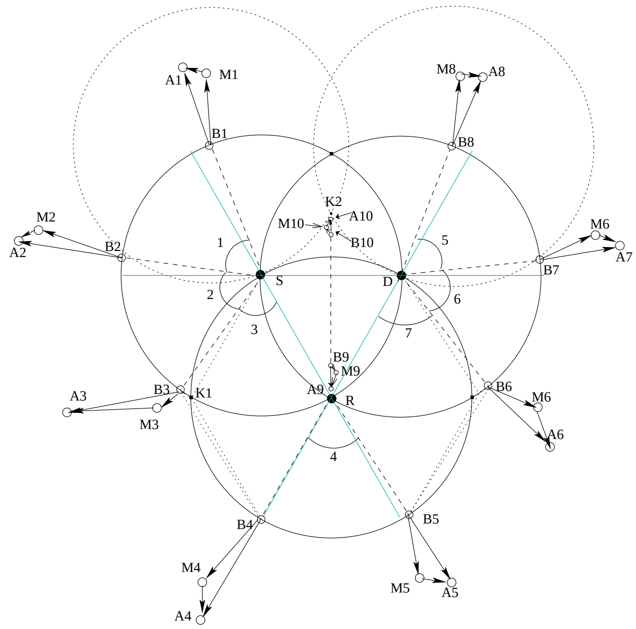

Fig. 4. Example of a network with $K(\mathcal{N}) = 10$.

However, based on our assumption, $|R_1R_2| > d_{max}$, which contradicts Equation (18). Thus, if $U_1 = 3$ and $U_2 = 4$, $U_3 \leq 1$ and $U_1 + U_2 + U_3 \leq 8$.

Since the equalities in Equation (4) cannot be achieved at the same time (i.e. $\overline{b(S_0)}\backslash\overline{b(D_0)}$ and $\overline{b(D_0)}\backslash\overline{b(S_0)}$ cannot each contain 4 nodes at the same time), there is no case where $U_1 = U_2 = 4$. Therefore, for any case, $U_1 + U_2 + U_3 \leq 8$ holds.

Step 2: Following Equation (*) and constrains in Equations (2) to (6), we now traverse the cases for U_1 to show that $K(\mathcal{N}) \leq 10$ always holds.

Case (i): $U_1 = 4$. Then $U_7 \leq 1$ by Equation (3), $U_4 = U_6 = 0$ by Equation (4) and $U_2 + U_3 \leq 4$ by Equation (*). Then we have the following scenarios:

1) If $U_2 \leq 1$, $K \leq 10$ since $U_3 + U_4 + U_5 + U_7 \leq 5$ from Equation (3).
2) If $3 \leq U_2 \leq 4$, $U_3 \leq 1$. Then $K \leq 9$ from Equation (2).
3) If $U_2 = 2$, $U_3 \leq 2$ and $U_5 \leq 1$ since the equalities in Equation (4) cannot be achieved at the same time. Thus $K \leq 4 + 2 + 2 + 1 + 1 = 10$.

Case (ii): $U_1 = 3$. Then $U_4 + U_6 + U_7 \leq 2$ by Equation (3), and $U2 + U_3 \leq 5$ by Equation (*). Thus $K \leq 3 + 2 + 5 = 10$.
Case (iii): $0 \leq U_1 \leq 2$. Since $\sum_{n=2}^{7} U_n \leq 8$ by Equation (2), $K \leq 2 + 8 = 10$.

In conclusion, $\max K(\mathcal{N}) \leq 10$ for all possible cases. ■

Next we show that the performance guarantee is tight by demonstrating the existence of a network with $K(\mathcal{N}) = 10$.

Lemma 2: There exists a wireless network with relay usage that uses a single frequency with equal power in all nodes and bi-directional communication such that $K(\mathcal{N}) = 10$.

Proof: We prove the result using construction. An example network with $K(\mathcal{N}) = 10$ is shown in Fig. 4. To construct the network, consider a session (I, S, D, R) with $|SD| = |SR| = |DR| = d_{max}$, d_{max} being the transmission range. Let B_i, A_i, M_i, $i = 1, \cdots, 10$ be the transmitter, receiver, and relay respectively of session I_i.

The nodes B_i, $i = 1, \cdots, 10)$ are located respectively at the edge of $\overline{b(S_i)} \cup \overline{b(D_i)} \cup \overline{b(R_i)}$ as shown. Thus, $I_i \in C(I)$. Specifically, $\angle B_1SD = 117°$ and $\angle B_1SB_2 = \angle B_2SB_3 = \angle B_3SR = 61°$. Thus, $|B_1B_2| = |B_2B_3| > d_{max}$. Also, we have $\angle B_4RS = 120°$ and $\angle B_4RB_5 = 61°$. Thus, $|B_4B_5| > d_{max}$. Finally, we have $\angle B_8DS = 116°$, $\angle B_8DB_7 = \angle B_7DB_6 = 61°$ and $\angle B_6DR = 62°$. Thus, $|B_8B_1| > d_{max}$, $|B_8B_7| = |B_7B_6| > d_{max}$.

We now show that $|B_3B_4| > d_{max}$ and $|B_5B_6| > d_{max}$. Denote one of the intersection points of $\overline{b(S)}$ and $\overline{b(R)}$ as K_1 (the other is D from the initial condition). Then SRB_4K_1 is a parallelogram and $|B_4K_1| = d_{max}$. Thus in triangle $K_1B_3B_4$, $\angle B_4K_1B_3 > \pi/2$ and $|B_4B_3| > |B_4K_1| = d_{max}$. Similarly, $|B_5B_6| > d_{max}$.

Next, suppose $\overline{b(B_1)}$ and $\overline{b(B_8)}$ intersect with each other at point K_2 as shown. Choose A_{10}, B_{10} and M_{10} such that none of them is in $\overline{b(B_1)}$ or $\overline{b(B_8)}$. More specifically, let A_{10} and B_{10} lie on the line K_2R with $|A_{10}K_2| = \epsilon_1$ and $|A_{10}B_{10}| = \epsilon_2$. Then we have $|B_1A_{10}| > d_{max}$, $|B_8A_{10}| > d_{max}$, $|B_1B_{10}| > d_{max}$, $|B_8B_{10}| > d_{max}$, $|B_1C_{10}| > d_{max}$ and $|B_8C_{10}| > d_{max}$.

In the same way, construct A_9, B_9 and M_9. Let $|B_9R| = \epsilon_3$ and $|A_9R| = \epsilon_4$. Choose ϵ_1, ϵ_2, ϵ_3 and ϵ_4 small enough such that $|B_{10}B_9| > d_{max}$.

Thus, I_i, $i = 1, \cdots, 10$ do not conflict with each other and can be scheduled at the same time, but all are in the conflict set of session I, so $K(\mathcal{N}) = 10$. ■

IV. Simulation and Results

In this section we present simulation results to evaluate the throughput achieved by maximal scheduling. The simulations were done using a simulator written in C. All simulations were run for a duration of 10000 time units and each result shown is the average of 10 simulation runs. For our simulations, we used a simple, distributed collision-free maximal scheduler based on the well known solution for maximal independent sets [6], as shown in Algorithm 1. For a graph of size n, the algorithm has a time complexity $O(log^2 n)$ [6].

Algorithm 1 Maximal Scheduling Algorithm

loop
  {comment: at each time slot, one single phase}
  Each undecided session I_i chooses a random number $r(I_i) \in (0, 1)$ and sends it to all its neighbors.
  If $r(I_i) < r(I_j)$ for all sessions in $C(I_i)$, session I_i is picked to be scheduled and informs all its neighbors.
  If one of I_i's neighbor is scheduled, I_i decides not to transmit.
  If all sessions reach their decisions, the scheduling is done. Otherwise, enter the next phase.
end loop

A. Bit Error Rate Evaluation

To evaluate the throughput, for each packet we first check if the transmission was successful or not by using the bit error rate (BER) associated with the transmission. For every session, we assume that the channels between the sensors are mutually independent Rayleigh fading channels with average channel powers $\lambda_{S,D}$, $\lambda_{S,R}$ and $\lambda_{R,D}$. Assuming that the signal at the destination is combined by using maximal ratio combining, we use the closed form expressions for the BER of Decode-and-Forward relaying for phase-shift keying (PSK) or quadrature amplitude modulation (QAM) given in [3] to evaluate the probability that a packet is successfully delivered.

The Log-distance Path Loss model is used to formulate the path loss between a transmitter and a receiver [8]. Using a path loss exponent α, the path loss can be expressed as $P(receiver) = P(transmitter)/d^{\alpha}$, where d is the distance between the transmitter and the receiver. In our simulations, α was set to 3. Finally, we assume equal power allocation between the source and the relay. If the power budget for a transmission is PmW, then we assume that the source and the relay each consume $P/2mW$.

B. Results

We simulate a network where nodes are randomly distributed in a $2000m \times 2000m$ square region. We report the results for nodes that have at least two neighbors, i.e. with at least one potential relay. We assume that the packet length is 64 bytes, 16-QAM is used, noise level N_0 is $-90dBm$, and transmission range $d_{max} = 100m$. We also assume that each node always has a packet to send in each slot.

Figure 5 shows the per node throughput per slot for two networks with 200 and 300 randomly distributed nodes. If a source has multiple relays to choose from, it picks one randomly in each slot. The results are shown as a function of the total transmission power as it is varied in the range $-10dBm$ to $20dBm$. We observe that as the node density increases, the throughput per node decreases due to higher channel contention. We also note that as the transmission power increases, the per node throughput saturates and higher node densities lead to lower throughput.

Figure 6 compares the performance of two different strategies for picking the relay in a network with 400 nodes. In addition to the random relay selection policy, we consider another policy where the source always chooses the relay that has the the smallest calculated BER. While choosing the relay with the lowest BER increases the throughput, it also leads to a faster battery consumption in the selected relays, as compared to random relay selection.

Figure 7 shows the impact of the packet size on the per node throughput in a 200 node network. At relatively smaller transmission powers, the throughput per node decreases as the packet size increases due to the higher packet error rate. When the transmission power is high enough, the packet error rate is negligible for all packet sizes and the throughput is limited by the requirement to pick conflict-free sessions for transmission.

V. Conclusions

This paper considers the problem of the achievable maximum throughput region of maximal schedulers in WSNs with cooperative relays. We show that distributed maximal scheduling algorithms can achieve a guaranteed fraction of the maximum throughput region in arbitrary wireless networks. It was also shown that the guarantees are tight in the sense that they cannot be improved any further with maximal scheduling.

References

[1] E. C. van der Meulen, “Three-terminal communication channels,” *Adv. Appl. Prob.,* vol. 3, pp. 120-154, 1971.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inf. Theory,* vol. 50, no. 12, pp. 3062-3080, December 2004.

[3] I. Lee, D. Kim, “BER Analysis for Decode-and-Forward Relaying in Dissimilar Rayleigh Fading Channels,” *IEEE Communications letters,* vol. 11, no. 1, pp. 52-54, Jan., 2007.

[4] L. Tassiulas and A. Ephremides, “Stability properties of constrained queueing systems and scheduling policies for maximal throughput in multihop radio networks,” *IEEE Transactions on Automatic Control,* vol.37,no.12, pp. 19361948, December 1992.

[5] M. Luby, “A simple parallel algorithm for the maximal independent set problem,” *Proc. ACM STOC,* pp. 1-10, 1985.

[6] M. Yves, J.M. Robson, S. Nasser, and A. Zemmari, “An Optimal Bit Complexity Randomized Distributed MIS Algorithm,” *Structural Information and Communication Complexity,* vol. 5869, pp. 323-337, Springer Berlin/Heidelberg, 2010.

[7] P. Chaporkar, K. Kar, S. Sarkar and X. Luo, “Throughput and Fairness Guarantees Through maximal Scheduling in Wireless Networks,” *IEEE Trans. on Inf. Theory,* vol.54, no.2, pp. 572-594, Feb. 2008.

[8] T. Rappaport, *Wireless Communications: Principles & Practice,* Prentice Hall, 1996.

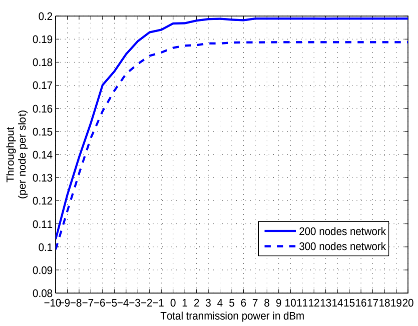

Fig. 5. Per node throughput in networks with 200 and 400 nodes.

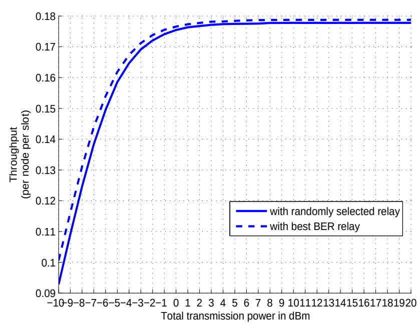

Fig. 6. Per node throughput in for two different relay selection policies in a network with 400 nodes.

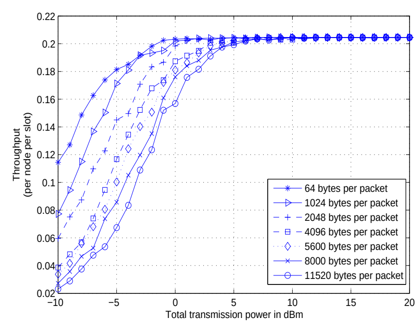

Fig. 7. Per node throughput for different packet lengths in a network with 200 nodes.