A Framework for Modeling the Lifetime and Residual Energy Distribution in Wireless Networks

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***Abstract*—A number of important characteristics of wireless sensor networks such as the lifetime, connectivity and coverage are determined the residual power levels of the nodes in the network. This paper presents a general framework for modeling the availability of power at sensor nodes as a function of time. Models are developed for sensors with and without battery recharging and expressions are derived for the network lifetime as well as the distribution and moments of random variables describing the number of sensors with different levels of residual energy as a function of time. Finally, the effect of the packet arrival rates and a sensor's geographical location are modeled.**

I. Introduction

Wireless sensor nodes, due to their low cost of operation coupled with the potential for remote deployment, have found a plethora of applications ranging from monitoring air, soil and water to seismic detection and military surveillance. A major constraint in the design and deployment of sensor networks is their limited battery capacity. The finite battery limits the lifetime of the network, and may also cause the network to become disconnected or lose coverage with time. To be able to provide guarantees on the performance of a sensor network and develop schemes to maximize the network lifetime, it is important to be able to characterize the available battery power at the sensors. In this paper, we present a general methodology for modeling the lifetime and the residual battery power of sensor nodes.

Existing research has primarily concentrated on developing algorithms, be it distributed or centralized, to optimize network longevity metrics. Works along the lines of actually building network models for energy consumption are addressed in [3], [8], [4] but these models fail to capture the interplay between a node's spatial location and it's energy consumption. A model for the network lifetime in a general form that is independent of the underlying network is proposed in [5]. The node density and the lifetime upper bound which ensures that a certain portion of network area is covered is studied in [6]. The effect of increasing the number of nodes on the network lifetime is examined in [7]. However, the existing literature fails to provide a unified framework for modeling the energy consumption and residual battery levels of sensor networks that simultaneously is capable of accounting for network and device related factors such as battery recharging, the traffic patterns, and the geographical location of the nodes. This paper tries to address these issues.

In this paper, we develop an unifying framework to characterize the lifetime and residual energy distribution of energy constrained networks. In particular, we use techniques similar to population models for biological systems to develop our framework. Our model allows the computation of the distribution of the network lifetime and its moments as well as the distribution of the available power at the nodes in the network and its moments. The proposed framework is general enough to accommodate scenarios with and without battery recharging. Our model also allows the inclusion of network related parameters in the energy calculations. We consider both *spatial* scenarios where a node's power consumption is governed by it's position in space as well as *non-spatial* scenarios where the node's location and power consumption model are independent entities. Extensive simulation results are presented to validate our results.

The rest of the paper is organized as follows. Our model for the scenarios where the sensors are incapable of recharging their batteries is presented in Section II while Section III extends to model for sensors with rechargeable batteries. Section V presents our simulation results and Section VI presents the concluding remarks.

II. Sensor Networks without Battery Recharging

In this section, we develop the formulation of the analytical framework to study the network lifetime and the distribution of residual power in sensor networks. At any time, we categorize each sensor in terms of its residual battery level.

To model the lifetime of energy constrained networks, we propose a generalization of Leslie's population matrix [1], which is used to study populations structured by age. The "age" of a node in our model corresponds to the amount of the battery power consumed, with one unit of power expended per packet transmitted, and the "age" of any node lies in one of the $m+1$ possible intervals; $0, 1, \cdots, m$. In other words, we assume that each sensor has enough energy to transmit m packets and the nodes in the network are structured based on this value. Our model makes the following assumptions

1) The power is mainly expended to transmit packets
2) The network lifetime is discretized into "cycles" and each cycle spans a communication round between nodes
3) The probability that a node receives i packets (its own and those it forwards), $i = 0, 1, \cdots, m$, to transmit is same in all cycles and we denote this probability by p_i.

Sleep-wake cycles used by many sensor networks to conserve energy can be incorporated in our model by choosing p_0 (the probability that no energy is consumed in a slot) appropriately. Further, the first assumption implies that the energy expended in sensing the environment is not incorporated into the model.

This energy is independent of the node's geographic location and impacts all nodes in the network uniformly, and hence is omitted. Additionally, the power consumption on communications dominates that for running the onboard circuitry [2]. Thus modeling the network lifetime based on the power spent on communications serves as a good approximation.

Let $n(t)$ be a $(m+1)$-dimensional vector whose i-th element, $n_i(t)$, denotes the number of nodes which have used up i units of the total battery capacity of m at time t. Note that the time t is discretized and is measured in units of cycles. Unlike biological population models where in each time step the age of each individual increases by 1, our model allows for arbitrary power consumption or increase in age in each time step. Recall that p_i, $0 \leq i \leq m$, denotes the probability that a node consumes i units of energy in a time unit (we derive expressions for p_i in Section IV). Then, the number of nodes at each energy level at an arbitrary time step is given by

$$\begin{aligned} n_0(t+1) &= p_0 n_0(t) \\ n_1(t+1) &= p_0 n_1(t) + p_1 n_0(t) \\ &\vdots \\ n_{m-1}(t+1) &= p_0 n_{m-1}(t) + p_1 n_{m-2}(t) + \cdots + p_{m-1} n_0(t) \\ n_m(t+1) &= n_m(t) + \sum_{i=1}^{m} p_i n_{m-1}(t) + \sum_{i=2}^{m} p_i n_{m-2}(t) \\ &\quad + \cdots + \sum_{i=m-1}^{m} p_i n_1(t) + p_m n_0(t) \end{aligned} \quad (1)$$

The rationale behind the above formulation can be justified as follows. A node with full power at time t (class n_0) will retain it's entire battery reserve only if it receives no packets to transmit for the duration of the cycle. The probability of this event is p_0, and since each node has the same probability distribution p_i, the expected of nodes who receive zero packets is $p_0 n_0(t)$, which in turn is the count of nodes with full battery power at time $t+1$. Similarly the number of nodes in class n_1 at time $t+1$ is the sum of nodes in class n_1 who transmit zero packets, and the nodes in class n_0 that spend one unit of energy at time t. For evaluating the number of nodes in class m, note that a sensor in class n_i, $i = 0, \cdots, m-1$ will expend all its energy if it transmits more than $m-i$ packets in a cycle and the probability of this event is given by $\sum_{k=(m-i)}^{m} p_i$, $i = 0, \cdots, m$. Also, since batteries are not capable of recharging, a sensor that had no battery power at time t will stay without power at time $t+1$ and hence the equation for $n_m(t+1)$.

The above formulation can also be expressed in a vector-matrix form. To this end, we first define the $(m+1) \times (m+1)$-dimensional "projection" matrix A as

$$A = \begin{bmatrix} p_0 & 0 & 0 & 0 & \dots & 0 & 0 \\ p_1 & p_0 & 0 & 0 & \dots & 0 & 0 \\ p_2 & p_1 & p_0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ p_m & \sum_{m-1}^{m} p_i & \sum_{m-2}^{m} p_i & \dots & \dots & \sum_{1}^{m} p_i & 1 \end{bmatrix} \quad (2)$$

The model then can be expressed as the vector difference equation

$$n(t+1) = An(t) \quad (3)$$

The recursive solution of this difference equation is given by

$$n(t+1) = A^{t+1} n(0) \quad (4)$$

where $n(0)$ is the initial distribution of nodes among the various energy levels. In practical situations, it is reasonable to assume that at time $t = 0$, all the nodes are fully powered, i.e. $n_i(0) = 0 \;\; \forall i > 0$ and $n_0(0) = N$. What now remains is determining the probabilities for the energy consumption during a cycle and this is done in Section IV.

A. Network Lifetime

When the batteries at sensor nodes do not have the capability to recharge, the network lifetime is an important quantity of interest. In this section we characterize the expected network lifetime using techniques that have been developed for calculating the extinction dynamics in biological populations.

We start by modeling the impact of the initial battery states on the network lifetime. From Eqn. (4), the dynamics of the energy model in the interval 0 to t can be represented as a product of t projection matrices A. Existing literature on population dynamics [9] has shown that asymptotically

$$n(t) \approx R(0,t)\langle v_0, n(0)\rangle u_0 \quad (5)$$

where $R(0,t)$ is a scalar representing the growth of the matrix product, v_0 and u_0 are the dominant left and right eigenvectors of the matrix product, normalized such that $\langle v_0, u_0\rangle = 1$ and the notation $\langle c, d\rangle$ is used to represent the scalar or dot product of vectors c and d. Consider the non-normalized dominant left eigenvector v of the matrix A. The impact of the initial battery states on the longevity of the network is then given by

$$V_0 = \langle v, n(0)\rangle \quad (6)$$

The rate at which the number of sensors without any remaining energy increases in the network is dependent on the dominant eigenvalue of the matrix A. In population studies, the size of the species under consideration varies with time. In contrast, the number of sensors in the network stays constant (in the absence of new nodes being added). Now state m in the model in Eqns. (2) and (3) corresponds to the state where a sensor has no remaining battery power. This is an absorbing state since the batteries do not have any recharging capability. Then we may consider the model

$$\hat{n}(t+1) = \hat{A}\hat{n}(t) \quad (7)$$

where $\hat{n}(t)$ is a m-dimensional vector corresponding to the number of sensors at time t in states 0 to $m-1$ of the original model in Eqns. (2) and (3) and $\hat{A}$ is a $m \times m$ matrix obtained from the matrix A by eliminating its $(m+1)$-th row and column. This modified model can now be used to evaluate the network lifetime by treating the model in Eqn. (7) as a population model and computing the extinction time of the "species" $\hat{n}$ modeled by the "population" projection matrix

$\hat{A}$. In [10] it has been shown that the infinitesimal long-run growth (or decay) rate of the population μ and its infinitesimal variance σ^2 are given by

$$\mu \approx \ln \lambda_0 - \frac{\sigma^2}{2} \tag{8}$$

$$\sigma^2 \approx \frac{1}{\lambda_0^2} \delta^T C \delta \tag{9}$$

where λ_0 is the dominant eigenvalue of the projection matrix $\hat{A}$ and δ is a column vector of the sensitivity coefficients $\frac{\partial \lambda_0}{\partial \hat{a}_{i,j}}$ with $\hat{a}_{i,j}$ being the (i,j)-th element of $\hat{A}$. The transpose of δ is denoted by δ^T and the sensitivity coefficients are given by $\frac{\partial \lambda_0}{\partial \hat{a}_{i,j}} = v_0^i u_0^j$ where v_0^i and u_0^j are the i-th and j-th elements of the normalized left and right eigenvectors of $\hat{A}$. The normalization is done such that $\sum_i u_0^i = 1$ and $\langle v_0, u_0 \rangle = 1$. Finally, C is the variance-covariance matrix of the elements in $\hat{A}$. Let x represent the natural logarithm of the total population $\sum_i \hat{n}_i$ representing the number of sensors in states 0 to $m-1$ and let $x_0 = \ln V_0$ be its adjusted initial value at time $t = 0$. Let $\varrho \triangleq \varrho(x, t|x_0)$ be the probability that the log population size is x at time t, given that its initial value was x_0. This probability ϱ quickly approaches the solution of the diffusion equation for the Weiner process ([11] p. 151)

$$\frac{\partial \varrho}{\partial t} = -\mu \frac{\partial \varrho}{\partial x} + \frac{\sigma^2}{2} \frac{\partial^2 \varrho}{\partial x^2} \tag{10}$$

with the initial condition $\varrho(x, 0|x_0) = \delta(x - x_0)$ where $\delta(x - x_0)$ is the Dirac delta function at x_0. Also, since the population becomes extinct (i.e. all sensors move to state m) when the population becomes less than one, we have the boundary condition

$$\varrho(0, t|x_0) = 0 \tag{11}$$

To obtain the solution for Eqn. (10) subject to the above initial and boundary conditions, we use the known solutions for Weiner processes with absorbing barriers [12]. This requires a linear transform of the coordinates and the solution to the system in Eqns. (10) and (11) is given by

$$\varrho(x, t|x_0) = \frac{1}{\sqrt{2\pi\sigma^2 t}} \left[e^{-\frac{(x - x_0 - \mu t)^2}{2\sigma^2 t}} - e^{-\frac{2\mu x_0}{\sigma^2} - \frac{(x + x_0 - \mu t)^2}{2\sigma^2 t}} \right] \tag{12}$$

Let $g(t|x_0)$ denote the probability that the population becomes extinct in an interval t and $t+dt$. Then $g(t|x_0)$ can be obtained by taking the derivative of the total probability of the event that the population is not extinct at time t:

$$g(t|x_0) = -\frac{d}{dt} \int_0^\infty \varrho(x, t|x_0) dx \tag{13}$$

$$= \frac{x_0}{\sqrt{2\pi\sigma^2 t^3}} e^{-\frac{(x_0 + \mu t)^2}{2\sigma^2 t}} \tag{14}$$

From Eqns. (12) and (14), the cumulative probability that the population is extinct before time t is then

$$G(t|x_0) = \int_0^t g(t'|x_0) dt' \tag{15}$$

$$= \Phi \left[-\frac{x_0 + \mu t}{\sigma \sqrt{t}} \right] + e^{-\frac{2\mu x_0}{\sigma^2}} \left[1 - \Phi \left[\frac{x_0 - \mu t}{\sigma \sqrt{t}} \right] \right] \tag{16}$$

where $\Phi[a]$ is the standard normal probability integral

$$\Phi[a] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^a e^{-\frac{z^2}{2}} dz \tag{17}$$

Note that when nodes cannot recharge their batteries, we have $\mu \le 0$ and thus $G(\infty|x_0) = 1$, i.e. the network eventually runs out of energy.

III. Sensors with Rechargeable Batteries

In this section we extend our model to accommodate sensors with rechargeable batteries. We consider an arbitrary recharge process governing the replenishing of the sensor batteries. We denote by α_i the probability that a sensor generates i units of energy in a cycle, with $i = 0, 1, \cdots m$. We assume that the recharge energy generated or harvested in a cycle becomes available for use at the end of the cycle. Also, the recharge process is assumed to be independent of the traffic at the node.

A sensor in state j at time t stays in the same state at time $t+1$ if the amount of energy it expends in time cycle t is the same as the amount of energy it generates. Since the traffic and energy generation processes are independent, this occurs with probability $\sum_{i=0}^m p_i \alpha_i$. Along the same lines, a sensor moves from state j to state i after a cycle, $j < i < m$, if the energy consumed in the cycle is $i - j$ units more than that generated in the cycle. The probability of this event is then $\sum_{k=0}^m p_{k+i-j} \alpha_k$. Similarly, the probability that a node in state j moves to state i after a cycle, $i < j < m$, is given by $\sum_{k=0}^m p_k \alpha_{k+j-i}$. For the boundary conditions where we consider the transition to states 0 and m, additional events need to be considered while calculating the transition probabilities. In particular, a sensor in state i at time t, $0 \le i < m$, moves to state m at time $t+1$ if at least $m - i$ more units of energy were consumed than generated in the time cycle. Similarly, a sensor in state i at time t, $0 < i \le m$, moves to state 0 at time $t+1$ if at least i more units of energy were generated than consumed in the cycle. Then, the number of nodes at each energy level at an arbitrary time step is given by

$$\begin{aligned}
n_0(t+1) &= n_0(t) \sum_{i=0}^m \alpha_i \sum_{j=0}^i p_j + n_1(t) \sum_{i=1}^m \alpha_i \sum_{j=0}^{i-1} p_j \\
&\quad + \cdots + n_{m-1}(t) \sum_{i=m-1}^m \alpha_i \sum_{j=0}^{i-m+1} p_j + n_m(t) \sum_{i=m}^m \alpha_i \sum_{j=0}^{i-m} p_j \\
n_1(t+1) &= n_0(t) \sum_{i=1}^m p_i \alpha_{i-1} + n_1(t) \sum_{i=0}^m p_i \alpha_i \\
&\quad + \cdots + n_{m-1}(t) \sum_{i=0}^m p_i \alpha_{i+m-2} + n_m(t) \sum_{i=0}^m p_i \alpha_{i+m-1} \\
&\vdots \\
n_{m-1}(t+1) &= n_0(t) \sum_{i=m-1}^m p_i \alpha_{i-m+1} + n_1(t) \sum_{i=m-2}^m p_i \alpha_{i-m+2} \\
&\quad + \cdots + n_{m-1}(t) \sum_{i=0}^m p_i \alpha_i + n_m(t) \sum_{i=0}^m p_i \alpha_{i+1}
\end{aligned}$$

$$n_m(t+1) = n_0(t)\sum_{i=m}^{m} p_i \sum_{j=0}^{i-m} \alpha_j + n_1(t)\sum_{i=m-1}^{m} p_i \sum_{j=0}^{i-m+1} \alpha_j + \cdots + n_{m-1}(t)\sum_{i=1}^{m} p_i \sum_{j=0}^{i-1} \alpha_j + n_m(t)\sum_{i=0}^{m} p_i \sum_{j=0}^{i} \alpha_j$$

The formulation above can be expressed in the form of a $(m+1) \times (m+1)$-dimensional projection matrix A:

$$A = \begin{bmatrix} \sum_{i=0}^{m}\alpha_i \sum_{j=0}^{i} p_j & \sum_{i=1}^{m}\alpha_i \sum_{j=0}^{i-1} p_j & \cdots & \sum_{i=m}^{m}\alpha_i \sum_{j=0}^{i-m} p_j \\ \sum_{i=1}^{m} p_i\alpha_{i-1} & \sum_{i=0}^{m} p_i\alpha_i & \cdots & \sum_{i=0}^{m} p_i\alpha_{i+m-1} \\ \vdots & \ddots & \ddots & \vdots \\ \sum_{i=m-1}^{m} p_i\alpha_{i-m+1} & \sum_{i=m-2}^{m} p_i\alpha_{i-m+2} & \cdots & \sum_{i=0}^{m} p_i\alpha_{i+1} \\ \sum_{i=m}^{m} p_i \sum_{j=0}^{i-m}\alpha_j & \sum_{i=m-1}^{m} p_i \sum_{j=0}^{i-m+1}\alpha_j & \cdots & \sum_{i=0}^{m} p_i \sum_{j=0}^{i}\alpha_j \end{bmatrix} \tag{18}$$

The model then can be expressed as the vector difference equation

$$n(t+1) = An(t) \tag{19}$$

whose recursive solution in terms of the initial distribution of nodes can again be written as $n(t+1) = A^{t+1}n(0)$.

A. Energy Distribution

In this section we characterize the distribution of the available energy at the sensors as a function of time. At each cycle, a sensor in any state i transits to any other state or stays in the same state according to the probabilities defined in the i-th column of A. In other words, the transition of a sensor in state i at the end of a cycle is determined according to a multinomial trial with $m+1$ possible outcomes with the probability of each outcome defined the entries in the i-th column of the matrix A. Then at time t, we have $n_i(t)$ multinomial trials corresponding to each sensor in class i that determines their transition at the start of time $t+1$. To characterize the vector $n(t+1)$, we start by evaluating the probability $Pr\{n(t+1) = \theta(t+1)|n(t)\}$ where $\theta(t+1)$ is a $(m+1)$-dimensional vector of non-negative integers. Since each sensor is assumed to operate independently, we have

$$Pr\{n(t+1) = \theta(t+1)|n(t)\} = \prod_{i=0}^{m} Pr\{n_i(t+1) = \theta_i(t+1)|n(t)\} \tag{20}$$

These conditional probabilities may be computed quite readily. However unconditioning the expression to obtain the unconditional distribution is quite laborious. Thus we use a multivariate probability generating function (PGF) to characterize the number of nodes at different power levels. We define

$$\rho_t(\nu_0, \nu_1, \cdots, \nu_m) = Pr\{n(t) = \{\nu_0, \nu_1, \cdots, \nu_m\}\} \tag{21}$$

and

$$H_t(z) = \sum_{\nu_0,\nu_1,\cdots,\nu_m} \rho_t(\nu_0, \nu_1, \cdots, \nu_m) z_0^{\nu_0} z_1^{\nu_1} \cdots z_m^{\nu_m} \tag{22}$$

Now consider the conditional PGF $H_{t+1|t}(z)$. Recall that at time t, the state transition of each sensor in class i occurs as per a multinomial trial. The PGF of the resulting vector from the multinomial trials on the $n_i(t)$ members of class i at time t is given by

$$(a_{0,i}z_0 + a_{1,i}z_1 + \cdots + a_{m,i}z_m)^{n_i(t)} = \left[\sum_{k=0}^{m} a_{k,i}z_k\right]^{n_i(t)} \tag{23}$$

Now, the number of sensors in class k at time $t+1$ is the sum of the number of sensors that move to class k from each of the m other classes at the end of time t as well as the sensors of class k that do not change their state. Since we are working with the transforms of the probability mass functions, the resulting PGF is the product of the individual PGFs. Thus we have

$$H_{t+1|t}(z) = \prod_{i=0}^{m} \left[\sum_{k=0}^{m} a_{k,i}z_k\right]^{n_i(t)} \tag{24}$$

Unconditioning on t, we have

$$\begin{aligned} H_{t+1}(z) &= \sum_{n_0(t),\cdots,n_m(t)} \rho_t(n_0(t), \cdots, n_m(t)) H_{t+1|t}(z) \\ &= \sum_{n(t)} \rho_t(n(t)) \prod_{i=0}^{m} \left[\sum_{k=0}^{m} a_{k,i}z_k\right]^{n_i(t)} \quad (25) \\ &= H_t(\xi_0, \xi_1, \cdots, \xi_m) \quad (26) \end{aligned}$$

where

$$\xi_i = \sum_{k=0}^{m} a_{k,i}z_k \tag{27}$$

Then given a starting state vector $n(0)$, we can recursively build the PGF of $n(t)$ and use it to obtain the exact distributions and its confidence intervals. As an illustration, we have

$$H_0(z) = z_0^{n_0(0)} z_1^{n_1(0)} \cdots z_m^{n_m(0)} = \prod_{i=0}^{m} z_i^{n_i(0)} \tag{28}$$

and

$$H_1(z) = H_0(\xi) = \prod_{i=0}^{m} \left[\sum_{k=0}^{m} a_{k,i}z_k\right]^{n_i(0)} \tag{29}$$

and so on.

IV. Impact of Network Parameters

In this section we highlight and investigate the interplay between a node's geographical co-ordinates in space and it's power consumption under the aegis of shortest path routing by considering two scenarios: (1) a spatial model and (2) a non-spatial model.

A. Spatial Network

To consider the impact of a node's spatial location on its energy consumption rates and node lifetime, we consider a deployment scenario where the sensor nodes are placed at the vertices of a finite grid. The co-ordinates of node i, $i = 1, \cdots, N$ in the grid (x_i, y_i) is determined as follows: $x_i = (i-1)/\sqrt{N}$ and $y_i = (i-1)\%\sqrt{N}$. The following probabilities are assumed known: p_s, the probability that in a given cycle a sensor node (say i) has a new packet to send to another node (say j) in the grid and p_c, the probability that any two given nodes in the grid communicate. The probability that a node i has a packet to transmit during a cycle is the probability of the union of two mutually exclusive events: the event of a node initiating a communication session and the event where it receives a routing request. The probability of the latter, p_{ri}, can be obtained by using the conditional probability of it receiving a packet given two nodes in the network communicate. Mathematically, for node i

$$p_{ri} = 2\left[\sum_{\substack{j=1\\ j\neq i}}^{N-1} \sum_{\substack{k=1\\ k\neq i, k>j}}^{N} \begin{array}{l} Pr\{\text{session for } j\text{-}k \text{ is through } i\} \\ \times Pr\{j \text{ and } k \text{ communicate}\}\end{array}\right] \tag{30}$$

Note that, for each pair (j,k), the expression for (k,j) communicating through node i has the same numerical value since the grid is symmetric and hence the summation in Eqn (30) is multiplied by a factor of two. Now, the probability that two particular nodes say j and k communicate is: $Pr\{j \text{ and } k \text{ communicate}\} = \frac{1-(1-p_s)^2}{\binom{N-1}{2}}$. In other words, the pair (j,k) can be selected from $(N-1)$ nodes (since node i is not a candidate) in $\binom{N-1}{2}$ ways and for nodes j and k to communicate, it is sufficient if either initiates a session. The expression for $Pr\{\text{session for } j \text{ and } k \text{ is through } i\}$ is derived as follows. Let $(x_i, y_i), (x_j, y_j), (x_k, y_k)$ denote the co-ordinates of nodes i, j and k respectively. Defining, $\Delta x_{i,j} \triangleq |x_i - x_j|$ and $\Delta y_{i,j} \triangleq |y_i - y_j|$, we obtain: $r_{i,j} = \Delta x_{i,j} + \Delta y_{i,j}$. Similar values for $r_{j,k}$ and $r_{k,i}$ can be obtained using the previous definition. Now,

$$Pr\{\text{session } j\text{-}k \text{ is through } i\} = \begin{cases} \frac{L_{j,k,i}}{L_{j,k}} & \text{if } r_{i,k} + r_{i,j} = r_{j,k} \\ 0 & \text{otherwise} \end{cases} \tag{31}$$

where

$$L_{j,k,i} = \binom{r_{i,j}}{\Delta x_{i,j}}\binom{r_{i,k}}{\Delta x_{i,k}} \quad \text{and} \quad L_{j,k} = \binom{r_{j,k}}{\Delta x_{j,k}}$$

Given the probability of a sensor to initiate a session, p_s, each cycle sees an average of Np_s sessions. To obtain the energy consumption probabilities, $p_i,\ i = 0, \cdots, m$, we again condition on the node's geographic location:

$$p_i = \sum_{k=1}^{N} Pr\{i \text{ packets transmitted} | \text{node id = k}\} \times \mathcal{P}_k \tag{32}$$

where $\mathcal{P}_k$ denotes $Pr\{\text{node id = k}\}$. Note that node k transmits $i, i > 0$ packets during a cycle if it either receives i routing packets and does not initiate a session or starts a communication session and receives $i-1$ routing requests. In our model we limit the number of communication sessions to Np_s, though theoretically the upper bound is N. The simulations validate our intuition that the expected number is a good approximation of the underlying communication process. The energy consumption probabilities can then be expressed as

$$p_i = \begin{cases} \left\{(1-p_s)(1-p_{rk})^{Np_s}\right\}\mathcal{P}_k & i = 0 \\ \left\{(1-p_s)\binom{Np_s}{i}p_{rk}^i(1-p_{rk})^{Np_s-i} + \right. & \\ \left. p_s\binom{Np_s-1}{i-1}p_{rk}^{(i-1)}(1-p_{rk})^{Np_s-i}\right\}\mathcal{P}_k & 0 < i \leq Np_s \\ 0 & \text{otherwise} \end{cases} \tag{33}$$

Also, the evaluation of $Pr\{\text{node id = k}\}$ has two possibilities: one where the choice of a node is equally likely among the N nodes present and the second, where the selection of the node is governed by it's location. Assuming shortest path routing, we approximate the likelihood of the node being chosen by the number of shortest paths it lies on. That is

$$Pr\{\text{node id = k}\} = \frac{\sum\limits_{\substack{i=1\\ i\neq k}}^{N-1} \sum\limits_{\substack{j=i+1\\ j\neq k}}^{N} \mathcal{I}\{r_{i,k} + r_{i,j} = r_{j,k}\}}{\sum\limits_{k=1}^{N} \sum\limits_{\substack{i=1\\ i\neq k}}^{N-1} \sum\limits_{\substack{j=i+1\\ j\neq k}}^{N} \mathcal{I}\{r_{i,k} + r_{i,j} = r_{j,k}\}} \tag{34}$$

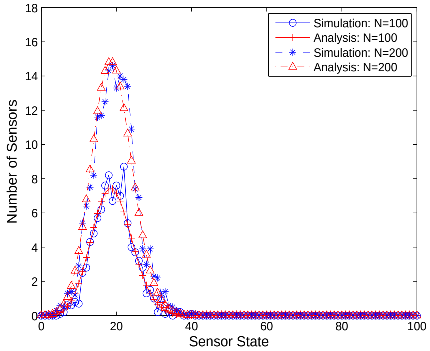
where $\mathcal{I}\{r_{i,k} + r_{i,j} = r_{j,k}\} = 1$ if $r_{i,k} + r_{i,j} = r_{j,k}$, 0 otherwise.

B. Non-spatial Homogeneous Networks

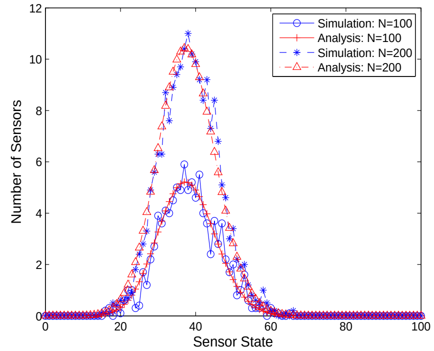
In the case of scenarios where the sensor network is homogeneous and is either assumed to span an extremely (ideally infinitely) large space or to be very densely deployed, the traffic conditions at each node can be approximated to be statistically identical. To qualitatively evaluate the node lifetimes in these scenarios, we consider a model where the number of packets transmitted by each node during a time cycle follows a Poisson distribution with mean λ, irrespective of its geographical location. The power consumption probabilities p_i in this case are given by: $p_i = \frac{e^{-\lambda}\lambda^i}{i!}$.

V. Results

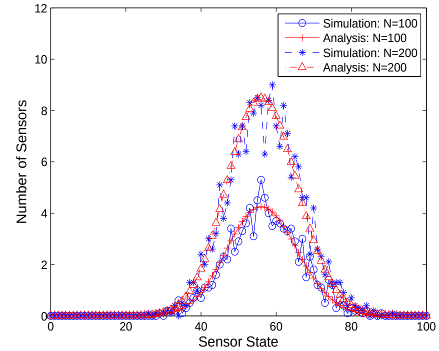
In this section, we evaluate the accuracy of the proposed framework by comparing the analytic results against simulations. The simulation results were generated using a custom built simulator written in C that, unlike existing simulation tools such as *ns-2*, allows the use of rechargeable batteries at nodes. Results are presented only for non-spatial or random networks. The results and those for spatial networks are similar. For each simulation result, ten runs of the simulation were conducted with different seeds and the average of these runs is presented in the figures.



(a) $t = 25$ cycles



(b) $t = 50$ cycles



(c) $t = 75$ cycles

Fig. 2. The residual power distribution at different times in a random network: Analytic versus simulation results.

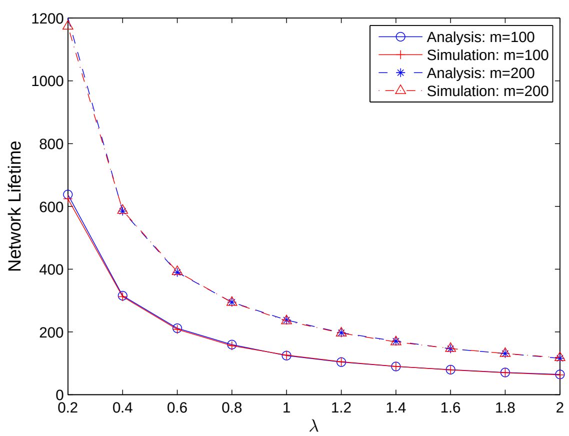


Fig. 1. Network Lifetime: Analysis versus simulation results.

We first consider the model in Section II where batteries do not have the capability to recharge. For this scenario, Fig. 1 shows the analytic and simulation values of the expected network lifetime (i.e. the time when all nodes run out of energy). The figure 1(b) shows the network lifetime for a non-spatial network where 100 nodes are distributed randomly in the network and the packet arrival process at each node is modeled according to a Poisson process. In this figure, the x-axis represents the parameter λ of the Poisson process. The analytic and simulation results match closely.

We next consider the model in Section III where each node has some capability to recharge its battery. For the results presented here we assume a simple model where a node generates a single unit of energy in a cycle with probability 0.25 and does not generate any energy with probability 0.75. The initial battery level of each sensor was kept at 100.

In Figure 2 we compare the analytic and simulation results for the number of sensors at different residual power levels after 25, 50 and 75 cycles of operation in a random network, with $\lambda = 1.0$. Results are presented for the case when there are 100 and 200 nodes in the network. We see that as time progresses, the number of nodes with lower residual energy increases. Also, there is a close match between the simulation and analytic results.

VI. Conclusion

In this paper we have motivated the need and importance of analyzing the network lifetime as a function of time and energy consumption. Using the work on population dynamics as the basis, we developed a general model for evaluating the residual battery power levels in networks with and without battery recharging. Expressions were derived for the network lifetime in the absence of battery recharging and the distribution and moments of the state occupancy of the sensors for the other cases. The impact of packet arrival rate at the sensor nodes and a sensor node's geographic location on the energy consumption was modeled.

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